Work, ENERGY AND POWER

5
Chapter

1. **The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:**

   a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

Ans: Positive.

Work done is the dot product of force and displacement. This means that the work will be positive if the projection of one of them on the other is in the same direction as the other. In other words, if the two vectors make an acute angle, then the work done is positive. Here, the bucket is being lifted upwards, which is in the same direction of the pull-force being applied. Hence the work done is positive.

b) work done by gravitational force in the above case,

Ans: Negative.

The gravitational force is acting downward, and the displacement is upwards, which means the angle these make $\sim 180^\circ$ (obtuse). This means that the dot product of the two vectors would be negative.

c) work done by friction on a body sliding down an inclined plane,

Ans: Negative.

The direction of frictional force is opposite to the direction of motion; hence the work done by the frictional force is negative in this case.

d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

Ans: Positive.

The frictional force is acting in the direction opposite to the motion of the body. However, to maintain the constant velocity, extra force is applied in the direction of the motion. So this means that the work done by this extra force is positive.

e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Ans: Negative.

The resistive force always acts in the direction opposite to the motion of the body (pendulum). Thus the work done by this force is also negative.

2. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7N on a table with coefficient of kinetic friction = 0.1. Compute the

a) work done by the applied force in 10s

Ans: Given the mass of the body, $m = 2kg$

The force applied on the body, $F = 7N$

The given coefficient of kinetic friction, $\mu = 0.1$

The initial velocity of the body, $u = 0$

Time, $t = 10s$

The acceleration produced in the body by the applied force is given by Newton's second law

of motion as:

$a' = F/m = 7/2 = 3.5\ ms^{-2}$

And the frictional force is given as: $f = \mu mg = 0.1 \times 2 \times 9.8 = -1.96N$

The acceleration produced by the frictional force:

$a'' = 1.96/2 = 0.98\ ms^{-2}$

Total acceleration of the body is the sum of the two accelerations: $a' + a'' = 3.5 + 0.98\ ms^{-2}$

Using the total acceleration, the distance traveled by the body is given by the equation of motion:

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.52 \times 10^2 = 126m$

Work done by the applied force (F),

$W_a = F.s = 7 \times 126 = 882J$.

Hence the work done by the applied force is 882J.

b) work done by friction in 10s,

Ans: The work done by the frictional force (f),

$W_f = f.s = -1.96 \times 126 = -247J$.

c) work done by the net force on the body in 10s,

Ans: The net force acting on the body $= 7 + (-1.96) = 5.04\text{N}$

Hence the work done by the net force in the 10s, is equal to the product of the force and the distance travelled by the body, $W_{Net} = 5.04 \times 126 = 635\text{J}$.

d) change in kinetic energy of the body in 10s, and interpret your results

Ans: Using the first equation of motion, final velocity can be calculated as:

$v = u + at = 0 + 2.52 \times 10 = 25.2 \text{ m/s}$

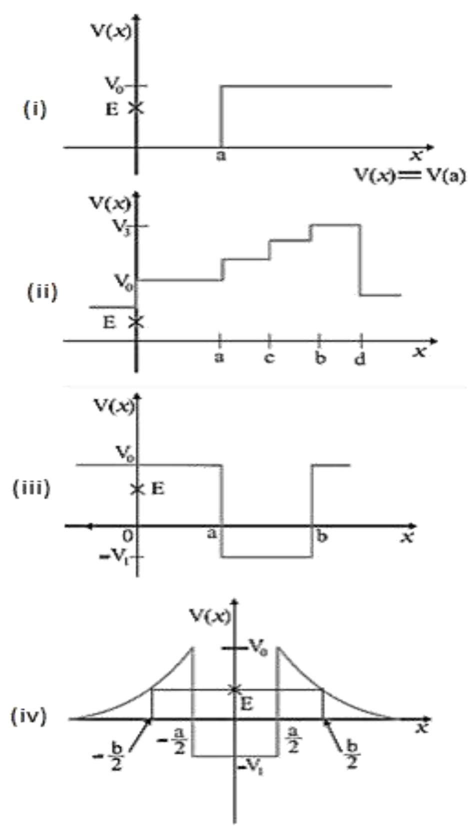
The change in kinetic energy is

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2) = 635\text{J}$

Hence the change in the kinetic energy is obtained as 635J.

3. Given in figure are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.

Ans.



The kinetic energy (T) of a body is given as the difference in the total energy of a system (E) and the potential energy (V):

$T = E - V$

Kinetic energy of a body is dependent on its mass and the square of its velocity. Neither of these quantities can be negative, hence Kinetic energy is also a positive quantity.

Therefore, the particle cannot exist in a region where T becomes negative or when $V > E$.

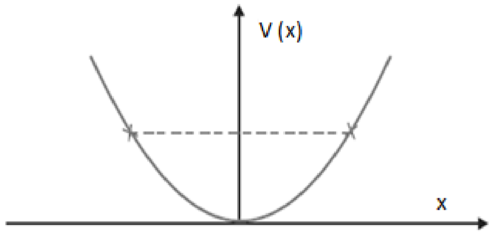
(i) For distance more than a, i.e., for $x > a$, the Kinetic energy becomes negative, hence the case is not possible.
The minimum energy required is equal to the maximum potential energy shown i.e., E should be at least equal to V_0

(ii) All throughout, the total energy E is smaller than the potential energy V. Hence the kinetic energy is negative, which isn't possible in a physical case. Hence a particle can't be found for the given energy. The minimum energy required is equal to the maximum potential energy shown i.e., the E should be at least equal to V_3

(iii) For distance $x < a$ and $x > b$, the kinetic energy becomes negative. Hence the scenario is not possible. However, for $b > x > a$, the kinetic energy is either positive or zero. This is possible.
But for the particle to stay anywhere in the system, the minimum energy required is equal to the maximum potential energy shown, i.e., E should be at least equal to V_0.

(iv) For $-b/2 < x < a/2;\ a/2 < x < b/2;$ the potential energy of the particle becomes greater than the total energy. Hence the particle can't stay here. However, it can stay everywhere else
In case the particle is needed in this region, then the minimum total energy in this case would be V_0.

4. The potential energy function for a particle executing linear simple harmonic motion is given by $V(x) = k x^2/2$, where k is the force constant of the oscillator. For $k = 0.5 N/m$, the graph of $V(x)$ versus x is shown in the given Fig. Show that a particle of total energy $1 J$ moving under this potential must 'turn back' when it reaches $x = \pm 2 m$.



Ans: As given, the total energy of the particle, $E1\ J =$

The potential energy of the particle $V(x) = k\,x^2/2$

The force constant of the oscillator is $k = 0.5 Nm^{-1}$

The total energy is: $E = V + T$ where T is the Kinetic energy of the particle. According to the conservation law, this energy does not change throughout the motion.

$\Rightarrow 1 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

When the pendulum 'turns back', velocity (and hence T) becomes zero.

$\Rightarrow 1 = \frac{1}{2}kx^2$

$\Rightarrow x = \pm 2$

Hence, the particle turns back when it reaches $x = \pm 2$.

5. Answer the following:

a) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?

Ans: Rocket.

Total energy is given by the relation:

$$TE = PE + KE = \frac{1}{2}mv^2 + mgh$$

Now, when the casing of a rocket burns due to friction, mass of the rocket gets reduced, which further causes the total energy to reduce. Clearly, it is at the cost of the rocket that the heat needed for burning is generated.

b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

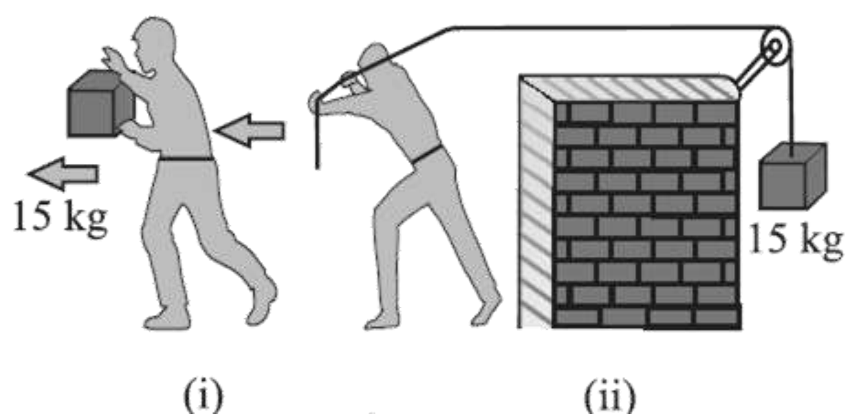
Ans: Gravitational force is a conserved force, which means that the work done by this force is independent of the path taken by the object. This means that if an object comes back to a place where it started from, i.e. when the total displacement is zero (i.e., a closed path), the work done is zero.

c) An artificial satellite orbiting the earth in a very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?

Ans: The atmospheric resistance costs the satellite its total energy. This means that the total energy keeps decreasing. So even when the kinetic energy increases due to the increase in the speed, the potential energy decreases much faster, leading to an overall drop in the energy.

So basically, the energy of the satellite is given as the sum of its kinetic energy, its potential energy, and the energy it loses to atmospheric resistance.

d) In (i), the man walks 2m carrying a mass of 15kg on his hands. In (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley and a mass of 15kg hangs at its other end. In which case is the work done greater?



Ans: In the case where the person is carrying the weight horizontally, which means he has applied the force (F) in the vertically upward direction, but the displacement (s) is horizontal. The work done (W) is given by $W = F.s = Fs\cos\theta$ and the θ is the angle between the two vectors. Since $\theta = 90°$ the work becomes zero.

Similarly, in the second case, the person is pulling the rope horizontally and the motion of the weight is upwards, and hence the two vectors are perpendicular to each other. Hence the work is zero.

6. Underline the correct alternative:

a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

Ans: decreases.

The body moves towards the lower potential hence decreasing the potential energy of the body.

b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

Ans: Kinetic energy.

Friction usually reduces the velocity of the body and hence reduces the Kinetic energy as well.

c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system.

Ans: External force.

Internal forces cannot produce any change in the total momentum of a body. It's a fact that the sum of all the internal forces is zero. Hence, the total momentum of a many- particle system is proportional to the external forces acting on the system.

d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies

Ans: Total linear momentum.

The momentum can neither be created nor destroyed. However the energies can change their forms.

7. State if each of the following statements is true or false. Give reasons for your answer.

a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

Ans: False

Law of conservation of momentum states that the total momentum of the system is conserved when there is no net external force applied on the system. The momentum of one body can be transferred to other body in the system.

b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

Ans: False

Any external force on a body can change the total energy of the body.

c) Work done in the motion of a body over a closed loop is zero for every force in nature.

Ans: False

Total energy over a closed-loop motion is only zero if the force involved is conservative. Eg. The friction would do zero work in a closed-loop.

d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Ans: True

In inelastic collisions, the total energy is usually lost as sound or heat or vibrations which means that the initial kinetic energy is more than the final kinetic energy

8. Answer carefully, with reasons:

a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?

Ans: No

At the point of contact, the moving billiard ball drops its velocity to zero and is still in the process of transferring its momentum and energy to the next ball. At this moment, the ball, which came to a sudden stop after hitting the next ball, has transferred its kinetic energy to the potential energy (of vibration), which is then transferred to the next ball as its kinetic energy.

b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

Ans: Yes

In an elastic collision, the total linear momentum is always conserved at every point in time.

c) What are the answers to (a) and (b) for an inelastic collision?

Ans: No, Yes.

There is a definite loss of kinetic in inelastic collisions, which happens during the collision. This is another reason why the total kinetic energy is not conserved.

The law of conservation of momentum is applicable to both elastic and inelastic collisions. Hence the total linear momentum is always conserved throughout the collision.

d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Ans: Elastic.

If the potential energy is only dependent on the separation distance, this means that the induced force is independent of the path that is taken by the balls. Hence the force is conservative. This means that the collisions here are also going to be elastic.

9. A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to (i)$t^{1/2}$ (ii)t (iii)$t^{3/2}$ (iv)t^2

Ans: It is given that the body is accelerated with a constant value- a hence the corresponding force experienced is $F = ma$.

Let's say that the initial velocity of the body is u.

Thus, the velocity- v of the body at any given point in time- t can be written as

$v - u = at$

And the power is

$P = F.v = (ma)v$

$\Rightarrow P = ma(at + u) = ma^2t + mau$

The mass, acceleration, and initial velocity are constant.

$\Rightarrow P \propto t$

Hence it can be noted that power is linearly dependent on t.

10. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to
(i)$t^{1/2}$ (ii)t (iii)$t^{3/2}$ (iv)t^2

Ans: In this case, the power is constant:
$P = F.v = (ma)v$

Since the mass of the body doesn't change, the acceleration and the velocity are inversely proportional.

$$\Rightarrow a = \frac{F}{mv}$$

Also,

$$\frac{dv}{dt} = a$$

Equating the two accelerations

$$\frac{dv}{dt} = a = \frac{F}{mv}$$

$$\Rightarrow \frac{m}{F}\int vdv = \int dt$$

$$\Rightarrow \frac{m}{F}v^2 = t + C$$

$$\Rightarrow v \propto t^{1/2}$$

Hence the option (i) is valid.

11. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by $F = (-\hat{i} + 2\hat{j} + 3\hat{k})N$
Where i, j, k are unit vectors along the x-, y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?

Ans: Force exerted on the body, $F = (-\hat{i} + 2\hat{j} + 3\hat{k})N$.

Displacement, $\hat{s} = 4km$.

Hence the work done, $W = F.s = (-\hat{i} + 2\hat{j} + 3\hat{k}).(4\hat{k}) = 12J$.

Therefore it is concluded that the work done is $12J$.

12. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy $10\ keV$, and the second with $100\ keV$. Which is faster, the

electron or the proton? Obtain the ratio of their speeds. (electron mass $m_e = 9.11 \times 10^{-31}$kg, proton mass $m_p = 1.67 \times 10^{-27}$kg, eV = 1.60×10^{-19} J) .

Ans: The given mass of the electron, $m_e = 9.11 \times 10^{-31}$kg and that of the proton is $m_p = 1.67 \times 10^{-27}$kg.

The given kinetic energy of the electron is $T_e = 10\text{keV} = 1.60 \times 10^{-15}$J, and the kinetic energy of the proton is $T_p = 100\text{keV} = 1.60 \times 10^{-14}$J.

The velocity of the electron is hence obtained as: $v_e = \left(\frac{2T_e}{m_e}\right)^{1/2} = 5.93 \times 10^7 \text{m/s}$

Similarly, the velocity of the proton can be obtained as

$$v_p = \left(\frac{2T_p}{m_p}\right)^{1/2} = 4.38 \times 10^6 \text{m/s}$$

The ratio of their velocities is $\frac{v_e}{v_p} = \frac{13.54}{1}$

Hence the electron moves 13.54 times faster.

13. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10m / s^{-1}?

Ans: The given radius of the raindrop, $r = 2 \times 10^{-3}$m

The volume of the drop is $V = (4/3)\pi r^3$

And hence the mass is $m = \rho V = \rho(4/3)\pi r^3$

Therefore the gravitational force is $F = mg = \rho(4/3)\pi r^3 g$

Thus the work by gravitation in the first half of the journey of the drop: $W_1 = F.s = 0.082\text{J}$ and here $s = 250\text{m}$

This is equal to the amount of work done in the second half of the journey, $W_2 = 0.082\text{J}$

If there is no resistance force, then the total energy of the raindrop will remain the same. Hence the total energy on the top is: $E_T = mgh + 0 = 0.164J$.

The resistive force makes the drop have a velocity of $10m/s$. Hence the total energy at the ground:

$E_G = (1/2)mv^2 = 1.675 \times 10^{-3} J$

Therefore the energy lost in the resistance is the difference between the energies: $= E_T - E_G = -0.162J$

Hence the energy due to the resistive force is -0.162J.

14. A molecule in a gas container hits a horizontal wall with speed $200m/s$ and angle 30^o with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Ans: The momentum of any system remains conserved as long as there aren't any external forces acting on the system. Similarly, the momentum of the system in the gas container remains conserved irrespective of the collision being elastic or inelastic.

As the gas molecules strike the stationary wall of the container, they rebound at the same speed. Thus the velocity of the wall remains zero. This means that the collision is elastic. Hence, the total kinetic energy of the molecule remains conserved during the collision.

15. A pump on the ground floor of a building can pump up water to fill a tank of volume $30m^3$ in $15 min$. If the tank is $40m$ above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Ans: Volume that can be filled in 15 minutes: $V = 30m^3$

And the time of operation, $t = 15 min = 900s$.

The tank is at $h = 40m$ height

And the given pump is of the efficiency $\eta = 30\% = 0.3$.

The average power required for lifting the water to this height is:

$$P = \frac{E}{t} = \frac{mgh}{t} = 13.067 \times 10^3 W$$

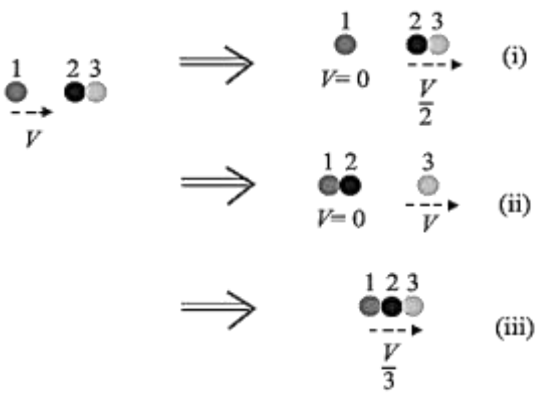
This is the required power output, so we need a pump whose power output needs to be equal to this. In other words, the 30% of the pump's output should be equal to the required power.

$P = 30\% P_{pump}$

$$\Rightarrow P_{pump} = \frac{100P}{30} = 43.6\times10^3 W = 43.6 kW$$

Hence we need a pump consumes $43.6kW$.

16. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V . If the collision is elastic, which of the following figure is a possible result after collision?



Ans: In all the cases of collisions where there is no external force, the linear momentum of the system is unchanged. In the case of elastic collision, the kinetic energy is also conserved.

Let's say that all the ball bearings have mass- m, so the total kinetic energy before collision:

$$T_i = \frac{1}{2}mV^2 + \frac{1}{2}m0^2 + \frac{1}{2}m0^2 = \frac{1}{2}mV^2$$

Now, one needs to check that the final kinetic energy is equal to the initial one.

Case i)

The final kinetic energy $T_f = \frac{1}{2}m0^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 = \frac{1}{4}mV^2$.

Case ii)

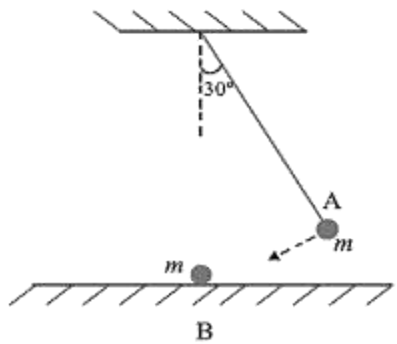
The final kinetic energy is $T_f = \frac{1}{2}m0^2 + \frac{1}{2}m0^2 + \frac{1}{2}mV^2 = \frac{1}{2}mV^2$.

Case iii)

The final kinetic energy is $T_f = \frac{1}{2}m\left(\frac{V}{3}\right)^2 + \frac{1}{2}m\left(\frac{V}{3}\right)^2 + \frac{1}{2}m\left(\frac{V}{3}\right)^2 = \frac{1}{6}mV^2$

Hence it is clearly seen that the kinetic energy is conserved only in case (ii).

17. The bob A of a pendulum released from 30^{o} to the vertical hits another bob B of the same mass at rest on a table as shown in figure. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



Ans: If the collision is elastic, then the bob-A will transfer all its momentum to the other bob. After the collision, bob A will come to rest, and bob B would move with the initial velocity of A in the horizontal direction. Thus the bob A will not rise after the collision.

18. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Ans: As provided, the length of the pendulum, $l = 1.5\text{m}$

The mass of the bob $= \text{m}$

Energy dissipated $=$ 5%

The law of conservation of momentum states the linear momentum of a system is unchanged in the absence of external force.

In the horizontal position

The PE of the bob is $E_P = mgl$, while the KE is $E_K = 0$ and hence the total energy is $E = mgl$

While at the lowermost point or the mean position, the potential energy is $E_P = 0$, the KE is $E_K = \frac{1}{2}mv^2$, and the total energy is $E = \frac{1}{2}mv^2$.

But the bob loses 5% of its energy.

Hence the total energy at the lowermost point is 95% of the total energy at the higher/horizontal position

$\Rightarrow \frac{1}{2}mv^2 = \frac{95}{100}mgl$

$\Rightarrow v = 5.28m/s$.

Hence the bob arrives with 5.28m/s at the lowest point.

19. A trolley of mass $300kg$ carrying a sandbag of $25kg$ is moving uniformly with a speed of $27km/h$ on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of. $0.05\ kg/s$. What is the speed of the trolley after the entire sand bag is empty?

Ans: As the trolley carrying the sand bag is moving without friction, which means, there is no friction which can act as an external force on the system. Hence the law of conservation of momentum is applicable.

The given initial mass of the bag $m_s(t=0) = 25kg$ and of the trolley is $m_{Tr} = 300kg$, hence the total initial mass is $M_{Tot}(t=0) = 325kg$. And the mass drop rate is $\frac{dM}{dt} = 0.05kg/s$.

The initial velocity of the trolley is $v(t=0) = 27km/h = 7.5m/s$

The momentum at any given time t is $p = (325 - 0.05t)v(t)$ and at $t = 0$, the momentum $p(t=0) = 325v(t=0) = 325 \times 27 = 8775kg\ m/s$

The time at which the sandbag empties, $t_{final} = \frac{25}{0.05} = 500s$

Thus the momentum at t_{final} can be written as

$8775 = (325 - 0.05 \times 500).v(t_{final} = 500)$

$\Rightarrow 8775 = 300.v(t_{final})$

$\Rightarrow v(t_{final}) = 29.25m/s$

Hence the final velocity when the bag is emptied is $29.25m/s$.

20. A body of mass $m = 0.5kg$ travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5m^{1/2}s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$?

Ans: The given mass of the body is $m = 0.5kg$ and is moving with velocity $v = ax^{3/2}$ where $a = 5m^{1/2}s^{-1}$.

The kinetic energy at $x = 0$ is $E_K(x=0) = \frac{1}{2}0.5 \times 0$ and at $x = 2$ is

$$E_K(x=2)=\frac{1}{2}0.5\times(5\times 2^{3/2})^2=50\text{J}.$$

The difference in these two values is equal to the work done.

$$W=E_K(x=2)-E_K(x=0)=50\text{J}$$

Hence the work done is 50J.

21. The blades of a windmill sweep out a circle of area $=A$

a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?

Ans: As provided, the circular area swept by the windmill $=A$

And the velocity of the wind is $=v$

And the density of the air is $=\rho$

The volume flow rate of wind through the windmill is $=Av$

And hence the mass flow rate is $m/t=\rho Av$

Hence the mass flowing in time t is $m=\rho Avt$.

b) What is the kinetic energy of the air?

Ans: Using this mass the kinetic energy is obtained as

$$=\frac{1}{2}mv^2=\frac{1}{2}\rho Av^2t=E_{wind}$$

This is also equal to the energy that the wind carries with itself.

c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A=30\text{m}^2$, $v=36\text{km/h}$ and the density of air is $\rho=1.2\text{kg/m}^3$. What is the electrical power produced?

Ans: The area of the circle swept is $A=30\text{m}^2$

And the given velocity is $v=36\text{km/h}=10\text{m/s}$

The density is $\rho=1.2\text{kg/m}^3$

Since the windmill converts only 25% of wind energy to electricity.

$$E_{electric}=25\%E_{wind}=\frac{1}{8}\rho Av^2t$$

Hence the electric power is obtained as $P_{electric}=E_{electric}/t=4.5\text{kW}$.

The electric power output from the windmill is 4.5kW.

22. A person trying to lose weight (dieter) lifts a 10kg mass, one thousand

times, to a height of 0.5m each time. Assume that the potential energy lost each time she lowers the mass is dissipated.

a) How much work does she do against the gravitational force?

Ans: Mass of the dumbbell $m = 10\text{kg}$

The height up to which it is lifted $h = 0.5\text{m}$

Hence the work done with every lift is mgh

Number of times the dumbbell is lifted $n = 1000$

Hence the total work done against gravity is $= n\,mgh = 49\text{kJ}$.

b) Fat supplies $3.8\times10^7\,\text{J}$ of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Ans: Energy supplied with every kilogram of fat is $3.8\times10^7\,\text{J}$

The efficiency is 20%

Hence the mechanical energy supplied by the body

$$= \frac{20}{100}3.8\times10^7\,\text{J} = 7.6\times10^6\,\text{J}$$

And the equivalent fat loss is $6.45\times10^{-3}\text{kg}$

Hence the body loses $6.45\times10^{-3}\text{kg}$ of fat.

23. A family uses 8kW of power.

a) Direct solar energy is incident on the horizontal surface at an average rate of 200W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8kW?

Ans: The given power requirement of a family is $P = 8\text{kW} = 8\times10^3\,\text{W}$

The solar energy per sq mts $P_{solar} / A = 200\text{W}$

Efficiency in converting solar to electrical energy $\eta = 20\%$

Let the area needed to generate the required electricity be A

Total solar power is $P_{solar} = (P_{solar} / A).A$

And the expected electrical power output can be written as

$$P = 20\%(P_{solar}) = \frac{20}{100}200\times A$$

$$\Rightarrow A = 200\text{m}^2$$

Hence the area required is $A = 200\text{m}^2$.

b) Compare this area to that of the roof of a typical house.

Ans: A typical house can have a roof of the dimensions $15\times15\text{m}^2$, hence the area is $\geq 200\text{m}^2$. And this is more than the required area.