## Units and Measurements



#### 1: Fill in the blanks.

a) The volume of a cube of side 1cm is equal to ..... m<sup>3</sup>. Ans: We know that,

 $1 \text{cm} = \frac{1}{100} \text{m}$ 

Volume of a cube of side 1cm would be,

 $V = 1 cm \times 1 cm \times 1 cm = 1 cm^3$ 

On converting it into unit of  $m^3$ , we get,

$$1 \text{cm}^{3} = \left(\frac{1}{100}\text{m}\right)^{3} = \left(10^{-2}\text{m}\right)^{3}$$
$$\therefore 1 \text{cm}^{3} = 10^{-6}\text{m}^{3}$$

Therefore, the volume of a cube of side 1cm is equal to  $10^{-6}$  m<sup>3</sup>.

### b) The surface area of a solid cylinder of radius 2.0cm and height 10.0cm is

equal to .....(mm)<sup>2</sup>

**Ans:** We know the formula for the total surface area of cylinder of radius r and height h to be,

 $S = 2\pi r (r + h)$ We are given: r = 2cm = 20mmh = 10cm = 100mmOn substituting the given values into the above expression, we get,  $S = 2\pi \times 20(20 + 100) = 15072mm^2 = 1.5 \times 10^4 mm^2$ Therefore, the surface area of a solid cylinder of radius 2.0cm and height 10.0cm is equal to  $1.5 \times 10^4 (mm)^2$ .

## c) A vehicle moving with a speed of 18kmh<sup>-1</sup> covers...... m in 1s.

**Ans:** We know the following conversion:

$$1 \text{km} / \text{h} = \frac{5}{18} \text{m} / \text{s}$$
$$\Rightarrow 18 \text{km} / \text{h} = 18 \times \frac{5}{18} = 5 \text{m} / \text{s}$$

Now we have the relation:  $Distance = speed \times time$ Substituting the given values, Distance =  $5 \times 1 = 5m$ Therefore, a vehicle moving with a speed of 18kmh<sup>-1</sup> covers 5m in 1s.

#### d) The relative density of lead is 11.3. Its density is ..... gcm<sup>-3</sup>

or..... kgm<sup>-3</sup>.

Ans: We know that the relative density of substance could be given by,

Relative density =  $\frac{\text{density of substance}}{\text{density of water}}$ 

density of water =  $1 \text{kg/m}^3$ 

density of lead = Relative density of lead  $\times$  density of water = 11.3 $\times$ 1= 11.3g/cm<sup>3</sup> But we know,

$$1g = 10^{-3} kg$$

$$1cm^{3} = 10^{-6} m^{3}$$

$$\Rightarrow 1g / cm^{3} = \frac{10^{-3}}{10^{-6}} kg / m^{3} = 10^{3} kg / m^{3}$$

$$\therefore 11.3g / cm^{3} = 11.3 \times 10^{3} kg / m^{3}$$

Therefore, the relative density of lead is 11.3. Its density is 11.3gcm<sup>-3</sup> or  $11.3 \times 10^{3} \text{kgm}^{-3}$ .

2. Fill ups.

a)  $1 \text{kgm}^2 \text{s}^{-2} = \dots \text{gcm}^2 \text{s}^{-2}$ 

**Ans:** We know that:

$$lkg = 10^{3}g$$
  

$$lm^{2} = 10^{4}cm^{2}$$
  

$$lkgm^{2}s^{-2} = 10^{3}g \times 10^{4}cm^{2} \times 1s^{-2} = 10^{7}gcm^{2}s^{-2}$$
  
Therefore,  $lkgm^{2}s^{-2} = 10^{7}gcm^{2}s^{-2}$ 

#### b)1m = .....ly



$$\Rightarrow 1 \text{ly} = (3 \times 10^8 \text{ m / s}) \times (365 \times 24 \times 60 \times 60 \text{s}) = 9.46 \times 10^{15} \text{ m}$$
$$\therefore 1 \text{m} = \frac{1}{9.46 \times 10^{15}} = 1.057 \times 10^{-16} \text{ly}$$
Therefore,  $1 \text{m} = 1.057 \times 10^{-16} \text{ly}$ 

c) 3.0m / s<sup>2</sup> = .....km / hr<sup>2</sup> Ans: 3.0m / s<sup>2</sup> = .....km / hr<sup>2</sup> We have, 1m = 10<sup>-3</sup> km 1hr = 3600s  $\Rightarrow 1s^{2} = \left(\frac{1}{3600}\right)^{2} hr^{2}$ 

Then,

$$3.0 \text{m}/\text{s}^{2} = \frac{3 \times 10^{-3}}{\left(\frac{1}{3600} \text{ h}\right)^{2}} \text{km}/\text{hr}^{2}$$
$$\therefore 3.0 \text{m}/\text{s}^{2} = 3.9 \times 10^{4} \text{ km}/\text{hr}^{2}$$

d)  $6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2 = \dots \text{g}^{-1} \text{cm}^3 \text{s}^{-2}$ 

Ans: We have,

$$1N = 1kgms^{-2}$$
  

$$1kg = 10^{-3}g$$
  

$$1m^{3} = 10^{6}cm^{3}$$
  

$$\Rightarrow 6.67 \times 10^{-11} Nm^{2}kg^{-2} = 6.67 \times 10^{-11} \times (1kgms^{-2})(1m^{2})(1s^{-2})$$
  

$$= 6.67 \times 10^{-11} \times (1kg \times 1m^{3} \times 1s^{-2})$$
  

$$= 6.67 \times 10^{-11} \times (10^{-3}g^{-1})(10^{6}cm^{3})(1s^{-2})$$
  

$$\therefore 6.67 \times 10^{-11} Nm^{2} / kg^{2} = 6.67 \times 10^{-8}cm^{3}s^{-2}g^{-1}$$

3. A calorie is a unit of heat or energy and is equivalent to 4.2 J where  $1J = 1 \text{kgm}^2 \text{s}^{-2}$ . Suppose we employ a system of units in which the unit of

mass equals  $\alpha$  kg, the unit of length equals  $\beta$  m, the unit of time is  $\gamma$  s. Show that a calorie has a magnitude  $4.2\alpha^{-1}\beta^{-2}\gamma^2$  in terms of the new unit.

Ans: We are given that,

 $1 \text{calorie} = 4.2(1 \text{kg})(1 \text{m}^2)(1 \text{s}^{-2})$ 

Let the new unit of mass  $= \alpha \text{ kg}$ .

So, one kilogram in terms of the new unit,  $1 \text{ kg} = \frac{1}{\alpha} = \alpha^{-1}$ .

One meter in terms of the new unit of length can be written as,  $1m = \frac{1}{\beta} = \beta^{-1}$  or

$$1m^2 = \beta^{-2}.$$

And, one second in terms of the new unit of time,

$$1 \ s = \frac{1}{\gamma} = \gamma^{-1}$$
  

$$1 \ s^{2} = \gamma^{-2}$$
  

$$1 \ s^{-2} = \gamma^{2}$$
  

$$\therefore 1 \text{ calorie} = 4.2 (1\alpha^{-1}) (1\beta^{-2}) (1\gamma^{2}) = 4.2\alpha^{-1}\beta^{-2}\gamma^{2}$$

Therefore, the value equivalent to one calorie in the mentioned new unit system is  $4.2\alpha^{-1}\beta^{-2}\gamma^{2}$ .

#### 4. Explain this statement clearly:

"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary:

Ans: The given statement is true because a dimensionless quantity may be large or small, but there should be some standard reference to compare that.For example, the coefficient of friction is dimensionless but we could say that the coefficient of sliding friction is greater than the coefficient of rolling friction, but less than static friction.

#### a) Atoms are very small objects.

Ans: An atom is a very small compared to a soccer ball.

#### b) A jet plane moves with great speed.

Ans: A jet plane moves with a speed greater than that of a bicycle.

#### c) The mass of Jupiter is very large.

Ans: Mass of Jupiter is very large compared to the mass of a cricket ball.

#### d) The air inside this room contains a large number of molecules.

**Ans:** The air inside this room contains a large number of molecules as compared to that contained by a geometry box.

#### e) A proton is much more massive than an electron.

**Ans:** A proton is more massive than an electron.

#### f) The speed of sound is much smaller than the speed of light.

- Ans: Speed of sound is less than the speed of light.
  - 5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

#### Ans: Distance between the Sun and the Earth:

x = Speed of light × Time taken by light to cover the distance

It is given that in the new system of units, the speed of light c = 1 unit.

Time taken,  $t = 8 \min 20 \text{ s} = 500 \text{ s}$ 

Thus, the distance between the Sun and the Earth in this system of units is given by  $x' = c \times t = 1 \times 500 = 500$  units

#### 6. Which of the following is the most precise device for measuring length?

**Ans:** A device which has the minimum least count is considered to be the most precise device to measure length.

#### a) A vernier calipers with 20 divisions on the sliding scale.

Ans: Least count of vernier calipers is given by

LC = 1 standard division(SD) - 1 vernier division(VD)

$$\Rightarrow$$
L.C=1- $\frac{19}{20} = \frac{1}{20} = 0.05$  cm

#### b) A screw gauge of pitch 1 mm and 100 divisions on the circular scale.

**Ans:** Least count of screw gauge  $=\frac{\text{Pitch}}{\text{No of divisions}}$ 

$$\Rightarrow L.C = \frac{1mm}{100} = \frac{0.1 \text{ cm}}{100}$$
$$\Rightarrow LC = \frac{1}{1000} = 0.001 \text{ cm}$$

## c) An optical instrument that can measure length to within a wavelength of light.

Ans: Least count of an optical device = Wavelength of light  $10^{-5}$  cm

 $\Rightarrow$  LC = 0.00001cm

Hence, it can be inferred that an optical instrument with the minimum least count among the given three options is the most suitable device to measure length.

7. A student measures the thickness of a human hair using a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. Estimate the thickness of hair.

**Ans:** We are given that:

Magnification of the microscope =100Average width of the hair in the field of view of the microscope =3.5 mm

: Actual thickness of the hair would be,  $\frac{3.5}{100} = 0.035$  mm.

#### 8. Answer the following:

## a) You are given a thread and a meter scale. How will you estimate the diameter of the thread?

**Ans:** Wrap the thread on a uniform smooth rod in such a way that the coils thus formed are very close to each other.

Measure the length that is wounded by the thread using a metre scale.

The diameter of the thread is given by the relation,

 $Diameter = \frac{Length of thread}{Number of turns}$ 

b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

**Ans:** Increasing the number divisions of the circular scale will increase its accuracy to a negligible extent only.

c) The mean diameter of a thin brass rod is to be measured by Vernier calipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

- **Ans:** A set of 100 measurements is more reliable than a set of 5 measurements because random errors involved will be reduced on increasing the number of measurements.
  - 9. The photograph of a house occupies an area of 1.75cm<sup>2</sup> on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55m<sup>2</sup>. What is the linear magnification of the projector-screen arrangement?

Ans: We are given,

$$A_0 = 1.75 \text{cm}^2$$
.

The area of the image of the house that is formed on the screen is given by,  $A_I = 1.55 \text{m}^2 = 1.55 \times 10^4 \text{cm}^2$ 

We know that areal magnification is given by,

$$m_a = \frac{A_I}{A_O}$$

Substituting the given values,

$$m_a = \frac{1.55 \times 10^4}{1.75}$$

Now, we have the expression for Linear magnification as,  $m_1 = \sqrt{m_a}$ 

$$\Rightarrow m_1 = \sqrt{\frac{1.55}{1.75} \times 10^4}$$
$$\therefore m_1 = 94.11$$

Thus, we found the linear magnification in the given case to be,  $m_1 = 94.11$ .

# 10. State the number of significant figures in the following:a) 0.007m<sup>2</sup>

**Ans:** We know that when the given number is less than one, all zeroes on the right of the decimal point are insignificant and hence for the given value, only 7 is the significant figure. So, the number of significant figures in this case is 1.

#### b) $2.64 \times 10^{24}$ kg

**Ans:** We know that the power of 10 is considered insignificant and hence, 2, 6 and 4 are the significant figures in the given case. So, the number of significant figures here is 3.

#### c) 0.2370gcm<sup>-3</sup>

**Ans:** For decimal number, the trailing zeroes are taken significant. 2, 3, 7 and 0 are the significant figures. So, the number of significant figures here is 4.

#### d) 6.320J

**Ans:** All figures present in the given case are significant. So, the number of significant figures here is 4.

#### e) 6.032Nm<sup>-2</sup>

**Ans:** Since all the zeroes between two non-zero digits are significant, the number of significant figures here is 4.

#### f) $0.0006032m^2$

**Ans:** For a decimal number less than 1, all the zeroes lying to the left of a non-zero number are insignificant. Hence, the number of significant digits here is 4.

# 11. The length, breadth and thickness of a rectangular sheet of metal are 4.234m, 1.005m and 2.01cm respectively. Give the area and volume of the sheet to correct significant figure.

#### Ans: We are given:

Length of sheet, 1 = 4.234m; number of significant figures: 4 Breadth of sheet, b = 1.005m; number of significant figures: 4 Thickness of sheet, h = 2.01cm = 0.0201m; number of significant figures: 3 So, we found that area and volume should have the least significant figure among the given dimensions, i.e., 3. Surface area,  $A = 2(1 \times b + b \times h + h \times 1)$ Substituting the given values,  $\Rightarrow A = 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) = 2(4.25517 + 0.02620 + 0.08510)$   $\therefore A = 8.72m^2$ Volume,  $V = 1 \times b \times h$ Substituting the given values,  $\Rightarrow V = 4.234 \times 1.005 \times 0.0201$   $\therefore V = 0.0855m^3$ Therefore, we found the area and volume with 3 significant figures to be  $A = 8.72m^2$ and  $V = 0.0855m^3$  respectively.

12. The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is:a) the total mass of the box?

Ans: We are given:

Mass of grocer's box = 2.300kg

Mass of gold piece I = 20.15g = 0.02015kg

Mass of gold piece II = 20.17g = 0.02017kg

Total mass of the box = 2.3 + 0.02015 + 0.02017 = 2.34032kg

In addition, the final result should retain as many decimal places as there are in the number with the least decimal places. Hence, the total mass of the box is 2.3kg.

#### b) the difference in the masses of the pieces to correct significant figures?

**Ans:** Difference in masses = 20.17 - 20.15 = 0.02g

While subtracting, the final result should retain as many decimal places as there are in the number with the least decimal places.

13. A famous relation in physics relates 'moving mass' m to the 'rest mass'  $m_0$  of a particle in terms of its speed v and speed of light c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = \frac{m_0}{\left(1 - v^2\right)^{\frac{1}{2}}}$$

Ans: We are given the following relation:

$$m = \frac{m_0}{\left(1 - v^2\right)^{\frac{1}{2}}}$$

Dimension of m,  $M^1L^0T^0$ 

Dimension of  $m_0$ ,  $M^1L^0T^0$ 

Dimension of v,  $M^0L^1T^{-1}$ 

Dimension of  $v^2$ ,  $M^0L^2T^{-2}$ 

Dimension of c,  $M^0L^1T^{-1}$ 

For the formula to be dimensionally correct, the dimensions on the LHS should be same as those on the RHS. In order to satisfy this condition,  $(1 - v^2)^{\frac{1}{2}}$  should be dimensionless and for that we require  $v^2$  be divided by  $c^2$ . So, the

dimensionally correct version of the above relation would be,

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}$$

#### 14. The unit of length convenient on the atomic scale is known as an

angstrom and is denoted by  $A: 1A = 10^{-10}$  m. The size of a hydrogen atom is about 0.5A. What is the total atomic volume in m<sup>3</sup> of a mole of hydrogen atoms?

Ans: Radius of hydrogen atom is given to be,

 $r = 0.5 \text{ Å} = 0.5 \times 10^{-10} \text{ m}$ The expression for the volume is,

$$V = \frac{4}{3}\pi r^3$$

Now on substituting the given values,

$$V = \frac{4}{3}\pi \left(0.5 \times 10^{-10}\right)^3 = 0.524 \times 10^{-30} \,\mathrm{m}^3$$

But we know that 1 mole of hydrogen would contain Avogadro number of hydrogen atoms, so volume of 1 mole of hydrogen atoms would be,

$$V' = N_A V = 6.023 \times 10^{23} \times 0.524 \times 10^{-30} = 3.16 \times 10^{-7} m^3$$

Therefore, we found the required volume to be  $3.16 \times 10^{-7} \text{ m}^3$ .

15. One mole of an ideal gas at standard temperature and pressure occupies 22.4L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule

to be about 1A). Why is this ratio so large?

Ans: Radius of hydrogen atom,  $r = 0.5 A = 0.5 \times 10^{-10} m$ 

Volume of hydrogen atom,  $V = \frac{4}{3}\pi r^3$ 

$$\Rightarrow V = \frac{4}{3} \times \frac{22}{7} \times \left(0.5 \times 10^{-10}\right)^3 = 0.524 \times 10^{-30} \,\mathrm{m}^3$$

Now, 1 mole of hydrogen contains  $6.023 \times 10^{23}$  hydrogen atoms. Volume of 1 mole of hydrogen atoms,

Volume of 1 mole of hydrogen atoms,  $V = (0.022 + 10^{23}) = 0.524 + 10^{-30} = 2.16 + 10^{-30}$ 

 $V_a = 6.023 \times 10^{23} \times 0.524 \times 10^{-30} = 3.16 \times 10^{-7} \text{ m}^3.$ 

Molar volume of 1 mole of hydrogen atoms at STP,  $V_m = 22.4L = 22.4 \times 10^{-3} m^3$ So, the required ratio would be,

$$\frac{V_{\rm m}}{V_{\rm a}} = \frac{22.4 \times 10^{-3}}{3.16 \times 10^{-7}} = 7.08 \times 10^4$$

Hence, we found that the molar volume is  $7.08 \times 10^4$  times higher than the atomic volume.

For this reason, the interatomic separation in hydrogen gas is much larger than the size of a hydrogen atom.

- 16. Explain this common observation clearly: If you look out of the window of a fast-moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).
- **Ans:** Line-of-sight is defined as an imaginary line joining an object and an observer's eye. When we observe nearby stationary objects such as trees, houses, etc., while sitting in a moving train, they appear to move rapidly in the opposite direction because the line-of-sight changes very rapidly. On the other hand, distant objects such as trees, stars, etc., appear stationary because of the large distance. As a result, the line-of-sight does not change its direction rapidly.
- 17. The sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding  $10^7$  K and its outer surface at a temperature of about 6000K. At these high temperatures no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the sun =  $2.0 \times 10^{30}$ kg,

radius of the sun =  $7.0 \times 10^8$  m.

**Ans:** We are given the following:

Mass of the sun,  $M = 2.0 \times 10^{30} \text{kg}$ 

Radius of the sun,  $R = 7.0 \times 10^8 m$ 

Now we find the volume of the sun to be,

$$V = \frac{4}{3}\pi R^{3} = \frac{4}{3}\pi (7.0 \times 10^{8})^{3} = 1437.3 \times 10^{24} m^{3}$$

Density of the sun is found to be,

$$\rho = \frac{M}{V} = \frac{2.0 \times 10^{30}}{1437.3 \times 10^{24}}$$
  
$$\therefore \rho \sim 1.4 \times 10^{3} \text{ kg} / \text{ m}^{3}$$

So, we found the density of sun to lie in the density range of solids and liquids. Clearly, the high intensity is attributed to the intense gravitational attraction of the inner layers on the outer layer of the sun.