

Thermodynamics



1. A geyser heats water flowing at the rate of 3.0 litres per minute from 27°C to 77°C . If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is $4.0 \times 10^4 \text{ J/g}$?

Ans: It is provided that,

Water flows at a rate of $3.0 \text{ litre/min} = 3 \times 10^{-3} \text{ m}^3 / \text{min}$

Density of water, $\rho = 10^3 \text{ kg} / \text{m}^3$.

Clearly, mass of water flowing per minute $= 3 \times 10^{-3} \times 10^3 \text{ kg} / \text{min} = 3 \text{ kg} / \text{min}$

The geyser heats the water, raising the temperature from 27°C to 77°C .

Initial temperature, $T_1 = 27^{\circ}\text{C}$

Final temperature, $T_2 = 77^{\circ}\text{C}$

Thus, rise in temperature,

$$\Delta T = T_2 - T_1$$

$$\Rightarrow \Delta T = 77^{\circ}\text{C} - 27^{\circ}\text{C}$$

$$\Rightarrow \Delta T = 50^{\circ}\text{C}$$

Now, heat of combustion $= 4 \times 10^4 \text{ J/g} = 4 \times 10^7 \text{ J/kg}$

Specific heat of water $= 4.2 \text{ J/g}^{\circ}\text{C}$

It is known that total heat used, $\Delta Q = mc\Delta T$

$$\Rightarrow \Delta Q = 3 \times 4.2 \times 10^3 \times 50$$

$$\Rightarrow \Delta Q = 6.3 \times 10^5 \text{ J/min}$$

Now, consider $m \text{ kg}$ of fuel to be used per minute.

Thus, the heat produced $= m \times 4 \times 10^7 \text{ J/min}$

However, the heat energy taken by water = heat produced by fuel

Thus, equating both the sides,

$$\Rightarrow 6.3 \times 10^5 = m \times 4 \times 10^7$$

$$\Rightarrow m = \frac{6.3 \times 10^5}{4 \times 10^4}$$

$$\Rightarrow m = 15.75 \text{ g/min}$$

Clearly, the rate of consumption of the fuel when its heat of combustion is $4.0 \times 10^4 \text{ J/g}$ supposing the geyser operates on a gas burner is 15.75 g/min .

2. What amount of heat must be supplied to 2.0×10^{-2} kg of Nitrogen (at room temperature) to raise its temperature by 45°C at constant pressure? (Molecular mass of $\text{N}_2 = 28$; $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$.)

Ans: Provided that,

Mass of Nitrogen, $m = 2.0 \times 10^{-2} \text{ kg} = 20 \text{ g}$.

Rise in temperature, $\Delta T = 45^\circ\text{C}$.

Molecular mass of N_2 , $M = 28$

Universal gas constant, $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$.

Number of moles, $n = \frac{m}{M}$

$$\Rightarrow n = \frac{2 \times 10^{-2} \times 10^3}{28}$$

$$\Rightarrow n = 0.714$$

Now, molar specific heat at constant pressure for nitrogen,

$$C_p = \frac{7}{2} R$$

$$\Rightarrow C_p = \frac{7}{2} \times 8.3$$

$$\Rightarrow C_p = 29.05 \text{ J mol}^{-1} \text{ K}^{-1}$$

The total amount of heat to be supplied is given by the relation:

$$\Delta Q = n C_p \Delta T$$

$$\Rightarrow \Delta Q = 0.714 \times 29.05 \times 45$$

$$\Rightarrow \Delta Q = 933.38 \text{ J}$$

Clearly, the amount of heat to be supplied is 933.38 J .

3. Explain why

- a) Two bodies at different temperatures T_1 and T_2 if brought in thermal contact do not necessarily settle to the mean temperature.

Ans: If two bodies at different temperatures T_1 and T_2 are brought in thermal contact, heat is said to flow from the body at higher temperature to the body at lower temperature until an equilibrium is obtained, i.e., until a point at which the temperatures of both the bodies becomes the same.

The equilibrium temperature turns out to be the same as the mean

temperature, which can be denoted as $\frac{(T_1 + T_2)}{2}$ only when thermal capacities of the two bodies are the same.

b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat.

Ans: The coolant in a chemical or nuclear plant must have a high specific heat as it is known that higher the specific heat of the coolant, higher is its heat-absorbing capacity and vice-versa.

Thus, a liquid which has a high specific heat is the best coolant to be utilized in a nuclear or chemical plant. This prevents different parts of the plant from becoming too hot.

c) Air pressure in a car tyre increases during driving.

Ans: When the car is in motion, the temperature of air inside the tyre rises due to motion of the air molecules. Charles's law suggests that pressure is directly proportional to the temperature, $P \propto T$.

Clearly, when the temperature inside a tyre rises, the air pressure inside the tyre would also increase.

d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude.

Ans: A harbour town has a more temperate climate than a town located in a desert at the same latitude.

In a harbour town, the relative humidity is greater than that in a desert town.

4. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?

Ans: Here, the cylinder is said to be completely insulated from its neighbourhood. As a result, no heat gets exchanged between the system (cylinder) and its neighbourhood. Clearly, the process turns out to be adiabatic.

Now, consider:

Initial pressure inside the cylinder = P_1

Final pressure inside the cylinder = P_2

Initial volume inside the cylinder = V_1

Final volume inside the cylinder = V_2

Ratio of specific heats, $\gamma = 1.4$

For an adiabatic process, it is known that $P_1 V_1^\gamma = P_2 V_2^\gamma$.

Also, the final volume is compressed to half of its initial volume.

$$\Rightarrow V_2 = \frac{V_1}{2}$$

Thus,

$$\Rightarrow P_1 V_1^\gamma = P_2 \left(\frac{V_1}{2} \right)^\gamma$$

$$\Rightarrow \frac{P_2}{P_1} = \frac{V_1^\gamma}{\left(\frac{V_1}{2} \right)^\gamma}$$

$$\Rightarrow \frac{P_2}{P_1} = 2^\gamma = 2^{1.4} = 2.639$$

Clearly, the pressure rises by a factor of 2.639.

- 5. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35cal, how much is the net work done by the system in the latter case? (Take 1cal = 4.19J)**

Ans: It is provided that the work done (W) on the system when the gas transforms from state A to state B is 22.3J .

This is an adiabatic process. Thus, the change in heat is zero.

$$\Rightarrow \Delta Q = 0$$

(As the work is done on the system)

Using the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

where,

ΔU = change in the internal energy of the gas

When the gas transforms from state A to state B via a process, the net heat absorbed by the system is given by

$$\Delta Q = 9.35\text{cal} = 9.35 \times 4.19\text{J} = 39.1765\text{J}$$

Heat absorbed can be given by the equation,

$$\Delta Q = \Delta U + \Delta W$$

$$\Rightarrow \Delta W = \Delta Q - \Delta U = 39.1765 - 22.3 = 16.8765\text{J}$$

Clearly, 16.88J work is done by the system.

- 6. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened. Answer the following:**

a) What is the final pressure of the gas in A and B?

Ans: The volume which can be availed by the gas at 1 atmospheric pressure would be doubled when the stopcock is opened instantly. As volume is inversely proportional to pressure, the pressure reduces to one-half times its original value. The initial pressure of the gas being 1 atm results in the pressure of each cylinder to be 0.5 atm.

b) What is the change in internal energy of the gas?

Ans: Change in the internal energy of any gas occurs when work is done by or on the gas. Here, as no work is done by or on the gas and thus, the internal energy of the gas would not change. Clearly, the change in internal energy of the gas is 0.

c) What is the change in the temperature of the gas?

Ans: Change in the temperature of any gas occurs when work is done by or on the gas. Here, as no work is done by the gas during the expansion of the gas, the temperature of the gas would not change. Clearly, the change in temperature of the gas is 0.

d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface?

Ans: No, the intermediate states of the system do not lie on the P-V-T surface since the process of free expansion is rapid and cannot be limited. The intermediate states are non-equilibrium states and do not follow the gas equation. In due course of time, the gas returns to its original state.

- 7. An electric heater supplies heat to a system at a rate of 100W. If system performs work at a rate of 75 Joules per second. At what rate is the internal energy increasing?**

Ans: It is provided that,
Heat is supplied to the system by an electric heater at a rate of 100W.
Thus, heat supplied, $Q = 100\text{J/s}$
The system operates at a rate of 75J/s .
Clearly, work done, $W = 75\text{J/s}$
Using the first law of thermodynamics,
 $Q = U + W$

where,

U = internal energy

$$\Rightarrow U = Q - W$$

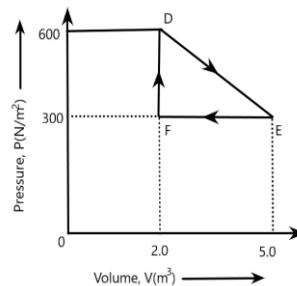
$$\Rightarrow U = 100 - 75$$

$$\Rightarrow U = 25 \text{ J / s}$$

$$\Rightarrow U = 25 \text{ W}$$

Clearly, the internal energy of the given electric heater rises at a rate of 25W or 25J / s .

8. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in figure.



Calculate the total work done by the gas from D to E to F .

Ans: Considering the given linear process in a thermodynamic system, it can be understood that,

Total work done by the gas from D to E to F is equal to the area of $\triangle DEF$.

$$\text{Area of } \triangle DEF = \frac{1}{2} \times DE \times EF$$

where,

DF = Change in pressure

$$\Rightarrow DF = 600 \text{ N / m}^2 - 300 \text{ N / m}^2$$

$$\Rightarrow DF = 300 \text{ N / m}^2$$

Also,

$$FE = \text{Change in volume} = 5 - 2 = 3 \text{ m}^3$$

Thus,

$$\text{Area of } \triangle DEF = \frac{1}{2} \times 300 \times 3 = 450 \text{ J}$$

Clearly, the total work done by the gas from D to E to F is 450J .