sequences and series

Chapter 8

Exercise 8.1

1. Write the first five terms of the sequences whose n^{th} term is $a_n = n(n+2)$.

Ans:

The given equation is $a_n = n(n+2)$.

Substitute $n=1$ in the equation.

$a_1 = 1(1+2)$

$\Rightarrow a_1 = 3$

Similarly, substitute $n = 2,3,4$ and 5 in the equation.

$a_2 = 2(2+2)$

$\Rightarrow a_2 = 8$

$a_3 = 3(3+2)$

$\Rightarrow a_3 = 15$

$a_4 = 4(4+2)$

$\Rightarrow a_4 = 24$

$a_5 = 5(5+2)$

$\Rightarrow a_5 = 35$

Therefore, the first five terms of $a_n = n(n+2)$ are $3,8,15,24$ and 35.

2. Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{n}{n+1}$.

Ans: The given equation is $a_n = \frac{n}{n+1}$.

Substitute $n=1$ in the equation.

$a_1 = \frac{1}{1+1}$

$\Rightarrow a_1 = \frac{1}{2}$

Similarly, substitute $n = 2, 3, 4$ and 5 in the equation.

$a_2 = \frac{2}{2+1}$

$\Rightarrow a_2 = \frac{2}{3}$

$a_3 = \frac{3}{3+1}$

$\Rightarrow a_3 = \frac{3}{4}$

$a_4 = \frac{4}{4+1}$

$\Rightarrow a_4 = \frac{4}{5}$

$a_5 = \frac{5}{5+1}$

$\Rightarrow a_5 = \frac{5}{6}$

Therefore, the first five terms of $a_n = \frac{n}{n+1}$ is $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$.

3. Write the first five terms of the sequences whose n^{th} term is $a_n = 2^n$.

Ans: The given equation is $a_n = 2^n$.

Substitute $n = 1$ in the equation.

$a_1 = 2^1$

$\Rightarrow a_1 = 2$

Similarly, substitute $n = 2, 3, 4$ and 5 in the equation.

$a_2 = 2^2$

$\Rightarrow a_2 = 4$

$a_3 = 2^3$

$\Rightarrow a_3 = 8$

$a_4 = 2^4$

$\Rightarrow a_4 = 16$

$a_5 = 2^5$

$\Rightarrow a_5 = 32$

Therefore, the first five terms of $a_n = 2^n$ is $2, 4, 8, 16$ and 32 .

4. Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{2n-3}{6}$.

Ans: The given equation is $a_n = \frac{2n-3}{6}$.

Substitute $n = 1$ in the equation.

$$a_1 = \frac{2(1)-3}{6}$$

$$\Rightarrow a_1 = -\frac{1}{6}$$

Similarly, substitute $n = 2, 3, 4$ and 5 in the equation.

$$a_2 = \frac{2(2)-3}{6}$$

$$\Rightarrow a_2 = \frac{1}{6}$$

$$a_3 = \frac{2(3)-3}{6}$$

$$\Rightarrow a_3 = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2(4)-3}{6}$$

$$\Rightarrow a_4 = \frac{5}{6}$$

$$a_5 = \frac{2(5)-3}{6}$$

$$\Rightarrow a_5 = \frac{7}{6}$$

Therefore, the first five terms of $a_n = \frac{2n-3}{6}$ is $-\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$ and $\frac{7}{6}$.

5. Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$.

Ans: The given equation is $a_n = (-1)^{n-1} 5^{n+1}$.
Substitute $n = 1$ in the equation.
$a_1 = (-1)^{1-1} 5^{1+1}$
$\Rightarrow a_1 = 5^2 = 25$
Similarly, substitute $n = 2, 3, 4$ and 5 in the equation.
$a_2 = (-1)^{2-1} 5^{2+1}$
$\Rightarrow a_2 = -5^3 = -125$
$a_3 = (-1)^{3-1} 5^{3+1}$
$\Rightarrow a_3 = 5^4 = 625$
$a_4 = (-1)^{4-1} 5^{4+1}$
$\Rightarrow a_4 = -5^5 = -3125$
$a_5 = (-1)^{5-1} 5^{5+1}$
$\Rightarrow a_5 = 5^6 = 15625$
Therefore, the first five terms of $a_n = (-1)^{n-1} 5^{n+1}$ is $25, -125, 625, -3125$ and 15625

6. Write the first five terms of the sequences whose n^{th} term is $a_n = n\frac{n^2+5}{4}$.

Ans: The given equation is $a_n = n\frac{n^2+5}{4}$.
Substitute $n = 1$ in the equation.
$a_1 = 1 \cdot \frac{1^2+5}{4}$
$\Rightarrow a_1 = \frac{6}{4} = \frac{3}{2}$
Similarly, substitute $n = 2, 3, 4$ and 5 in the equation.
$a_2 = 2 \cdot \frac{2^2+5}{4}$
$\Rightarrow a_2 = \frac{18}{4} = \frac{9}{2}$
$a_3 = 3 \cdot \frac{3^2+5}{4}$

$\Rightarrow a_3 = \frac{42}{4} = \frac{21}{2}$

$a_4 = 4 \cdot \frac{4^2 + 5}{4}$

$\Rightarrow a_4 = \frac{84}{4} = 21$

$a_5 = 5 \cdot \frac{5^2 + 5}{4}$

$\Rightarrow a_5 = \frac{150}{4} = \frac{75}{2}$

Therefore, the first five terms of $a_n = n\frac{n^2 + 5}{4}$ is $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21$ and $\frac{75}{2}$.

7. Find the 17^{th} and 24^{th} term in the following sequence whose n^{th} term is $a_n = 4n - 3$.

Ans: The given equation is $a_n = 4n - 3$.

Substitute $n = 17$ in the equation.

$a_{17} = 4(17) - 3$

$\Rightarrow a_{17} = 65$

Similarly, substitute $n = 24$ in the equation.

$a_{24} = 4(24) - 3$

$\Rightarrow a_{24} = 93$

Therefore, the 17^{th} and 24^{th} term of $a_n = 4n - 3$ is 65 and 93 respectively.

8. Find the 7^{th} term in the following sequence whose n^{th} term is $a_n = \frac{n^2}{2^n}$.

Ans: The given equation is $a_n = \frac{n^2}{2^n}$.

Substitute $n = 7$ in the equation.

$a_7 = \frac{7^2}{2^7}$

$\Rightarrow a_7 = \frac{49}{128}$

Therefore, the 7^{th} term of $a_n = \frac{n^2}{2^n}$ is $\frac{49}{128}$.

9. Find the 9^{th} term in the following sequence whose n^{th} term is $a_n = (-1)^{n-1} n^3$.

Ans: The given equation is $a_n = (-1)^{n-1} n^3$.

Substitute $n = 9$ in the equation.

$a_9 = (-1)^{9-1} 9^3$

$\Rightarrow a_9 = 729$

Therefore, the 9^{th} term of $a_n = (-1)^{n-1} n^3$ is 729.

10. Find the 20^{th} term in the following sequence whose n^{th} term is $a_n = \frac{n(n-2)}{n+3}$

Ans: The given equation is $a_n = \frac{n(n-2)}{n+3}$.

Substitute $n = 20$ in the equation.

$a_{20} = \frac{20(20-2)}{20+3}$

$\Rightarrow a_{20} = \frac{360}{23}$

Therefore, the 20^{th} term of $a_n = \frac{n(n-2)}{n+3}$ is $\frac{360}{23}$.

11. Write the first five terms of the following sequence and obtain the corresponding series: $a_1 = 3$, $a_n = 3a_{n-1} + 2$ for all $n > 1$.

Ans: The given equation is $a_n = 3a_{n-1} + 2$ where $a_1 = 3$ and $n > 1$.

Substitute $n = 2$ and $a_1 = 3$ in the equation.

$a_2 = 3a_{2-1} + 2 = 3(3) + 2$

$\Rightarrow a_2 = 11$

Similarly, substitute $n = 3, 4$ and 5 in the equation.

$a_3 = 3a_{3-1} + 2 = 3(11) + 2$

$\Rightarrow a_3 = 35$

$a_4 = 3a_{4-1} + 2 = 3(35) + 2$

$\Rightarrow a_4 = 107$

$a_5 = 3a_{5-1} + 2 = 3(107) + 2$

$\Rightarrow a_5 = 323$

Therefore, the first five terms of $a_n = 3a_{n-1} + 2$ are $3, 11, 35, 107$ and 323.

The corresponding series obtained from the sequence is $3 + 11 + 35 + 107 + 323 + ...$

12. Write the first five terms of the following sequence and obtain the corresponding series: $a_1 = -1$, $a_n = \frac{a_{n-1}}{n}$, $n \geq 2$.

Ans: The given equation is $a_n = \frac{a_{n-1}}{n}$ where $a_1 = -1$ and $n \geq 2$.

Substitute $n = 2$ and $a_1 = -1$ in the equation.

$$a_2 = \frac{a_{2-1}}{2} = \frac{-1}{2}$$

$$\Rightarrow a_2 = -\frac{1}{2}$$

Similarly, substitute $n = 3, 4$ and 5 in the equation.

$$a_3 = \frac{a_{3-1}}{3} = \frac{-1/2}{3}$$

$$\Rightarrow a_3 = -\frac{1}{6}$$

$$a_4 = \frac{a_{4-1}}{4} = \frac{-1/6}{4}$$

$$\Rightarrow a_4 = -\frac{1}{24}$$

$$a_5 = \frac{a_{5-1}}{5} = \frac{-1/24}{5}$$

$$\Rightarrow a_5 = -\frac{1}{120}$$

Therefore, the first five terms of $a_n = \frac{a_{n-1}}{n}$ is $-1, -\frac{1}{2}, -\frac{1}{6}, -\frac{1}{24}$ and $-\frac{1}{120}$.

The corresponding series obtained from the sequence is

$$(-1)+\left(-\frac{1}{2}\right)+\left(-\frac{1}{6}\right)+\left(-\frac{1}{24}\right)+\left(-\frac{1}{120}\right)...$$

13. Write the first five terms of the following sequence and obtain the corresponding series: $\mathbf{a_1 = a_2 = 2}$, $\mathbf{a_n = a_{n-1} - 1}$, $\mathbf{n > 2}$.

Ans: The given equation is $a_n = a_{n-1} - 1$ where $a_1 = a_2 = 2$ and $n > 2$.

Substitute $n = 3$ and $a_2 = 2$ in the equation.

$a_3 = a_{3-1} - 1 = 2 - 1$

$\Rightarrow a_3 = 1$

Similarly, substitute $n = 4$ and 5 in the equation.

$a_4 = a_{4-1} - 1 = 1 - 1$

$\Rightarrow a_4 = 0$

$a_5 = a_{5-1} - 1 = 0 - 1$

$\Rightarrow a_5 = -1$

Therefore, the first five terms of $a_n = a_{n-1} - 1$ is $2, 2, 1, 0$ and -1 .

The corresponding series obtained from the sequence is $2 + 2 + 1 + 0 + (-1) + ...$

14. The Fibonacci sequence is defined by $\mathbf{1 = a_1 = a_2}$, $\mathbf{a_n = a_{n-1} + a_{n-2}}$, $\mathbf{n > 2}$. Find $\mathbf{\frac{a_{n+1}}{a_n}}$, for $\mathbf{n = 1, 2, 3, 4, 5}$.

Ans: The given equation is $a_n = a_{n-1} + a_{n-2}$ where $1 = a_1 = a_2$ and $n > 2$.

Substitute $n = 3$ and $1 = a_1 = a_2$ in the equation.

$a_3 = a_{3-1} + a_{3-2} = 1 + 1$

$\Rightarrow a_3 = 2$

Similarly, substitute $n = 4, 5$ and 6 in the equation.

$a_4 = a_{4-1} + a_{4-2} = 2 + 1$

$\Rightarrow a_4 = 3$

$a_5 = a_{5-1} + a_{5-2} = 3 + 2$

$\Rightarrow a_5 = 5$

$a_6 = a_{6-1} + a_{6-2} = 5 + 3$

$\Rightarrow a_6 = 8$

Substitute the values of a_1 and a_2 in the expression $\frac{a_{n+1}}{a_n}$ for $n=1$.

$$\Rightarrow \frac{a_{1+1}}{a_1} = \frac{1}{1} = 1$$

Similarly, when $n=2$,

$$\Rightarrow \frac{a_{2+1}}{a_2} = \frac{2}{1} = 2$$

When $n=3$,

$$\Rightarrow \frac{a_{3+1}}{a_3} = \frac{3}{2}$$

When $n=4$,

$$\Rightarrow \frac{a_{4+1}}{a_4} = \frac{5}{3}$$

When $n=5$,

$$\Rightarrow \frac{a_{5+1}}{a_5} = \frac{8}{5}$$

Therefore, the value of $\frac{a_{n+1}}{a_n}$ for $n=1,2,3,4,5$ is $1,2,\frac{3}{2},\frac{5}{3}$ and $\frac{8}{5}$ respectively.

Exercise 8.2

1. Find the 20^{th} and n^{th} term of the G.P. $\frac{5}{2},\frac{5}{4},\frac{5}{8},\ldots$

Ans: $\frac{5}{2},\frac{5}{4},\frac{5}{8},\ldots$ is the given G.P.

The first term of the G.P. is $a=\frac{5}{2}$ and the common ratio is $r=\frac{5/4}{5/2}=\frac{1}{2}$.

The n^{th} term of the G.P. is given by the equation $a_n=ar^{n-1}$.

Substituting the values of a and r we get

$$a_n=\frac{5}{2}\left(\frac{1}{2}\right)^{n-1}=\frac{5}{(2)(2)^{n-1}}=\frac{5}{(2)^n}$$

Similarly, the 20^{th} term of the G.P. is $a_{20}=ar^{20-1}$

$$\Rightarrow a_{20}=\frac{5}{2}\left(\frac{1}{2}\right)^{19}=\frac{5}{(2)(2)^{19}}=\frac{5}{(2)^{20}}$$

Therefore, the 20^{th} and n^{th} term of the given G.P. is $\frac{5}{(2)^{20}}$ and $\frac{5}{(2)^n}$ respectively.

2. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2

Ans: Let the first term of the G.P. be a and the common ratio $r=2$.

The 8^{th} term of the G.P. is given by the equation $a_8=ar^{8-1}$.

Substituting the values of a_8 and r we get

$$\Rightarrow 192=a(2)^7$$

$$\Rightarrow (2)^6(3)=a(2)^7$$

$$\Rightarrow a=\frac{(2)^6(3)}{(2)^7}=\frac{3}{2}$$

Then 12^{th} term of the G.P. is given by the equation $a_{12}=ar^{12-1}$.

Substitute the values of a and r in the equation.

$$a_{12}=\frac{3}{2}(2)^{11}$$

$$=3(2)^{10}$$

$= 3072$

Therefore, the 12^{th} term of the G.P. is 3072 .

3. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s , respectively. Show that $q^2 = ps$.

Ans: Let the first term and the common ratio of the G.P. be a and r respectively.

According to the conditions given in the question,

$a_5 = ar^{5-1} = ar^4 = p$

$a_8 = ar^{8-1} = ar^7 = q$

$a_{11} = ar^{11-1} = ar^{10} = s$

Dividing a_8 by a_5 we get

$\frac{ar^7}{ar^4} = \frac{q}{p}$

$\Rightarrow r^3 = \frac{q}{p}$

Dividing a_{11} by a_8 we get

$\frac{ar^{10}}{ar^7} = \frac{s}{q}$

$\Rightarrow r^3 = \frac{s}{q}$

Equate both the values of r^3 obtained.

$\frac{q}{p} = \frac{s}{q}$

$\Rightarrow q^2 = ps$

Therefore, $q^2 = ps$ is proved.

4. The 4^{th} term of a G.P. is square of its second term, and the first term is -3 . Determine its 7^{th} term.

Ans: Let the first term and the common ratio of the G.P. be a and r respectively.

It is given that $a = -3$.

The n^{th} term of the G.P. is given by the equation $a_n = ar^{n-1}$.

Then,

$a_4 = ar^3 = (-3)r^3$

$a_2 = ar^1 = (-3)r$

According to the conditions given in the question,

$(-3)r^3 = [(-3)r]^2$

$\Rightarrow -3r^3 = 9r^2$

$\Rightarrow r = -3a_7$

$= ar^6$

$= (-3)(-3)^6$

$= -(3)^7$

$= -2187$

Therefore, -2187 is the seventh term of the G.P.

5. Which term of the following sequences:

a) $2, 2\sqrt{2}, 4...$ is 128 ?

Ans: $2, 2\sqrt{2}, 4...$ is the given sequence.

The first term of the G.P. $a = 2$ and the common ratio $r = \left(2\sqrt{2}\right)/2 = \sqrt{2}$.

128 is the n^{th} term of the given sequence.

The n^{th} term of the G.P. is given by the equation $a_n = ar^{n-1}$.

Therefore, $ar^{n-1} = 128$

$\Rightarrow (2)\left(\sqrt{2}\right)^{n-1} = 128$

$\Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7$

$\Rightarrow (2)^{\frac{n-1}{2}+1} = (2)^7$

$\Rightarrow \frac{n-1}{2} + 1 = 7$

$\Rightarrow \frac{n-1}{2} = 6$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

Therefore, 128 is the 13^{th} term of the given sequence.

b) $\boldsymbol{\sqrt{3},3,3\sqrt{3}...}$ **is 729 ?**

Ans: $\sqrt{3},3,3\sqrt{3}...$ is the given sequence.

The first term of the G.P. $a=\sqrt{3}$ and the common ratio $r=3/\sqrt{3}=\sqrt{3}$.

729 is the n^{th} term of the given sequence.

The n^{th} term of the G.P. is given by the equation $a_n=ar^{n-1}$.

Therefore, $ar^{n-1}=729$

$$\Rightarrow\left(\sqrt{3}\right)\left(\sqrt{3}\right)^{n-1}=729$$

$$\Rightarrow(3)^{1/2}(2)^{\frac{n-1}{2}}=(3)^6$$

$$\Rightarrow(3)^{\frac{1}{2}+\frac{n-1}{2}}=(3)^6$$

$$\Rightarrow\frac{1}{2}+\frac{n-1}{2}=6$$

$$\Rightarrow\frac{1+n-1}{2}=6$$

$$\Rightarrow\frac{n}{2}=6$$

$$\Rightarrow n=12$$

Therefore, 729 is the 12^{th} term of the given sequence.

c) $\boldsymbol{\frac{1}{3},\frac{1}{9},\frac{1}{27},...}$ **is** $\boldsymbol{\frac{1}{19683}}$ **?**

Ans: $\frac{1}{3},\frac{1}{9},\frac{1}{27},...$ is the given sequence.

The first term of the G.P. $a=\frac{1}{3}$ and the common ratio $r=\frac{1}{9}\div\frac{1}{3}=\frac{1}{3}$.

$\frac{1}{19683}$ is the n^{th} term of the given sequence.

The n^{th} term of the G.P. is given by the equation $a_n=ar^{n-1}$.

Therefore, $ar^{n-1}=\frac{1}{19683}$

$$\Rightarrow\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1}=\frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9$$

$$\Rightarrow n = 9$$

Therefore, $\frac{1}{19683}$ is the 9^{th} term of the given sequence.

6. For what values of x, the numbers $-\frac{2}{7}, x, -\frac{7}{2}$ are in G.P.?

Ans: $-\frac{2}{7}, x, -\frac{7}{2}$ are the given numbers and the common ratio $= \frac{x}{-2/7} = \frac{-7x}{2}$

We also know that, common ratio $= \frac{-7/2}{x} = \frac{-7}{2x}$

Equating both the common ratios we get

$$\frac{-7x}{2} = \frac{-7}{2x}$$

$$\Rightarrow x^2 = \frac{-2\times 7}{-2\times 7} = 1$$

$$\Rightarrow x = \sqrt{1}$$

$$\Rightarrow x = \pm 1$$

Therefore, the given numbers will be in G.P. for $x = \pm 1$.

7. Find the sum up to 20 terms in the geometric progression $0.15, 0.015, 0.0015...$

Ans: $0.15, 0.015, 0.0015...$ is the given G.P.

The first term of the G.P. $a = 0.15$ and the common ratio $r = \frac{0.015}{0.15} = 0.1$.

The sum of first n terms of the G.P. is given by the equation $S_n = \frac{a\left(1-r^n\right)}{1-r}$.

Therefore, the sum of first 20 terms of the given G.P. is

$$S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$

$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$

$=\frac{15}{90}\left[1-(0.1)^{20}\right]$

$=\frac{1}{6}\left[1-(0.1)^{20}\right]$

Therefore, the sum up to 20 terms in the geometric progression $0.15, 0.015, 0.0015...$ is $\frac{1}{6}\left[1-(0.1)^{20}\right]$.

8. Find the sum of n terms in the geometric progression $\sqrt{7}, \sqrt{21}, 3\sqrt{7}...$

Ans: $\sqrt{7}, \sqrt{21}, 3\sqrt{7}...$ is the given G.P.

The first term of the G.P. $a=\sqrt{7}$ and the common ratio $r=\frac{\sqrt{21}}{\sqrt{7}}=\sqrt{3}$.

The sum of the first n terms of the G.P. is given by the equation $S_n=\frac{a(1-r^n)}{1-r}$.

The sum of the first n terms of the given G.P. is

$$S_n=\frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}}$$

$$=\frac{\sqrt{7}\left[1-(\sqrt{3})^n\right]}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$=\frac{\sqrt{7}(\sqrt{3}+1)\left[1-(\sqrt{3})^n\right]}{1-3}$$

$$=\frac{-\sqrt{7}(\sqrt{3}+1)\left[1-(\sqrt{3})^n\right]}{2}$$

$$=\frac{-\sqrt{7}(1+\sqrt{3})}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

Therefore, the sum of n terms of the geometric progression $\sqrt{7}, \sqrt{21}, 3\sqrt{7}...$ is $\frac{-\sqrt{7}(1+\sqrt{3})}{2}\left[(3)^{\frac{n}{2}}-1\right]$.

9. Find the sum of n terms in the geometric progression $1,-a,a^2,-a^3...$ (if $a \neq -1$)

Ans: $1,-a,a^2,-a^3...$ is the given G.P.

The first term of the G.P. $a_1 = 1$ and the common ratio $r = -a$.

The sum of first n terms of the G.P. is given by the equation $S_n = \frac{a_1(1-r^n)}{1-r}$.

The sum of first n terms of the given G.P. is

$$S_n = \frac{1\left[1-(-a)^n\right]}{1-(-a)}$$

$$= \frac{\left[1-(-a)^n\right]}{1+a}$$

Therefore, the sum of n terms of the geometric progression $1,-a,a^2,-a^3...$ is $\frac{\left[1-(-a)^n\right]}{1+a}$.

10. Find the sum of n terms in the geometric progression $x^3,x^5,x^7...$ (if $a \neq -1$)

Ans: $x^3,x^5,x^7...$ is the given G.P.

The first term of the G.P. $a = x^3$ and the common ratio $r = x^2$.

The sum of first n terms of the G.P. is given by the equation $S_n = \frac{a_1(1-r^n)}{1-r}$.

The sum of first n terms of the given G.P. is

$$S_n = \frac{x^3\left[1-(x^2)^n\right]}{1-x^2}$$

$$= \frac{x^3\left[1-x^{2n}\right]}{1-x^2}$$

Therefore, the sum of n terms of the geometric progression $x^3,x^5,x^7...$ is $\frac{x^3\left[1-x^{2n}\right]}{1-x^2}$

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11. Evaluate $\sum_{k=1}^{11}(2+3k)$

Ans: $\sum_{k=1}^{11}(2+3k)=\sum_{k=1}^{11}(2)+\sum_{k=1}^{11}(3k)=22+\sum_{k=1}^{11}(3^k)$ $\quad\ldots(1)$

We know that,

$$\sum_{k=1}^{11}(3^k)=3^1+3^2+\ldots+3^{11}$$

This sequence $3,3^2,3^3,\ldots,3^{11}$ forms a G.P. Therefore,

$$S_n=\frac{a\left(r^n-1\right)}{(r-1)}$$

Substituting the values to the above equation we get,

$$\Rightarrow S_n=\frac{3\left[(3)^{11}-1\right]}{(3-1)}$$

$$\Rightarrow S_n=\frac{3}{2}\left(3^{11}-1\right)$$

Therefore,

$$\Rightarrow \sum_{k=1}^{11}3^k=\frac{3}{2}\left(3^{11}-1\right)$$

Substitute this value in equation (1).

$$\sum_{k=1}^{11}(2+3k)=22+\frac{3}{2}\left(3^{11}-1\right)$$

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Ans: Let the first three terms of a G.P. be $\frac{a}{r},a,ar$.

Then, its sum is

$$\frac{a}{r}+a+ar=\frac{39}{10} \quad\ldots(1)$$

And the product is

$$\left(\frac{a}{r}\right)(a)(ar)=1 \qquad \ldots(2)$$

Solving equation (2) we will get,

$a^3=1$

Considering the real roots,

$a=1$

Substitute the value of a in the equation.

$$\frac{1}{r}+1+r=\frac{39}{10}$$

$$\Rightarrow 1+r+r^2=\frac{39}{10}r$$

$$\Rightarrow 10+10r+10r^2=39r$$

$$\Rightarrow 10r^2-29r+10=0$$

$$\Rightarrow 10r^2-25r-4r+10=0$$

$$\Rightarrow 5r(2r-5)-2(2r-5)=0$$

$$\Rightarrow (5r-2)(2r-5)=0$$

$$\Rightarrow r=\frac{2}{5} \text{ or } \frac{5}{2}$$

Therefore, $\frac{5}{2}$,1 and $\frac{2}{5}$ are the first three terms of the G.P.

13. How many terms of G.P. $3,3^2,3^3...$ are needed to give the sum 120?

Ans: Given G.P. $3,3^2,3^3,\ldots,3^{11}$.

Let there be n terms to get the sum as 120.

Then using the formula, we get,

$$S_n=\frac{a\left(r^n-1\right)}{(r-1)} \qquad \ldots(1)$$

Given that,

$S_n=120$

$a=3$

$r=3$

Substituting the given values in equation (1),

$$S_n = 120 = \frac{3\left(3^n - 1\right)}{3-1}$$

$$\Rightarrow 120 = \frac{3\left(3^n - 1\right)}{2}$$

$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = 3^4$$

$$\Rightarrow n = 4$$

Therefore, for getting the sum as 120 the given G.P. should have 4 terms.

14. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio, and the sum to n terms of the G.P.

Ans: Let $a, ar, ar^2, ar^3...$ be the G.P.

According to the conditions given in the question,

$$a + ar + ar^2 = 16 \qquad \ldots(1)$$

$$ar^3 + ar^4 + ar^5 = 128 \qquad \ldots(2)$$

Equation (1) and (2) can also be written as,

$$a\left(1 + r + r^2\right) = 16$$

$$ar^3\left(1 + r + r^2\right) = 128$$

Divide equation (2) by (1) .

$$\frac{(2)}{(1)} \Rightarrow \frac{ar^3\left(1 + r + r^2\right)}{a\left(1 + r + r^2\right)} = \frac{128}{16}$$

$$\Rightarrow r^3 = 8$$

$$\Rightarrow r = 2$$

Substituting the value of r in equation (1), we get

$$a\left(1 + r + r^2\right) = 16$$

$$\Rightarrow a\left(1 + 2 + 4\right) = 16$$

$$\Rightarrow 7a = 16$$

$\Rightarrow a=\frac{16}{7}$

Sum of n terms of the G.P. is,

$S_n=\frac{a\left(r^n-1\right)}{\left(r-1\right)}$

$\Rightarrow S_n=\frac{16}{7}\frac{\left(2^n-1\right)}{2-1}$

$\Rightarrow S_n=\frac{16}{7}\left(2^n-1\right)$

Therefore, the first term of the G.P. is $a=\frac{16}{7}$, the common ratio $r=2$ and the sum of terms $S_n=\frac{16}{7}\left(2^n-1\right)$.

15. Given a G.P. with $a=729$ and 7^{th} term 64, determine S_7 .

Ans: Given that $a=729$ and $a_7=64$

Let the common ratio of the G.P be r. Then,

$a_n=ar^{n-1}$

$\Rightarrow a_7=ar^{6-1}$

$\Rightarrow 64=729\left(r^6\right)$

$\Rightarrow r^6=\left(\frac{2}{3}\right)^6$

$\Rightarrow r=\frac{2}{3}$

We know that,

$S_n=\frac{a\left(1-r^n\right)}{\left(1-r\right)}$

Therefore,

$$S_7 = \frac{729\left(1-\left(\frac{2}{3}\right)^7\right)}{\left(1-\frac{2}{3}\right)}$$

$$= 729 \times 3\left(\frac{(3)^7-(2)^7}{(3)^7}\right)$$

$$= (3)^7\left(\frac{(3)^7-(2)^7}{(3)^7}\right)$$

$$= (3)^7-(2)^7$$

$$= 2187-128$$

$$= 2059$$

Therefore, the value of S_7 is 2059 .

16. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

Ans: Let a and r be the first term and common ratio of the G.P. respectively.
According to the conditions given in the question,

$$a_5 = 4 \times a_3$$

$$\Rightarrow ar^4 = 4 \times ar^2$$

$$\Rightarrow r^2 = 4$$

$$\Rightarrow r = \pm 2$$

Given that,

$$S_2 = -4 = \frac{a\left(1-r^2\right)}{(1-r)}$$

Substituting $r = 2$ in the above equation,

$$-4 = \frac{a\left[1-(2)^2\right]}{1-2}$$

$$\Rightarrow -4 = \frac{a(1-4)}{-1}$$

$$\Rightarrow -4 = a(3)$$

$\Rightarrow a = \frac{-4}{3}$

Now, taking $r = -2$, we get,

$-4 = \frac{a\left[1-(-2)^2\right]}{1-(-2)}$

$\Rightarrow -4 = \frac{a(1-4)}{1+2}$

$\Rightarrow -4 = \frac{a(-3)}{3}$

$\Rightarrow a = 4$

Therefore , $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, ...$ or $4, -8, -16, -32...$ is the required G.P.

17. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z , respectively. Prove that x, y, z are in G.P.

Ans: Let the first term of the G.P be a and the common ratio be r .

According to the conditions given in the question,

$a_4 = ar^3 = x \quad \ldots(1)$

$a_{10} = ar^9 = y \quad \ldots(2)$

$a_{16} = ar^{15} = z \quad \ldots(3)$

Then divide equation (2) by (1) .

$\frac{y}{x} = \frac{ar^9}{ar^3}$

$\Rightarrow \frac{y}{x} = r^6$

Now, divide equation (3) by (1).

$\frac{z}{y} = \frac{ar^{15}}{ar^9}$

$\Rightarrow \frac{z}{y} = r^6$

Therefore,

$\frac{y}{x} = \frac{z}{y}$

Therefore, it is proved that x, y, z are in G. P.

18. Find the sum to n terms of the sequence, $8, 88, 888, 8888...$

Ans: $8, 88, 888, 8888...$ is the given sequence

The given sequence is not in G.P. In order to make the sequence in G.P., it has to be changed to the form,

$S_n = 8 + 88 + 888 + 8888 + ...$ to n terms

$= \frac{8}{9}[9 + 99 + 999 + 9999 + ...\text{to } n \text{ terms}]$

$= \frac{8}{9}[(10-1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \text{to } n \text{ terms}]$

$= \frac{8}{9}[(10 + 10^2 + ...n \text{ terms}) - (1 + 1 + 1 + ...n \text{ terms})]$

$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$

$= \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$

$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$

Therefore, the sum of n terms the given sequence is $\frac{80}{81}(10^n - 1) - \frac{8}{9}n$.

19. Find the sum of the products of the corresponding terms of the sequences $2, 4, 8, 16, 32$ and $128, 32, 8, 2, 1/2$.

Ans: $2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2}$

$= 64\left[4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2}\right]$

is the required sum.

We can see that, $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}$ is a G.P.

The first term of the G.P. is $a = 4$ and the common ratio is $r = \frac{1}{2}$.

We know that,

$$S_n = \frac{a\left(1-r^n\right)}{\left(1-r\right)}$$

Therefore,

$$S_3 = \frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}}$$

$$= \frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}}$$

$$= 8\left(\frac{32-1}{32}\right)$$

$$= \frac{31}{4}$$

Therefore, the required sum $= 64\left(\frac{31}{4}\right) = \left(16\right)\left(31\right) = 496$.

20. Show that the products of the corresponding terms of the sequences form $a, ar, ar^2, ...ar^{n-1}$ and A, AR, AR^2, AR^{n-1} a G.P. and find the common ratio.

Ans: The sequence $aA, arAR, ar^2AR^2, ...ar^{n-1}AR^{n-1}$ forms a G.P. is to be proved.

Second term / First term $= \frac{arAR}{aA} = rR$

Third term / Second term $= \frac{ar^2AR^2}{aA} = rR$

Therefore, the $aA, arAR, ar^2AR^2, ...ar^{n-1}AR^{n-1}$ forms a G.P. and the common ratio is rR.

21. Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4^{th} by 18 .

Ans: Let the first term be a and the common ratio be r of the G.P.

$a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$

According to the conditions given in the question,

$a_3 = a_1 + 9$

$\Rightarrow ar^2 = a + 9$

$\Rightarrow a\left(r^2 - 1\right) = 9 \quad ...(1)$

$a_2 = a_4 + 9$

$\Rightarrow ar = ar^3 + 18$

$\Rightarrow ar\left(1 - r^2\right) = 18 \quad ...(2)$

Divide (2) by (1).

$$\frac{ar\left(1 - r^2\right)}{a\left(r^2 - 1\right)} = \frac{18}{9}$$

$\Rightarrow -r = 2$

$\Rightarrow r = -2$

Substitute $r = -2$ in equation (1).

$a\left(4 - 1\right) = 9$

$\Rightarrow a\left(3\right) = 9$

$\Rightarrow a = 3$

Therefore, $3, 3\left(-2\right), 3\left(-2\right)^2$ and $3\left(-2\right)^3$,i.e., $3, -6, 12$ and -24 are the first four numbers of the G.P.

22. If p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c, respectively. Prove that $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$.

Ans: Let the first term be A and the common ration be R of the G.P.

According to the conditions given in the question,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

Then,

$a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{q-r} \times R^{(r-1)(p-q)}$

$= A^{q-r+r-p+p-q} \times R^{(pq-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Therefore, $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$ is proved.

23. If the first and n^{th} the term of a G.P. are a and b, respectively, and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Ans: a is the first term and b is the last term of the G.P.

Therefore, is the G.P. $a, ar, ar^2, ar^3 ... ar^{n-1}$, where the common ratio is r .

$b = ar^{n-1} \qquad \ldots(1)$

P is the product of n terms. Therefore,

$P = (a)(ar)(ar^2)...(ar^{n-1})$

$= (a \times a \times ...a)(r \times r^2 \times ...r^{n-1})$

$= a^n r^{1+2+...(n-1)} \qquad \ldots(2)$

We can see that, $1, 2, ...(n-1)$ is an A.P. Therefore,

$1 + 2 + ... + (n-1)$

$= \frac{n-1}{2}\left[2 + (n-1-1) \times 1\right]$

$= \frac{n-1}{2}[2 + n - 2]$

$= \frac{n(n-1)}{2}$

So, equation (2) can be written as $P = a^n r^{\frac{n(n-1)}{2}}$.

Therefore,

$P^2 = a^{2n} r^{n(n-1)}$

$= \left[a^2 r^{(n-1)}\right]^n$

$= \left[a \times ar^{n-1}\right]^n$

Substituting (1) in the equation,

$P^2 = (ab)^n$

Therefore, $P^2 = (ab)^n$ is proved.

24. Show that the ratio of the sum of first $\mathbf{n}$ terms of a G.P. to the sum of terms from $\mathbf{(n+1)^{th}}$ to $\mathbf{(2n)^{th}}$ term is $\mathbf{\frac{1}{r^n}}$.

Ans: Let the first term be a and the common ration be r of the G.P.

$\frac{a(1-r^n)}{(1-r)}$ is the sum of first n terms.

From $(n+1)^{th}$ to $(2n)^{th}$ term there are n terms.

From $(n+1)^{th}$ to $(2n)^{th}$ term the sum of the terms is

$$S_n = \frac{a_{n+1}(1-r^n)}{1-r}$$

$$a^{n+1} = ar^{n+1-1} = ar^n$$

Therefore, the required ratio is $= \frac{a(1-r^n)}{1-r} \times \frac{1-r}{ar^n(1-r^n)} = \frac{1}{r^n}$

Therefore, $\frac{1}{r^n}$ is the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term .

25. If $\mathbf{a,b,c}$ and $\mathbf{d}$ are in G.P. show that:

$$\mathbf{(a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2}$$

Ans: Let us assume a,b,c,d are in G.P.

Therefore,

$bc = ad \quad \ldots(1)$

$b^2 = ac \quad \ldots(2)$

$c^2 = bd \quad \ldots(3)$

To prove :

$$(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$$

R.H.S.

$= (ab+bc+cd)^2$

Substitute (1) in the equation.

$= (ab+ad+cd)^2$

$=\left(ab+d\left(a+c\right)\right)^2$

$=a^2b^2+2abd\left(a+c\right)+d^2\left(a+c\right)^2$

$=a^2b^2+2a^2bd+2acbd+d^2\left(a^2+2ac+c^2\right)$

Substitute (1) and (2) in the equation.

$=a^2b^2+2a^2c^2+2b^2c^2+d^2a^2+d^2b^2+d^2b^2+d^2c^2$

$=a^2b^2+a^2c^2+a^2c^2+b^2c^2+b^2c^2+d^2a^2+d^2b^2+d^2b^2+d^2c^2$

$=a^2b^2+a^2c^2+a^2d^2+b^2\times b^2+b^2c^2+b^2d^2+c^2b^2+c^2\times c^2+c^2d^2$

Substitute (2) and (3) in the equation and rearrange the terms.

$=a^2\left(b^2+c^2+d^2\right)+b^2\left(b^2+c^2+d^2\right)+c^2\left(b^2+c^2+d^2\right)$

$=\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$

$=\text{L.H.S.}$

Therefore, $\text{L.H.S.}=\text{R.H.S.}$

Therefore, $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=\left(ab+bc+cd\right)^2$ is proved.

26. Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Ans: Let the two numbers between 3 and 81 be G_1 and G_2 such that the series, $3,G_1,G_2,81$, forms a G.P.

Let the first term be a and the common ration be r of the G.P.

Therefore,

$81=\left(3\right)\left(r\right)^3$

$\Rightarrow r^3=27$

Taking the real roots, we get $r=3$.

When $r=3$,

$G_1=ar=\left(3\right)\left(3\right)=9$

$G_2=ar^2=\left(3\right)\left(3\right)^2=27$

Therefore, 9 and 27 are the two required numbers.

27. Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b .

Ans: The geometric mean of a and b is $\sqrt{ab}$.

According to conditions given in the question,

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$$

Square on both the sides.

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2}=ab$$

$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=(ab)\left(a^{2n}+2a^nb^n+b^{2n}\right)$$

$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2}=a^{2n+1}b+2a^{n+1}b^{n+1}+ab^{2n+1}$$

$$\Rightarrow a^{2n+2}+b^{2n+2}=a^{2n+1}b+ab^{2n+1}$$

$$\Rightarrow a^{2n+2}-a^{2n+1}b=ab^{2n+1}-b^{2n+2}$$

$$\Rightarrow a^{2n+1}(a-b)=b^{2n+1}(a-b)$$

$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1}=1=\left(\frac{a}{b}\right)^0$$

$$\Rightarrow 2n+1=0$$

$$\Rightarrow n=\frac{-1}{2}$$

28. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Ans: Let a and b be the two numbers.

$\sqrt{ab}$ is the geometric mean.

According to the conditions given in the question,

$$a+b=6\sqrt{ab} \qquad \ldots(1)$$

$$\Rightarrow (a+b)^2=36ab$$

Also,

$$(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$$

$$\Rightarrow a-b=\sqrt{32}\sqrt{ab}$$

$$=4\sqrt{2}\sqrt{ab} \qquad \ldots(2)$$

Add (1) and (2).

$$2a=\left(6+4\sqrt{2}\right)\sqrt{ab}$$

$$\Rightarrow a=\left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substitute $a=\left(3+2\sqrt{2}\right)\sqrt{ab}$ in equation (1) .

$b=6\sqrt{ab}-\left(3+2\sqrt{2}\right)\sqrt{ab}$

$\Rightarrow b=\left(3-2\sqrt{2}\right)\sqrt{ab}$

Divide a by b .

$$\frac{a}{b}=\frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Therefore, it is proved that the numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

29. If A and B be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A\pm\sqrt{\left(A+G\right)\left(A-G\right)}$.

Ans: Given: The two positive numbers between A.M. and G.M. are A and G.
Let a and b be these two positive numbers.

Therefore, $AM=A=\dfrac{a+b}{2}$ $\quad\ldots(1)$

$GM=G=\sqrt{ab}$ $\quad\ldots(2)$

Simplifying (1) and (2) , we get

$a+b=2A$ $\quad\ldots(3)$

$ab=G^2$ $\quad\ldots(4)$

Substituting (3) and (4) in the identity,

$\left(a-b\right)^2=\left(a+b\right)^2-4ab$,

We get

$\left(a-b\right)^2=4A^2-4G^2=4\left(A^2-G^2\right)$

$\left(a-b\right)^2=4\left(A+G\right)\left(A-G\right)$

$\left(a-b\right)=2\sqrt{\left(A+G\right)\left(A-G\right)}$ $\quad\ldots(5)$

Adding (3) and (5) we get ,

$2a=2A+2\sqrt{\left(A+G\right)\left(A-G\right)}$

$\Rightarrow a=A+\sqrt{\left(A+G\right)\left(A-G\right)}$

Substitute $a=A+\sqrt{\left(A+G\right)\left(A-G\right)}$ in equation (3).

$b = 2A - A - \sqrt{(A+G)(A-G)}$

$= A - \sqrt{(A+G)(A-G)}$

Therefore, $A \pm \sqrt{(A+G)(A-G)}$ are the two numbers.

30. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and n^{th} hour?

Ans: The number of bacteria after every hour will form a G.P. as it is given that the number of bacteria doubles every hour.

Given: $a = 30$ and $r = 2$

Therefore,

$a_3 = ar^2 = (30)(2)^2 = 120$

That is, 120 will be the number of bacteria at the end of 2^{nd} hour.

$a_5 = ar^4 = (30)(2)^4 = 480$

That is, 480 will be the number of bacteria at the end of 4^{th} hour.

$a_{n+1} = ar^n = (30)2^n$

Therefore, $30(2)^n$ will be the number of bacteria at the end of n^{th} hour.

31. What will Rs.500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

Ans: Rs.500 is the amount deposited in the bank.

The amount $=$ Rs.$500\left(1+\frac{1}{10}\right) =$ Rs.$500(1.1)$, at the end of first year.

The amount $=$ Rs.$500(1.1)(1.1)$, at the end of 2^{nd} year.

The amount $=$ Rs.$500(1.1)(1.1)(1.1)$, at the end of 3^{rd} year and so on.

Therefore, the amount at the end of 10 years

$=$ Rs.$500(1.1)(1.1)...(10\,\text{times})$

$=$ Rs.$500(1.1)^{10}$

32. If A.M. and G.M. of roots of a quadratic equation are 8 and 5 , respectively, then obtain the quadratic equation.

Ans: Let a and b be the root of the quadratic equation.

According to the conditions given in the question,

$$\text{A.M.} = \frac{a+b}{2} = 8$$

$$\Rightarrow a+b=16 \qquad \ldots(1)$$

$$\text{G.M.} = \sqrt{ab} = 5$$

$$\Rightarrow ab=25 \qquad \ldots(2)$$

The quadratic equation is given by the equation,

$$x^2 - x\,(\text{Sum of roots}) + (\text{Product of roots}) = 0$$

$$x^2 - x(a+b) + (ab) = 0$$

Substituting (1) and (2) in the equation.

$$x^2 - 16x + 25 = 0$$

Therefore, $x^2 - 16x + 25 = 0$ is the required quadratic equation.

Miscellaneous Exercise

1. If is a function satisfying $f(x+y)=f(x).f(y)$ for all $x,y \in N$, such that $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$ find the value of n.

Ans: According to the given conditions in the question,

$f(x+y)=f(x)\times f(y)$ for all $x, y, \in N$

$f(1)=3$

Let $x=y=1$.

Then,

$$f(1+1)=f(1+2)=f(1)f(2)=3\times 3=9$$

We can also write

$$f(1+1+1)=f(3)=f(1+2)=f(1)f(2)=3\times 9=27$$

$$f(4)=f(1+4)=f(1)f(3)=3\times 27=81$$

Both the first term and common ratio of $f(1), f(2), f(3), \ldots,$ that is $3, 9, 27, \ldots,$ that forms s G.P. is equal to 3

We know that, $S_n = \dfrac{a(r^n - 1)}{r-1}$

Given that, $\sum_{k=1}^{n} f(x) = 120$

Then,

$$120=\frac{3\left(3^n-1\right)}{3-1}$$

$$\Rightarrow 120=\frac{3}{2}\left(3^n-1\right)$$

$$\Rightarrow 3^n-1=80$$

$$\Rightarrow 3^n=80=3^4$$

$$\Rightarrow 3^n-1=80$$

$$n=4$$

Therefore, 4 is the value of n.

2. The sum of some terms of G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

Ans: Let 315 be the sum of n terms of the G.P.

We know that, $S_n=\dfrac{a\left(r^n-1\right)}{r-1}$

The first term a of the A.P. is 5 and the common difference r is 2.

Substitute the values of a and r in the equation

$$315=\frac{5\left(2^n-1\right)}{2-1}$$

$$\Rightarrow 2^n-1=63$$

$$\Rightarrow 2^n=63=\left(2\right)^2$$

$$\Rightarrow n=6$$

Therefore, the 6^{th} term is the last term of the G.P.

6^{th} term $=ar^{6-1}=\left(5\right)\left(2\right)^5=\left(5\right)\left(32\right)=160$

Therefore, 160 is the last term of the G.P and the number of terms is 6.

3. The first term of a G.P. is 1 . The sum of the third term and fifth term is 90. Find the common ratio of G.P.

Ans: Let the first term of the G.P. be a and the common ratio be r .

Then, $a=1$

$$a_3=ar^2=r^2$$

$$a_5=ar^4=r^4$$

Therefore,

$$r^2+r^4=90$$

$$\Rightarrow r^4+r^2-90=0$$

$$\Rightarrow r^2=\frac{-1+\sqrt{1+360}}{2}$$

$= \frac{-1+\sqrt{361}}{2}$

$= -10 \text{ or } 9$

$\Rightarrow r = \pm 3$

Therefore, ± 3 is the common ratio of the G.P.

4. The sum of the three numbers in G.P. is 56. If we subtract $1, 7, 21$ from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

Ans: Let a, ar and ar^2 be the three numbers in G.P.

According to the conditions given in the question,

$a + ar + ar^2 = 56$

$\Rightarrow a\left(1 + r + r^2\right) = 56 \qquad \ldots(1)$

An A.P. is formed by

$a - 1, ar - 7, ar^2 - 21$

Therefore,

$\left(ar - 7\right) - \left(a - 1\right) = \left(ar^2 - 21\right) - \left(ar - 7\right)b$

$\Rightarrow ar - a - 6 = ar^2 - ar - 14$

$\Rightarrow ar^2 - 2ar + a = 8$

$\Rightarrow ar^2 - ar - ar + a = 8$

$\Rightarrow a\left(r^2 + 1 - 2r\right) = 8$

$\Rightarrow a\left(r^2 - 1\right)^2 = 8 \qquad \ldots(2)$

Equating (1) and (2), we get

$\Rightarrow 7\left(r^2 - 2r + 1\right) = 1 + r + r^2$

$\Rightarrow 7r^2 - 14r + 7 - 1 - r - r^2$

$\Rightarrow 6r^2 - 15r + 6 = 0$

$\Rightarrow 6r^2 - 12r - 3r + 6 = 0$

$\Rightarrow 6\left(r - 2\right) - 3\left(r - 2\right) = 0$

$\Rightarrow \left(6r - 3\right)\left(r - 2\right) = 0$

Then, $8, 16$ and 32 are the three numbers when $r = 2$ and $32, 16$ and 8 are the numbers when $r = \frac{1}{2}$.

Therefore, $8, 16$ and 32 are the three required numbers in either case.

5. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

Ans: Let $T_1, T_2, T_3, T_4, ...T_{2n}$ be the G.P.

$2n$ is the number of terms.

According to the conditions given in the question,

$T_1 + T_2 + T_3 + ... + T_{2n} = 5[T_1 + T_3 + ... + T_{2n-1}]$

$\Rightarrow T_1 + T_2 + T_3 + ... + T_{2n} - 5[T_1 + T_3 + ... + T_{2n-1}] = 0$

$\Rightarrow T_2 + T_4 + ... + T_{2n} = 4[T_1 + T_3 + ... + T_{2n-1}]$

Let a, ar, ar^2, ar^3 be the G.P.

Therefore,

$$\frac{ar(r^n - 1)}{r - 1} = \frac{4 \times a(r^n - 1)}{r - 1}$$

$\Rightarrow ar = 4a$

$\Rightarrow r = 4$

Therefore, 4 is the common ratio of the G.P.

6: If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx} (x \neq 0)$ then show that a, b, c and d are in G.P.

Ans: Given ,

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$$

$\Rightarrow (a+bx)(b-cx) = (b+cx)(a-bx)$

$\Rightarrow ab - acx + b^2x - bcx^2 = ab - b^2x + -acx - bcx^2$

$\Rightarrow 2b^2x = 2acx$

$\Rightarrow b^2 = ac$

$\Rightarrow \frac{b}{a} = \frac{c}{b}$

It is also given that,

$$\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

$\Rightarrow (b+cx)(c-dx) = (b-cx)(c+dx)$

$\Rightarrow bc - bdx + c^2x - cdx^2 = bc + bdx - c^2x - cdx^2$

$\Rightarrow 2c^2x = 2bdx$

$\Rightarrow c^2 = bd$

$\Rightarrow \frac{c}{d} = \frac{d}{c}$

Equating both the results, we get

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{b}$$

Therefore, it is proved that a, b, c and d are in G.P.

7. Let S be the sum, P the product and R the sum of reciprocals of terms in a G.P. Prove that $P^2R^n = S^n$.

Ans: Let $a, ar, ar^2, ar^3 ... ar^{n-1}$ be the G.P.

According to the conditions given in the question,

$$S = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$P = a^n \times r^{1+2+...+n-1}$$

Since the sum of first n natural numbers is $n\frac{(n+1)}{2}$

$$\Rightarrow P = a^n r^{\frac{n(n-1)}{2}}$$

$$R = \frac{1}{a} + \frac{1}{ar} + ... + \frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1} + r^{n-2} + ...r + 1}{ar^{n-1}}$$

Since $1, r, ...r^{n-1}$ forms a G.P.,

$$\Rightarrow R = \frac{1\left(r^n - 1\right)}{(r-1)} \times \frac{1}{ar^{n-1}}$$

$$= \frac{r^n - 1}{ar^{n-1}(r-1)}$$

Then,

$$P^2R^n = a^{2n} r^{n(n-1)} \frac{\left(r^n - 1\right)^n}{a^n r^{n(n-1)} (r-1)^n}$$

$$= \frac{a^n \left(r^n - 1\right)^n}{(r-1)^n}$$

$$= \left[\frac{a\left(r^n - 1\right)}{(r-1)}\right]^n$$

$$= S^n$$

Therefore, $P^2R^n = S^n$.

8. If a,b,c,d are in G.P., prove that $\left(a^n + b^n\right), \left(b^n + c^n\right), \left(c^n + d^n\right)$ are in G.P

Ans: Given:

a, b, c and d are in G.P.

Therefore,

$b^2 = ac$
$c^2 = bd$
$ad = bc$
To prove:
$\left(a^n + b^n\right), \left(b^n + c^n\right), \left(c^n + d^n\right)$ are in G.P.

That is, $\left(b^n + c^n\right)^2 = \left(a^n + b^n\right), \left(c^n + d^n\right)$

Then,

L.H.S $= \left(b^n + c^n\right)^2$

$= b^{2n} + 2b^n c^n + c^{2n}$

$= \left(b^2\right)^n + 2b^n c^n + \left(c^2\right)^n$

$= \left(ac\right)^n + 2b^n c^n + \left(bd\right)^n$

$= a^n c^n + b^n c^n + b^n c^n + b^n d^n$
$= a^n c^n + b^n c^n + a^n d^n + b^n d^n$
$= c^n \left(a^n + b^n\right) + d^n \left(a^n + b^n\right)$

$= \left(a^n + b^n\right)\left(a^n + d^n\right)$

$=$R.H.S
Therefore,

$\left(b^n + c^n\right)^2 = \left(a^n + b^n\right)\left(c^n + d^n\right)$

Therefore, $\left(b^n + c^n\right), \left(b^n + c^n\right)$ and $\left(c^n + d^n\right)$ are in G.P.

9. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$, where a, b, c, d form a G.P. Prove that $\left(q + p\right) : \left(q - p\right) = 17 : 15$.

Ans: Given: a and b are the roots of $x^2 - 3x + p = 0$.
Therefore,
$a + b = 3$ and $ab = p \quad \ldots(1)$

We also know that c and d are the roots of $x^2 - 12x + q = 0$.
Therefore,
$c + d = 12$ and $cd = q \quad \ldots(2)$

Also, a, b, c, d are in G.P.
Let us take $a = x, b = xr, c = xr^2$ and $d = xr^3$
We get from (1) and (2) that,
$x + xr = 3$
$\Rightarrow x\left(1 + r\right) = 3$
Also,
$xr^2 + xr^3 = 12$
$\Rightarrow xr^2 + \left(1 + r\right) = 12$

Divide both the equations obtained.

$$\frac{xr^2(1+r)}{x(1+r)}=\frac{12}{3}$$

$$\Rightarrow r^2=4$$

$$\Rightarrow r=\pm 2$$

$x=\frac{3}{1+2}=\frac{3}{3}=1$, when $r=2$ and

$x=\frac{3}{1-2}=\frac{3}{-1}=-3$, when $r=-2$.

Case I:

$ab=x^2r=2$, $cd=x^2r^5=32$ when $r=2$ and $x=1$.

Therefore,

$$\frac{q+p}{q-p}=\frac{32+2}{32-2}=\frac{34}{30}=\frac{17}{15}$$

$$\Rightarrow (q+p):(q-p)=17:15$$

Case II:

$ab=x^2r=18$, $cd=x^2r^5=-288$ when $r=-2$ and $x=-3$.

Therefore,

$$\frac{q+p}{q-p}=\frac{-288-18}{-288+18}=\frac{-306}{-270}=\frac{17}{15}$$

$$\Rightarrow (q+p):(q-p)=17:15$$

Therefore, it is proved that $(q+p):(q-p)=17:15$ as we obtain the same for both the cases.

10. The ratio of the A.M and G.M. of two positive numbers a and b is $m:n$. Show that $a:b=\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$.

Ans: Let a and b be the two numbers.

The arithmetic mean, A.M $=\frac{a+b}{2}$ and the geometric mean, G.M $=\sqrt{ab}$

According to the conditions given in the question,

$$\frac{a+b}{2\sqrt{ab}}=\frac{m}{n}$$

$$\Rightarrow \frac{(a+b)^2}{4(ab)}=\frac{m^2}{n^2}$$

$$\Rightarrow (a+b)=\frac{4abm^2}{n^2}$$

$$\Rightarrow (a+b)=\frac{2\sqrt{ab}m}{n} \qquad \ldots(1)$$

Using the above equation in the identity $(a-b)^2=(a+b)^2-4ab$, we obtain

$$(a-b)^2=\frac{4abm^2}{n^2}-4ab=\frac{4ab\left(m^2-n^2\right)}{n^2}$$

$$\Rightarrow (a-b)=\frac{2\sqrt{ab}\sqrt{m^2-n^2}}{n} \qquad \ldots(2)$$

Add equation (1) and (2)

$$2a=\frac{2\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

$$\Rightarrow a=\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

Substitute in (1) the value of a.

$$b=\frac{2\sqrt{ab}}{n}m-\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)$$

$$=\frac{\sqrt{ab}}{n}m-\frac{\sqrt{ab}}{n}\sqrt{m^2-n^2}$$

$$=\frac{\sqrt{ab}}{n}\left(m-\sqrt{m^2-n^2}\right)$$

Therefore,

$$a:b=\frac{a}{b}=\frac{\frac{\sqrt{ab}}{n}\left(m+\sqrt{m^2-n^2}\right)}{\frac{\sqrt{ab}}{n}\left(m-\sqrt{m^2-n^2}\right)}=\frac{\left(m+\sqrt{m^2-n^2}\right)}{\left(m-\sqrt{m^2-n^2}\right)}$$

Therefore, it is proved that $a:b=\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$.

11. Find the sum of the following series up to n terms:

i. $5+55+555+\ldots$

Ans: Let $S_n=5+55+555\ldots$ to n terms.

$$=\frac{5}{9}[9+99+999+\ldots \text{ to } n \text{ terms}]$$

$$=\frac{5}{9}[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\ldots \text{ to } n \text{ terms}]$$

$$=\frac{5}{9}[(10+10^2+10^3\ldots \text{ to } n \text{ terms})-(1+1+ \text{ to } n \text{ terms})]$$

$$=\frac{5}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$$

$$=\frac{5}{9}\left[\frac{10\left(10^{n}-1\right)}{9}-n\right]$$

$$=\frac{50}{81}\left(10^{n}-1\right)-\frac{5n}{9}$$

Therefore, the sum of n terms of the given series is $\frac{50}{81}\left(10^{n}-1\right)-\frac{5n}{9}$.

ii. .6+ .66+ .666.+ ...

Ans: Let $S_n = 0.6+0.66+0.666+$ to n terms.

$$=6[0.1+0.11+0.111+\ldots \text{ to } n \text{ terms}]$$

$$=\frac{6}{9}[0.9+0.99+0.999+\ldots \text{ to } n \text{ terms}]$$

$$=\frac{6}{9}[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^{2}}\right)+\left(1-\frac{1}{10^{3}}\right)+\ldots \text{ to } n \text{ terms}]$$

$$=\frac{2}{3}[(1+1+\ldots \text{ to } n \text{ terms})-\frac{1}{10}(1+\frac{1}{10}+\frac{1}{10^{2}} \text{ to } n \text{ terms})]$$

$$=\frac{2}{3}\left[n-\frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^{n}}{1-\frac{1}{10}}\right)\right]$$

$$=\frac{2}{3}n-\frac{2}{30}\times\frac{10}{9}\left(1-10^{-n}\right)$$

$$=\frac{2}{3}n-\frac{2}{27}\left(1-10^{-n}\right)$$

Therefore, the sum of n terms of the given series is $\frac{2}{3}n-\frac{2}{27}\left(1-10^{-n}\right)$.

12. Find the 20^{th} term of the series $2\times4+4\times6+6\times8+\ldots+n$ terms.

Ans: $2\times4+4\times6+6\times8+\ldots+n$ is the given series,

Therefore the n^{th} term $a_n=2n\times\left(2n+2\right)=4n^{2}+4n$

Then,

$$a_{20}=4\left(20\right)^{2}+4\left(20\right)$$

$$=4\left(400\right)+80$$

$$=1600+80$$

$$=1680$$

Therefore, 1680 is the 20^{th} term of the series.

13. A farmer buys a used tractor for Rs.12000. He pays Rs.6000 cash and agrees to pay the balance in annual installments of Rs.500 plus 12% interest on the unpaid amount. How much will be the tractor cost him?

Ans: It is given that Rs. 6000 is paid in cash by the farmer.

Therefore, the unpaid amount is given by

$\text{Rs.}12000 - \text{Rs.}6000 = \text{Rs.}6000$

According to the conditions given in the question, the interest to be paid annually by the farmer is

$12\% \text{ of } 6000$, $12\% \text{ of } 5500$, $12\% \text{ of } 5000 ... 12\% \text{ of } 500$

Therefore, the total interest to be paid by the farmer

$= 12\% \text{ of } 6000 + 12\% \text{ of } 5500 + 12\% \text{ of } 5000 + ... + 12\% \text{ of } 500$

$= 12\% \text{ of } \left(6000 + 5500 + 5000 + ... + 500\right)$

$= 12\% \text{ of } \left(500 + 1000 + 1500 + ... + 6000\right)$

With both the first term and common difference equal to 500, the series $500, 1000, 1500 ... 6000$ is an A.P.

Let n be the number of terms of the A.P.

Therefore,

$6000 = 500 + \left(n - 1\right)500$

$\Rightarrow 1 + \left(n - 1\right) = 12$

$\Rightarrow n = 12$

Therefore, the sum of the given A.P.

$= \frac{12}{2}\left[2\left(500\right) + \left(12 - 1\right)\left(500\right)\right]$

$= 6\left[1000 + 5500\right]$

$= 6\left(6500\right)$

$= 39000$

Therefore, the total interest to be paid by the farmer

$= 12\% \text{ of } \left(500 + 1000 + 1500 + ... + 6000\right)$

$= 12\% \text{ of Rs.}39000$

$= \text{Rs.}4680$

Therefore, the total cost of tractor

$= (\text{Rs.}12000 + \text{Rs.}4680)$

$= \text{Rs.}16680$

Therefore, the total cost of the tractor is Rs.16680.

14. Shamshad Ali buys a scooter for Rs. 22000. He pays Rs. 4000 cash and agrees to pay the balance in annual installment of Rs. 1000 plus 10% interest on the unpaid amount. How much will the scooter cost him?

Ans: It is given that for Rs. 22000 Shamshad Ali buys a scooter and Rs. 4000 is paid in cash.

Therefore, the unpaid amount is given by

$\text{Rs.}\,22000 - \text{Rs.}\,4000 = \text{Rs.}18000$

According to the conditions given in the question, the interest to be paid annually is

10% of 18000 , 10% of 17000 , 10% of $16000 ... 10\%$ of 1000

Therefore, the total interest to be paid by the farmer

$= 10\% \text{ of } 18000 + 10\% \text{ of } 17000 + 10\% \text{ of } 16000 + ... + 10\% \text{ of } 1000$

$= 10\% \text{ of } (18000 + 17000 + 16000 + ... + 1000)$

$= 10\% \text{ of } (1000 + 2000 + 3000 + ... + 18000)$

With both the first term and common difference equal to 1000, the series $1000, 2000, 3000 ... 18000$ is an A.P.

Let n be the number of terms of the A.P.

Therefore,

$18000 = 1000 + (n-1)1000$

$\Rightarrow 1 + (n-1) = 18$

$\Rightarrow n = 18$

Therefore, the sum of the given A.P.

$= \frac{18}{2}\left[2(1000) + (18-1)(1000)\right]$

$= 9[2000 + 17000]$

$= 9(19000)$

$= 171000$

Therefore, the total interest to be paid

$= 10\% \text{ of } (18000 + 17000 + 16000 + ... + 1000)$

$= 10\% \text{ of Rs.}171000$

$= \text{Rs.}17100$

Therefore, the total cost of scooter

$= (\text{Rs.}\,22000 + \text{Rs.}17100)$

$= \text{Rs.}\,39100$

Therefore, the total cost of the scooter is Rs. 39100 .

15. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spent on the postage when 8^{th} set of letter is mailed.

Ans: $4, 4^2, ... 4^8$ is the number of letters mailed and it forms a G.P.

The first term $a = 4$, the common ratio $r = 4$ and the number of terms $n = 8$ of the G.P.

We know that the sum of n terms of a G.P. is

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

Therefore,

$$S_8 = \frac{4\left(4^8 - 1\right)}{4 - 1}$$

$$= \frac{4\left(65536 - 1\right)}{3}$$

$$= \frac{4\left(65535\right)}{3}$$

$$= 4\left(21845\right)$$

$$= 87380$$

50 paisa is the cost to mail one letter.

Therefore,

Cost of mailing 87380 letters $= \text{Rs.}\,87380 \times \frac{50}{100}$ $= \text{Rs.}\,43690$

Therefore, Rs. 43690 is the amount spent when 8^{th} set of letter is mailed.

16. A man deposited Rs. 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

Ans: Rs. 10000 is deposited by the man in a bank at the rate of 5% simple interest annually

$$= \frac{5}{100} \times \text{Rs.}\,10000 = \text{Rs.}\,500$$

Therefore,

$10000 + 500 + 500 + ... + 500$ is the interest in 15^{th} year. (500 is 14 added times)

Therefore, the amount in 15^{th} year

$$= \text{Rs.}\,10000 + 14 \times \text{Rs.}\,500$$

$$= \text{Rs.}\,10000 + \text{Rs.}\,7000$$

$$= \text{Rs.}\,17000$$

$\text{Rs.}10000+500+500+...+500$ is the amount after 20 years. (500 is 20 added times)

Therefore, the amount after 20 years

$=\text{Rs.}10000+20\times\text{Rs.}500$

$=\text{Rs.}10000+\text{Rs.}10000$

$=\text{Rs.}20000$

The total amount after 20 years is Rs. 20000.

17. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Ans: The cost of the machine is Rs. 15625.

Every year machine depreciates by 20%.

Therefore, 80% of the original cost ,i.e., $\frac{4}{5}$ of the original cost is its value after every year.

Therefore, the value at the end of 5 years

$=15626\times\frac{4}{5}\times\frac{4}{5}\times...\times\frac{4}{5}$

$=5\times1024$

$=5120$

Therefore, Rs. 5120 is the value of the machine at the end of 5 years.

18. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on. It took 8 more days to finish the work. Find the number of days in which the work was completed.

Ans: Let the number of days in which 150 workers finish the work be x.

According to the conditions given in the question,

$150x=150+146+142+...(x+8)$ terms

With first term $a=146$, common difference $d=-4$ and number of turns as $(x+8)$, the series $150+146+142+...(x+8)$ terms is an A.P.

$\Rightarrow 150x=\frac{(x+8)}{2}\left[2(150)+(x+8-1)(-4)\right]$

$\Rightarrow 150x=(x+8)\left[150+(x+7)(-2)\right]$

$\Rightarrow 150x=(x+8)(150-2x-14)$

$\Rightarrow 150x=(x+8)(136-2x)$

$\Rightarrow 75x=(x+8)(68-x)$

$\Rightarrow 75x=68x-x^2+544-8x$

$\Rightarrow x^2+75x-60x-544=0$

$\Rightarrow x^2+15x-544=0$

$\Rightarrow x^2 + 32x - 17x - 544 = 0$
$\Rightarrow x(x+32) - 17(x+32) = 0$
$\Rightarrow (x-17)(x+32) = 0$
$\Rightarrow x = 17$ or $x = -32$
We know that x cannot be negative.
So, $x = 17$.
Therefore, 17 is the number of days in which the work was completed. Then the required number of days $= (17 + 8) = 25$.