linear inequalities

5
Chapter

Exercise 5.1

1. Solve $24x<100$, when

Ans: The given inequality is $24x<100$.

$$\Rightarrow \frac{24x}{24}<\frac{100}{24}$$

[Dividing both sides by same positive number]

$$\Rightarrow x<\frac{25}{6}$$

(i) x is a natural number-

1,2,3 and 4 are the only natural numbers that are smaller than 25\ 6 . When x is a natural number, the above inequality's solutions are 1,2,3 and 4. As a result, the solution set in this example is(1, 2 ,3 ,4).

(ii) x is an integer-

When x is an integer.

Suppose the given x is an integer then by the integer definition we have $(-\infty,\infty)$ which means there is no end at positive and negative values and then we have $\{\ldots,-3,-2,-1,0,1,2,3,\ldots.\}\}$

Thus, the value of x as integers is $\{\ldots,-3,-2,-1,0,1,2,3,4\}$ after four which is a greater value.

2. Solve $-12x > 30$, when

Ans: The given inequality is $-12x > 30$.

$\Rightarrow \frac{-12x}{-12} < \frac{30}{-12}$

[Dividing both sides by same negative number]

$\Rightarrow x < -\frac{5}{2}$

(i) x is a natural number-There isn't a natural number that isn't less than $\left(-\frac{5}{2}\right)$. As a result there is no solution to the given inequality when it is a natural number.

(ii) x is an integer-

The integers less than $\left(-\frac{5}{2}\right)$ are$\ldots -5,-4,-3$. As a result, when an integer is an integer, the above inequality's solutions are $\ldots\ldots -5,-4,-3$ As a result, the solution set in this situation is $\{\ldots\ldots,-5,-4,-3\}$.

3. Solve $5x-3<7$, when

Ans: The given inequality is $5x-3<7$.

$\Rightarrow 5x-3+3<7+3$

$\Rightarrow 5x<10$

$\Rightarrow \frac{5x}{5}<\frac{10}{5}$

$\Rightarrow x<2$

(i) x is an integer-

The integers less than 2 are ..., $-4,-3,-2,-1,0,1$. As a result, the solutions to the above inequality are when is an integer$\ldots,-4,-3,-2,-1,0,1$. As a result, the solution set in this example is $\{\cdots,-4,-3,-2,-1,0,1\}$.

(ii) $\mathbf{x}$ is a real number-

When x is a real number, the inequality's solutions are x<2, that is, all real values x less than 2. As a result, the given inequality's solution set is $x\in(-\infty,2)$.

4. Solve $3x+8>2$, when

Ans: The given inequality is $3x+8>2$ $3x+8>2$

$\Rightarrow 3x+8-8>2-8$

$\Rightarrow 3x>-6$

$\Rightarrow \frac{3x}{3}>\frac{-6}{3}$

$\Rightarrow x>-2$

(i) $\mathbf{x}$ is an integer-

The integers higher than -2 are $-1,0,1,2,\ldots$ Hence, when is an integer, the solutions of the above inequality are $-1,0,1,2,\ldots$ Hence, the solution set in this case is $\{-1,0,1,2,\ldots\}$ -

(ii) $\mathbf{x}$ is a real number-

When x is a real number, the above inequality's solutions are all real numbers that are bigger than -2. As a result, the solution set in this situation is $(-2,\infty)$.

5. Solve the given inequality for real $x: 4x+3<5x+7$

Ans: $4x+3<5x+7$

$\Rightarrow 4x+3-7<5x+7-7$

$\Rightarrow 4x-4<5x$

$\Rightarrow 4x-4-4x<5x-4x$

$\Rightarrow -4<x$

As a result, all real numbers x bigger than -4 are solutions to the given inequality. As a result, the given inequality's solution set is $(-4,\infty)$.

6. Solve the given inequality for real $x: 3x-7>5x-1$

Ans: $3x-7>5x-1$

$\Rightarrow 3x-7+7>5x-1+7$

$\Rightarrow 3x>5x+6$

$\Rightarrow 3x-5x>5x+6-5x$

$\Rightarrow -2x>6$

$\Rightarrow \frac{-2x}{-2}<\frac{6}{-2}$

$\Rightarrow x<-3$

As a result, all real numbers x less than -3 are solutions to the given inequality. As a result, the given inequality's solution set is $(-\infty,-3)$.

7. Solve the given inequality for real $x: 3(x-1)\leqslant 2(x-3)$

Ans: $3(x-1)\leqslant 2(x-3)$

$\Rightarrow 3x-3 \leqslant 2x-6$

$\Rightarrow 3x-3+3 \leqslant 2x-6+3$

$\Rightarrow 3x \leqslant 2x-3$

$\Rightarrow 3x-2x \leqslant 2x-3-2x$

$\Rightarrow x \leqslant -3$

As a result, any real numbers x less than or equal -3 to are solutions to the specified inequality. As a result, the given inequality's solution set is $(-\infty,-3]$.

8. Solve the given inequality for real $x: 3(2-x) \geqslant 2(1-x)$

Ans: $3(2-x) \geqslant 2(1-x)$

$\Rightarrow 6-3x \geqslant 2-2x$

$\Rightarrow 6-3x+2x \geqslant 2-2x+2x$

$\Rightarrow 6-x \geqslant 2$

$\Rightarrow 6-x-6 \geqslant 2-6$

$\Rightarrow -x \geqslant -4$

$\Rightarrow x \leqslant 4$

As a result, the solutions to the following inequality are all real values x higher than or equal to 4. As a result, the given inequality's solution set is $(-\infty,4]$.

9. Solve the given inequality for real $x: x+\frac{x}{2}+\frac{x}{3}<11$

Ans: $x+\frac{x}{2}+\frac{x}{3}<11$

$$\Rightarrow x\left(1+\frac{1}{2}+\frac{1}{3}\right)<11$$

$$\Rightarrow x\left(\frac{6+3+2}{6}\right)<11$$

$$\Rightarrow \frac{11x}{6}<11$$

$$\Rightarrow \frac{11x}{6\times 11}<\frac{11}{11}$$

$$\Rightarrow \frac{x}{6}<1$$

$$\Rightarrow x<6$$

As a result, all real numbers x less than 6 are solutions to the specified inequality. As a result, the given inequality's solution set is $(-\infty,6)$.

10. Solve the given inequality for real $x: \frac{x}{3}>\frac{x}{2}+1$

Ans: $\frac{x}{3}>\frac{x}{2}+1$

$$\Rightarrow \frac{x}{3}-\frac{x}{2}>1$$

$$\Rightarrow \frac{2x-3x}{6}>1$$

$$\Rightarrow -\frac{x}{6}>1$$

$$\Rightarrow -x>6$$

$$\Rightarrow x<-6$$

As a result, all real numbers less than equal the specified inequality's solution. As a result, the solution set for the given inequality is $(-\infty,-6)$.

11. Solve the given inequality for real $x:\dfrac{3(x-2)}{5}\leqslant\dfrac{5(2-x)}{3}$

Ans: $\dfrac{3(x-2)}{5}\leqslant\dfrac{5(2-x)}{3}$

$\Rightarrow 9(x-2)\leqslant 25(2-x)$

$\Rightarrow 9x-18\leqslant 50-25x$

$\Rightarrow 9x-18+25x\leqslant 50$

$\Rightarrow 34x-18\leqslant 50$

$\Rightarrow 34x\leqslant 50+18$

$\Rightarrow 34x\leqslant 68$

$\Rightarrow \dfrac{34x}{34}\leqslant\dfrac{68}{34}$

$\Rightarrow x\leqslant 2$

As a result, any real numbers x less than or equal to 2 are solutions of the above inequality, and hence the solution set of the given inequality is $(-\infty,2]$.

12. Solve the given inequality for real $x:\dfrac{1}{2}\left(\dfrac{3x}{5}+4\right)\geqslant\dfrac{1}{3}(x-6)$

Ans: $\dfrac{1}{2}\left(\dfrac{3x}{5}+4\right)\geqslant\dfrac{1}{3}(x-6)$

$\Rightarrow 3\left(\dfrac{3x}{5}+4\right)\geqslant 2(x-6)$

$$\Rightarrow \frac{9x}{5}+12 \geqslant 2x-12$$

$$\Rightarrow 12+12 \geqslant 2x-\frac{9x}{5}$$

$$\Rightarrow 24 \geqslant \frac{10x-9x}{5}$$

$$\Rightarrow 24 \geqslant \frac{x}{5}$$

$$\Rightarrow 120 \geqslant x$$

As a result, the solutions of the following inequality are all real values x less than or equal to 120. As a result, the given inequality's solution set is $(-\infty,120]$.

13. Solve the given inequality for real $x: 2(2x+3)-10<6(x-2)$

Ans: $2(2x+3)-10<6(x-2)$

$$\Rightarrow 4x+6-10<6x-12$$

$$\Rightarrow 4x-4<6x-12$$

$$\Rightarrow -4+12<6x-4x$$

$$\Rightarrow 8<2x$$

$$\Rightarrow 4<x$$

As a result, any real numbers x bigger than 4 are solutions to the specified inequality. As a result, the given inequality's solution set is $(4,-\infty)$.

14. Solve the given inequality for real $x: 37-(3x+5) \geqslant 9x-8(x-3)$

Ans: $37-(3x+5) \geqslant 9x-8(x-3)$

$\geqslant$

$\Rightarrow 37-3x-5 \quad 9x-8x+24$

$\Rightarrow 32-3x \geqslant x+24$

$\Rightarrow 32-24 \geqslant x+3x$

$\Rightarrow 8 \geqslant 4x$

$\Rightarrow 2 \geqslant x$

As a result, the solutions of the following inequality are all real values x less than or equal to 2. As a result, the given inequality's solution set is $(-\infty, 2]$.

15. Solve the given inequality for real $x : \frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Ans: $\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

$\Rightarrow \frac{x}{4} < \frac{5(5x-2)-3(7x-3)}{15}$

$\Rightarrow \frac{x}{4} < \frac{25x-10-21x+9}{15}$

$\Rightarrow \frac{x}{4} < \frac{4x-1}{15}$

$\Rightarrow 15x < 4(4x-1)$

$\Rightarrow 15x < 16x-4$

$\Rightarrow 4 < 16x-15x$

$\Rightarrow 4 < x$

As a result, any real numbers x bigger than 4 are solutions to the specified inequality. As a result, the given inequality's solution set is $(4, \infty)$.

16. Solve the given inequality for real $x: \frac{(2x-1)}{3} \geqslant \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Ans: $\frac{(2x-1)}{3} \geqslant \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

$\Rightarrow \frac{(2x-1)}{3} \geqslant \frac{5(3x-2)-4(2-x)}{20}$

$\Rightarrow \frac{(2x-1)}{3} \geqslant \frac{15x-10-8+4x}{20}$

$\Rightarrow \frac{(2x-1)}{3} \geqslant \frac{19x-18}{20}$

$\Rightarrow 20(2x-1) \geqslant 3(19x-18)$

$\Rightarrow 40x-20 \geqslant 57x-54$

$\Rightarrow -20+54 \geqslant 57x-40x$

$\Rightarrow 34 \geqslant 17x$

$\Rightarrow 2 \geqslant x$

As a result, the solutions of the following inequality are all real values x less than or equal to 2. As a result, the given inequality's solution set is $(-\infty, 2]$.

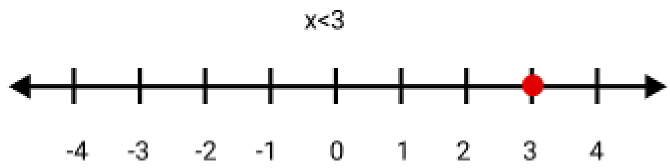
17. Solve the given inequality and show the graph of the solution on number line:

$\mathbf{3x-2<2x+1}$

Ans: $3x-2<2x+1$

$\Rightarrow 3x-2x<1+2$

The following is a graphical illustration of the solutions to the given inequality:



18. **Solve the given inequality and show the graph of the solution on number line:**

$\mathbf{5x-3\geqslant 3x-5}$

Ans: $5x-3\ \ 3x-5\geqslant$

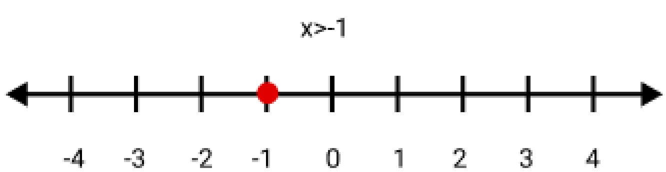
$\Rightarrow 5x-3x\geqslant -5+3$

$\Rightarrow 2x\geqslant -2$

$\Rightarrow \frac{2x}{2}\geqslant \frac{-2}{2}$

$\Rightarrow x\geqslant -1$

The answers to the given inequality are shown graphically as follows :



19. **Solve the given inequality and show the graph of the solution on number line:**

$\mathbf{3(1-x)<2(x+4)}$

Ans: $3(1-x) < 2(x+4)$

$\Rightarrow 3-3x < 2x+8$

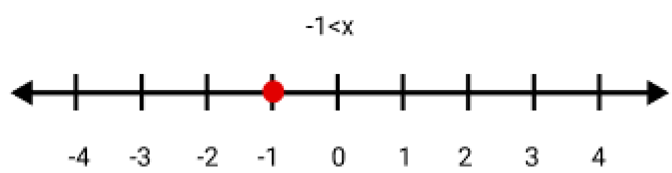
$\Rightarrow 3-8 < 2x+3x$

$\Rightarrow -5 < 5x$

$\Rightarrow \frac{-5}{5} < \frac{5x}{5}$

$\Rightarrow -1 < x$

The following is a graphical illustration of the solutions to the given inequality:



20. Solve the given inequality and show the graph of the solution on number line:

$$\boldsymbol{\frac{x}{2} \geqslant \frac{(5x-2)}{3} - \frac{(7x-3)}{5}}$$

Ans:

$$\frac{x}{2} \geqslant \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

$$\Rightarrow \frac{x}{2} \geqslant \frac{5(5x-2)-3(7x-3)}{15}$$

$$\Rightarrow \frac{x}{2} \geqslant \frac{25x-10-21x+9}{15}$$

$\Rightarrow \frac{x}{2} \geqslant \frac{4x-1}{15}$

$\Rightarrow 15x \geqslant 2(4x-1)$

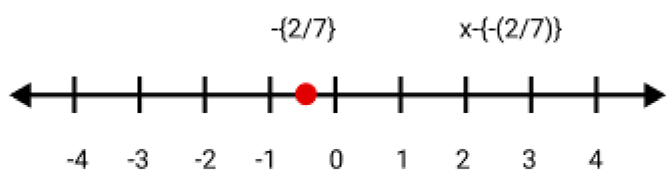
$\Rightarrow 15x \geqslant 8x-2$

$\Rightarrow 15x-8x \geqslant 8x-2-8x$

$\Rightarrow 7x \geqslant -2$

$\Rightarrow x \geqslant -\frac{2}{7}$

The following is a graphical illustration of the solutions to the given inequality.



21. Ravi obtained 70 and 75 marks in first two-unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Ans: Let x represent Ravi's score on the third unit test. Because the student must have a grade point average of at least 60, $\frac{70+75+x}{3} \geqslant 60$

$\Rightarrow 145+x \geqslant 180$

$\Rightarrow x \geqslant 180-145$

$\Rightarrow x \geqslant 35$

As a result, the student must score at least 35 points to achieve a 60-point average.

22. To receive Grade ' A^{*} in a course, one must obtain an average of $90 \mathrm{marks}$ or more in five examinations (each of 100 marks). If Sunita"s marks in first four examinations are $87,92,94$ and 95, find minimum marks that Sunita must obtain in fifth examination to get grade ' A " in the course.

Ans: Let x represent Sunita's grade in the fifth examination. She must receive an average of 90 or above in five examinations in order to receive a grade of in the course,

$$\frac{87+92+94+95+x}{5} \geqslant 90$$

$$\Rightarrow \frac{368+x}{5} \geqslant 90$$

$$\Rightarrow 368+x \geqslant 450$$

$$\Rightarrow x \geqslant 450-368$$

$$\Rightarrow x \geqslant 82$$

Sunita must therefore achieve a score of at least 82 in the fifth examination.

23. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11 .

Ans: Let x be the lesser of the two odd positive integers that follow. Then there's x + 2 as the other integer. Because both integers are less than 10 , $x+2<10$

$$\Rightarrow x<10-2$$

$$\Rightarrow x<8 \ldots .(t)$$

Furthermore, the sum of the two integers exceeds 11. $\therefore x+(x+2)>11$

$$\Rightarrow 2 x+2>11$$

$$\Rightarrow 2 x>11-2$$

$\Rightarrow 2x > 9$

$\Rightarrow x > \frac{9}{2}$

$\Rightarrow x > 4.5$

We get I and (ii) from I and (ii).

Because x is an odd number, the values 5 and 7 can be used.

As a result, the required pairs are $(5,7)$ and $(7,9)$.

24. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23 .

Ans: Let be the smaller of the two even positive integers that follow. The other integer is x + 2

because both integers are greater than 5, $x > 5 \ldots\ldots(1)$

Also, the sum of the two integers is less than 23 $x+(x+2) < 23$

$\Rightarrow 2x+2 < 23$

$\Rightarrow 2x < 23-2$

$\Rightarrow 2x < 21$

$\Rightarrow x < \frac{21}{2}$

$\Rightarrow x < 10.5 \ldots\ldots(2)$

From (1) and (2), we obtain $5 < x < 10.5$

Because x is an even number, it can have any of the following values: 6,8, or 10.

As a result, the required pairs are $(6,8),(8,10)$ and $(10,12)$.

25. The longest side of a triangle is 3 times the shortest side and the third side is $2 \mathrm{~cm}$ shorter than the longest side. If the perimeter of the triangle is at least $61 \mathrm{~cm}$, find the minimum length of the shortest side.

Ans: Determine the length of the triangle's shortest side xcm. Then, length of the longest side $=3 \mathrm{xcm}$ Length of the third side $(3 x-2) \mathrm{cm}$ Since the perimeter of the triangle is at least $61 \mathrm{~cm}$, $x c m+3 x c m+(3 x-2) c m \geqslant 61 \mathrm{~cm}$

$\Rightarrow 7 x-2 \geqslant 61$

$\Rightarrow 7 x \geqslant 61+2$

$\Rightarrow 7 x \geqslant 63$

$\Rightarrow \frac{7 x}{7} \geqslant \frac{63}{7}$

$\Rightarrow x \geqslant 9$

As a result, the shortest side's minimal length is $9 \mathrm{~cm}$.

26. A man wants to cut three lengths from a single piece of board of length $91 \mathrm{~cm}$. The second length is to be $3 \mathrm{~cm}$ longer than the shortest and the third length is to be twice as bang as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least $5 \mathrm{~cm}$ longer than the second? [Hint: It x is the length of the shortest board, then $x,(x+3)$ and $2 x$ are the lengths of the second and third piece, respectively. Thus, $x=(x+3)+2 x \leqslant 91$ and $2 x \geqslant(x+3)+5]$

Ans: The shortest piece's length should be xcm. Then, length of the second piece and the third piece are $(\mathrm{x}+3) \mathrm{cm}$ and $2 \mathrm{xcm}$ respectively. Since the three lengths are to be cut from a single piece of board of length $91 \mathrm{~cm}$, $x \mathrm{~cm}+(x+3) \mathrm{cm}+2 x \mathrm{~cm} \leqslant 91 \mathrm{~cm}$

$\Rightarrow 4 x+3 \leqslant 91$

$\Rightarrow 4 x \leqslant 91-3$

$\Rightarrow 4 x \leqslant 88$

$\Rightarrow \frac{4 x}{4} \leqslant \frac{88}{4}$

$\Rightarrow x \leqslant 22 \ldots .(1)$

In addition, the third component is at least 5 cm as long as the second. $\therefore 2x \geqslant (x+3)+5$

$\Rightarrow 2x \geqslant x+8$

$\Rightarrow x \geqslant 8$

From (1) and (2), we obtain $8 \leqslant x \leqslant 22$

As a result, the smallest board's potential length is larger than or equal to but less than or equal to 22 cm.

Miscellaneous Exercise

1. Solve the inequality $2 \leqslant 3x-4 \leqslant 5$

Ans: $2 \leqslant 3x-4 \leqslant 5$

$\Rightarrow 2+4 \leqslant 3x-4+4 \leqslant 5+4$

$\Rightarrow 6 \leqslant 3x \leqslant 9$

$\Rightarrow 2 \leqslant x \leqslant 3$

As a result, the solutions of the following inequality are all real values higher than or equal to 2 but less than or equal to 3. For the given inequality, the solution set is [2,3].

2. Solve the inequality $6 \leqslant -3(2x-4) < 12$

Ans: $6 \leqslant -3(2x-4) < 12$

$\Rightarrow 2 \leqslant -(2x-4) < 4$

$\Rightarrow -2 \geqslant 2x-4 > -4$

$\Rightarrow 4-2 \geqslant 2x > 4-4$

$\Rightarrow 2 \geqslant 2x > 0$

$\Rightarrow 1 \geqslant x > 0$

As a result, the set of solutions for the given inequality is [1 ,0).

3. **Solve the inequality** $-3 \leqslant 4 - \frac{7x}{2} \leqslant 18$

Ans: $-3 \leqslant 4 - \frac{7x}{2} \leqslant 18$

$\Rightarrow -3 - 4 \leqslant -\frac{7x}{2} \leqslant 18 - 4$

$\Rightarrow -7 \leqslant -\frac{7x}{2} \leqslant 14$

$\Rightarrow 7 \geqslant \frac{7x}{2} \geqslant -14$

$\Rightarrow 1 \geqslant \frac{x}{2} \geqslant -2$

$\Rightarrow 2 \geqslant x \geqslant -4$

As a result, the set of solutions for the given inequality is $[-4, 2]$.

4. **Solve the inequality** $\mathbf{-15 < \frac{3(x-2)}{5} \leqslant 0}$

Ans: $-15 < \frac{3(x-2)}{5} \leqslant 0$

$\Rightarrow -75 < 3(x-2) \leqslant 0$

$\Rightarrow -25 < x - 2 \leqslant 0$

$\Rightarrow -25 + 2 < x \leqslant 2$

$\Rightarrow -23 < x \leqslant 2$

As a result, the set of solutions for the given inequality is (-23, 2]

5. **Solve the inequality** $\mathbf{-12 < 4 - \frac{3x}{-5} \leqslant 2}$

Ans:

$$-12 < 4 - \frac{3x}{-5} \leqslant 2$$

$$\Rightarrow -12 - 4 < \frac{-3x}{-5} \leqslant 2 - 4$$

$$\Rightarrow -16 < \frac{3x}{5} \leqslant -2$$

$$\Rightarrow -80 < 3x \leqslant -10$$

$$\Rightarrow \frac{-80}{3} < x \leqslant \frac{-10}{3}$$

As a result, the set of solutions for the given inequality is $\left(\frac{-80}{3}, \frac{-10}{3}\right]$.

6. Solve the inequality $7 \leqslant \frac{(3x+11)}{2} \leqslant 11$

Ans: $7 \leqslant \frac{(3x+11)}{2} \leqslant 11$

$$\Rightarrow 14 \leqslant 3x + 11 \leqslant 22$$

$$\Rightarrow 14 - 11 \leqslant 3x \leqslant 22 - 11$$

$$\Rightarrow 3 \leqslant 3x \leqslant 11$$

$$\Rightarrow 1 \leqslant x \leqslant \frac{11}{3}$$

As a result, the set of solutions for the given inequality is $\left[1, \frac{11}{3}\right]$.

7. Solve the inequalities and represent the solution graphically on number line:

$\mathbf{5x + 1 > -24, 5x - 1 < 24}$

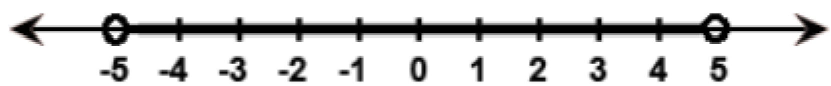
Ans: $5x+1>-24 \Rightarrow 5x>-25$

$\Rightarrow x>-5 \ldots.(1)$

$5x-1<24 \Rightarrow 5x<25$

$\Rightarrow x<5$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be $(-5,5)$. On a number line, the solution to the above system of inequalities can be expressed as



8. Solve the inequalities and represent the solution graphically on number line:

$\mathbf{2(x-1)<x+5, 3(x+2)>2-x}$

Ans: $2(x-1)<x+5 \Rightarrow 2x-2<x+5 \Rightarrow 2x-x<5+2$

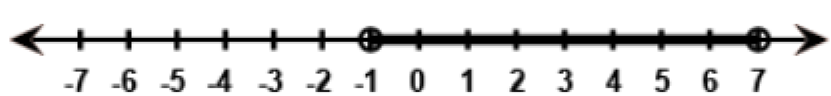
$\Rightarrow x<7$

(1) $3(x+2)>2-x \Rightarrow 3x+6>2-x \Rightarrow 3x+x>2-6$

$\Rightarrow 4x>-4$

$\Rightarrow x>-1 \ldots\ldots(2)$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be $(-1,7)$. On a number line, the solution to the above system of inequalities can be expressed as



9. **Solve the following inequalities and represent the solution graphically on number line:**

$\mathbf{3x-7>2(x-6), 6-x>11-2x}$

Ans: $3x-7>2(x-6) \Rightarrow 3x-7>2x-12 \Rightarrow 3x-2x>-12+7$

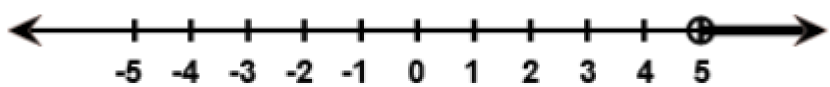
$\Rightarrow x>-5 \ldots\ldots\ldots(1)$

$6-x>11-2x \Rightarrow -x+2x>11-6$

$\Rightarrow x>5$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be

$(5,\infty)$. On a number line, the solution to the above system of inequalities can be expressed as



10. **Solve the inequalities and represent the solution graphically on number line:**

$\mathbf{5(2x-7)-3(2x+3)\leqslant 0, 2x+19\leqslant 6x+47}$

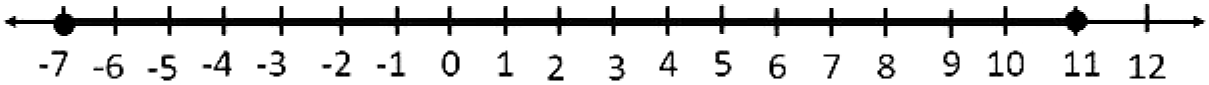
Ans: $5(2x-7)-3(2x+3)\leqslant 0 \Rightarrow 10x-35-6x-9\leqslant 0 \Rightarrow 4x-44\leqslant 0 \Rightarrow 4x\leqslant 44$

$\Rightarrow x\leqslant 11$

$2x+19\leqslant 6x+47 \Rightarrow 19-47\leqslant 6x-2x \Rightarrow -28\leqslant 4x$

$\Rightarrow -7\leqslant x$

From (1) and (2), The solution set for the given system of inequalities can be deduced to be $[-7,11]$. On a number line, the solution to the above system of inequalities can be expressed as



11. A solution is to be kept between $68^{\circ}\mathrm{F}$ and $77^{\circ}\mathrm{F}$. What is the range in temperature in degree Celsius (C) if the Celsius/Fahrenheit (F) conversion formula is given by $F = \frac{9}{5}C + 32$?

Ans: Because the solution must be preserved somewhere in the middle , $68^{\circ}\mathrm{F}$ and $77^{\circ}\mathrm{F}, 68 < F < 77$ Putting $F = \frac{9}{5}C + 32$, we obtain $68 < \frac{9}{5}C + 32 < 77$

$$\Rightarrow 68 - 32 < \frac{9}{5}C < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

As a result, the required temperature range in degrees Celsius is between $20^{\circ}\mathrm{C}$ and $25^{\circ}\mathrm{C}$.

12. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Ans: Let 2% of x litres of boric acid solution is required to be added. Then, total mixture $= (x + 640)$ litres.

This resulting mixture is to be more than 4% but less than 6% boric acid.

$\therefore$ 2% of $x+8\%$ of $640 > 4\%$ of $(x+640)$

And, 2% of $x+8\%$ of $640 < 6\%$ of $(x+640)$`

$\Rightarrow 2x+5120 > 4x+2560$

$\Rightarrow 5120-2560 > 4x-2x$

$\Rightarrow 5120-2560 > 2x$

$\Rightarrow 2560 > 2x$

$\Rightarrow 1280 > x$

$2\%x+8\%$ of $640 < 6\%$ of $(x+640)$ $\frac{2}{100}x+\frac{8}{100}(640) < \frac{6}{100}(x+640)$

$\Rightarrow 2x+5120 < 6x+3840$

$\Rightarrow 5120-3840 < 6x-2x$

$\Rightarrow 5120-3840 < 6x-2x$

$\Rightarrow 1280 < 4x$

$\Rightarrow 320 < x$

$\therefore 320 < x < 1280$

As a result, the total amount of boric acid solution to be added must be greater than 320 litres but less than 1280 litres**.**

13. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Ans: Allow for the addition of x litres of water. The entire mixture is then calculated $=(x+1125)$ litres It is clear that the amount of acid in the final mixture is excessive.

45% of 1125 litres. The resulting mixture will have a higher concentration of 25% but less than 30% acid content.

$\therefore 30\%$ of $(1125+x) > 45\%$ of 1125

And, 25% of $(1125+x) < 45\%$ of 1125

$$\Rightarrow \frac{30}{100}(1125+x) > \frac{45}{100}\times 1125$$

$$\Rightarrow 30(1125+x) > 45\times 1125$$

$$\Rightarrow 30\times 1125 + 30x > 45\times 1125$$

$$\Rightarrow 30 > 45\times 1125 - 30\times 1125$$

$$\Rightarrow 30x > (45-30)\times 1125$$

$$\Rightarrow x > \frac{15\times 1125}{30} = 562.5$$

25% of $(1125+x) < 45\%$ of 1125 $\Rightarrow \frac{25}{100}(1125+x) < \frac{45}{100}\times 1125$

$$\Rightarrow 25(1125+x) > 45\times 1125$$

$$\Rightarrow 25\times 1125 + 25x > 45\times 1125$$

$$\Rightarrow 25x > 45\times 1125 - 25\times 1125$$

$$\Rightarrow 25x > (45-25)\times 1125$$

$$\Rightarrow x > \frac{20\times 1125}{25} = 900$$

$$\therefore 562.5 < x < 900$$

As a result, the required number of litres of water must be greater than 562.5 but less than 900.

14. **IQ of a person is given by the formula** $IQ = \frac{MA}{CA} \times 100$**, Where MA is mental age and CA is chronological age. If** $80 \leqslant 1Q \leqslant 140$ **for a group of 12 years old children, find the range of their mental age**.

Ans: It is reported that for a group of twelve-year-olds $80 \leqslant IQ \leqslant 140 \ldots (i)$

For a group of 12 years old children, $\mathrm{CA} = 12$ years $\mathrm{IQ} = \frac{\mathrm{MA}}{12} \times 100$

Putting this value of IQ in (i), we obtain $80 \leqslant \frac{\mathrm{MA}}{12} \times 100 \leqslant 140$

$$\Rightarrow 80 \times \frac{12}{100} \leqslant \mathrm{MA} \leqslant 140 \times \frac{12}{100}$$

$$\Rightarrow 9.6 \leqslant \mathrm{MA} \leqslant 16.8$$

As a result, the mental age range of the 12-year-olds has widened. $\Rightarrow 9.6 \leqslant \mathrm{MA} \leqslant 16.8$.