trigonometric functions

3 Chapter

Exercise 3.1

1. Find the radian measures corresponding to the following degree measures:

(i) 25^o

Ans: We know that $180^o = \pi$ radian

Therefore $1^\circ = \frac{\pi}{180}$ radian

hence,

$$25^o = \frac{\pi}{180} \times 25 \text{ radian}$$
$$= \frac{5\pi}{36} \text{radian}$$

(ii) $-47^o 30'$

Ans: Here we have,

$$-47^o 30' = -47\frac{1}{2}^o$$
$$= -\frac{95}{2} \text{ degree}$$

Since we know that, $180^o = \pi$ radian

Therefore $1^o = \frac{\pi}{180}$ radian

Hence,

$$-\frac{95}{2} \text{ degree} = \frac{\pi}{180} \times \left(\frac{-95}{2}\right) \text{ radian}$$
$$= \left(\frac{-19}{36\times 2}\right)\pi \text{ radian}$$
$$= \frac{-19}{72}\pi \text{ radian}$$

Therefore,

$$-47^{o}30' = -\frac{19}{72}\pi \text{ radian}$$

(iii) 240^{o}

Ans: We know that,

$180^{o} = \pi$ radian

Therefore $1^{o} = \frac{\pi}{180}$ radian

Hence,

$$240^{o} = \frac{\pi}{180} \times 240 \text{ radian}$$

$$= \frac{4}{3}\pi \text{ radian}$$

(iv) 520^{o}

Ans: We know that,

$180^{o} = \pi$ radian

Therefore $1^{o} = \frac{\pi}{180}$ radian

Hence,

$$520^{o} = \frac{\pi}{180} \times 520 \text{ radian}$$

$$= \frac{26\pi}{9} \text{ radian}$$

2. Find the degree measures corresponding to the following radian measures (Use $\pi = \frac{22}{7}$)

(i) $\frac{11}{16}$

Ans: nWe know that,

π radian $= 180^{o}$

Therefore $1 \text{ radian} = \frac{180^{o}}{\pi}$

Hence,

$$\frac{11}{16}\text{ radian}=\frac{180}{\pi}\times\frac{11}{16}\text{ degree}$$

$$=\frac{45\times11}{\pi\times4}\text{degree}$$

$$=\frac{45\times11\times7}{22\times4}\text{ degree}$$

$$=\frac{315}{8}\text{degree}$$

Further computing,

$$\frac{11}{16}\text{ radian}=39\frac{3}{8}\text{ degree}$$

$$=39^{o}+\frac{3\times60}{8}\text{ minutes}$$

Since $1^{o}=60'$

$$\frac{11}{16}\text{ radian }=39^{o}+22'+\frac{1}{2}\text{minutes}$$

Since $1'=60''$

$$\frac{11}{16}\text{ radian }=39^{o}22'30''$$

(ii) -4

Ans: We know that,

π radian $=180^{o}$

Therefore $1\text{ radian }=\frac{180}{\pi}^{o}$

Hence,

$$\text{-4 radian}=\frac{180}{\pi}\times(-4)\text{ degree}$$

$$=\frac{180\times7(-4)}{22}\text{degree}$$

$$=\frac{-2520}{11}\text{ degree}$$

$$=-229\frac{1}{11}\text{degree}$$

Since $1^{o}=60'$

We have,

$$-4 \text{ radian}=-229^{o}+\frac{1\times60}{11} \text{ minutes}$$

$$=-229^{o}+5'+\frac{5}{11} \text{ minutes}$$

Since $1'=60''$

$$-4 \text{ radian}=-229^{o}5'27''$$

(iii) $\boldsymbol{\frac{5\pi}{3}}$

Ans: We know that,

$\pi \text{ radian}=180^{o}$

Therefore $1 \text{ radian} =\frac{180^{o}}{\pi}$

Hence,

$$\frac{5\pi}{3} \text{ radian}=\frac{180}{\pi}\times\frac{5\pi}{3} \text{ degree}$$

$$=300^{o}$$

(iv) $\boldsymbol{\frac{7\pi}{6}}$

Ans: We know that,

$\pi \text{ radian}=180^{o}$

Therefore $1 \text{ radian} =\frac{180^{o}}{\pi}$

Hence,

$$\frac{7\pi}{6} \text{ radian}=\frac{180}{\pi}\times\frac{7\pi}{6}$$

$$=210^{o}$$

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Ans: Number of revolutions the wheel makes in 1 minute $=360$

Number of revolutions the wheel make in 1 second $=\frac{360}{60}$

$=6$

In one complete revolution, the wheel turns an angle of 2π radian.
Hence, it will turn an angle of $6\times2\pi=12\pi$ radian, in 6 complete revolutions.
Therefore, the wheel turns an angle of 12π radian in one second.

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

(Use $\pi=\frac{22}{7}$)

Ans: We know that,
in a circle of radius r unit, if an angle θ radian at the centre is subtended by an arc of length l unit then

$$\theta=\frac{l}{r} \quad \ldots\ldots(1)$$

Therefore,
Substituting $r=100\text{cm}$, $l=22\text{cm}$ in the formula (1) , we have,

$$\theta=\frac{22}{100} \text{ radian}$$

Since 1 radian $=\frac{180}{\pi}$

Therefore,

$$\theta=\frac{180}{\pi}\times\frac{22}{100} \text{ degree}$$

$$=\frac{180\times7\times22}{22\times100} \text{ degree}$$

$$=\frac{63}{5} \text{ degree}$$

$$=12\frac{3}{5} \text{ degree}$$

Since $1^{o}=60'$, we have,

$\theta=12^{o}36'$

Hence , the required angle is $12^{o}36'$.

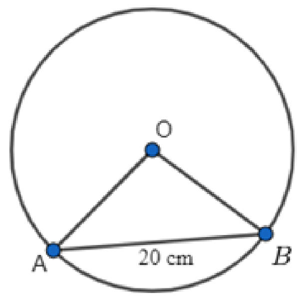
5. **In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.**

Ans: Given that, diameter of the circle $= 40$ cm

Hence Radius (r) of the circle $= \frac{40}{2}$ cm

$= 20$cm

Let AB be a chord of the circle whose length is 20 cm.



circle

In ΔOAB,

$OA=OB$

$=$ Radius of the circle

$=20$cm

Now also, $AB=20$cm

Therefore, ΔOAB is an equilateral triangle.

$\therefore \theta=60^{o}$

$= \frac{\pi}{3}$ radian

We know that,

in a circle of radius r unit, if an angle θ radian at the centre is subtended by an arc of length l unit then

$\theta=\frac{l}{r}$ ……(1)

Substituting $\theta=\frac{\pi}{3}$ in the formula (1),

$$\frac{\pi}{3}=\frac{\text{arc } AB}{20}$$

$\text{arc AB}=\frac{20\pi}{3}\text{cm}$

Therefore, the length of the minor arc of the chord is $\frac{20\pi}{3}\text{cm}$.

6. If in two circles, arcs of the same length subtend angles 60^o and 75^o at the centre, find the ratio of their radii.

Ans: Let the radii of the two circles be r_1 and r_2 . Let an arc of length l_1 subtends an angle of 60^o at the centre of the circle of radius r_1 , whereas let an arc of length l_2 subtends an angle of 75^o at the centre of the circle of radius r_2 .

Now, we have,

$60^o=\frac{\pi}{3}$ radian and

$75^\circ=\frac{5\pi}{12}$ radian

We know that,

in a circle of radius r unit, if an angle θ radian at the centre is subtended by an arc of length l unit then

$\theta=\frac{l}{r}$

$l=r\theta$

Hence we obtain,

$l=\frac{r_1\pi}{3}$

and $l=\frac{r_2 5\pi}{12}$

according to the question $l_1=l_2$

thus we have,

$\frac{r_1\pi}{3\pi}=\frac{r_2 5\pi}{12}$

$r_1=\frac{r_2 5}{4}$

$\frac{r_1}{r_2}=\frac{5}{4}$

Hence , the ratio of the radii is $5:4$.

7. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length.

(i) 10 cm

Ans: We know that,

in a circle of radius r unit, if an angle θ radian at the centre is subtended by an arc of length l unit then

$$\theta=\frac{l}{r}$$

Given that $r=75cm$

And here, $l=10cm$

Hence substituting the values in the formula,

$$\theta=\frac{10}{75}\text{ radian}$$

$$=\frac{2}{15}\text{radian}$$

(ii) 15 cm

Ans: We know that,

in a circle of radius r unit, if an angle θ radian at the centre is subtended by an arc of length l unit then

$$\theta=\frac{l}{r}$$

Given that $r=75cm$

And here, $l=15cm$

Hence substituting the values in the formula,

$$\theta=\frac{15}{75}\text{ radian}$$

$$=\frac{1}{5}\text{radian}$$

(iii) 21 cm

Ans: We know that,

in a circle of radius r unit, if an angle θ radian at the centre is subtended by an arc of length l unit then

$\theta=\frac{l}{r}$

And here, $l=21cm$

Hence substituting the values in the formula,

$$\theta=\frac{21}{75} \text{ radian}$$

$$=\frac{7}{25} \text{radian}$$

Exercise 3.2

1. Find the values of other five trigonometric functions if $\cos x=-\frac{1}{2}$, x lies in third quadrant.

Ans: Here given that, $\cos x=-\frac{1}{2}$

Therefore we have,

$$\sec x=\frac{1}{\cos x}$$

$$=\frac{1}{\left(-\frac{1}{2}\right)}$$

$$=-2$$

Now we know that, $\sin^2x+\cos^2x=1$

Therefore we have, $\sin^2x=1-\cos^2x$

Substituting $\cos x=-\frac{1}{2}$ in the formula, we obtain,

$$\sin^2x=1-\left(-\frac{1}{2}\right)^2$$

$$\sin^2x=1-\frac{1}{4}$$

$$=\frac{3}{4}$$

$$\sin x=\pm\frac{\sqrt{3}}{2}$$

Since x lies in the 3^{rd} quadrant, the value of $\sin x$ will be negative.

$$\sin x=-\frac{\sqrt{3}}{2}$$

Therefore, $\operatorname{cosec}x=\frac{1}{\sin x}$

$$=\frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}$$

$$=-\frac{2}{\sqrt{3}}$$

Hence ,

$$\tan x=\frac{\sin x}{\cos x}$$

$$=\frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}$$

$$=\sqrt{3}$$

And

$$\cot x=\frac{1}{\tan x}$$

$$=\frac{1}{\sqrt{3}}$$

2. Find the values of other five trigonometric functions if $\sin x=\frac{3}{5}$, x lies in second quadrant.

Ans: Here given that, $\sin x=\frac{3}{5}$

Therefore we have,

$$\operatorname{cosec}x=\frac{1}{\sin x}$$

$$=\frac{1}{\left(\frac{3}{5}\right)}$$

$$=\frac{5}{3}$$

Now we know that , $\sin^2 x+\cos^2 x=1$

Therefore we have, $\cos^2 x=1-\sin^2 x$

Substituting $\sin x=\frac{3}{5}$ in the formula, we obtain,

$$\cos^2 x=1-\left(\frac{3}{5}\right)^2$$

$$\cos^2 x=1-\frac{9}{25}$$

$$=\frac{16}{25}$$

$$\cos x=\pm\frac{4}{5}$$

Since x lies in the 2^{nd} quadrant, the value of $\cos x$ will be negative.

$$\cos x=-\frac{4}{5}$$

Therefore, $\sec x=\frac{1}{\cos x}$

$$=\frac{1}{\left(-\frac{4}{5}\right)}$$

$$=-\frac{5}{4}$$

Hence ,

$$\tan x=\frac{\sin x}{\cos x}$$

$$=\frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)}$$

$$=-\frac{3}{4}$$

And

$$\cot x=\frac{1}{\tan x}$$

$$=-\frac{4}{3}$$

3. Find the values of other five trigonometric functions if $\cot x=\frac{3}{4}$, x lies in third quadrant.

Ans: Here given that, $\cot x=\frac{3}{4}$

Therefore we have,

$$\tan x=\frac{1}{\cot x}$$

$$=\frac{1}{\left(\frac{3}{4}\right)}$$

$$=\frac{4}{3}$$

Now we know that , $\sec^2 x-\tan^2 x=1$

Therefore we have, $\sec^2 x=1+\tan^2 x$

Substituting $\tan x=\frac{4}{3}$ in the formula, we obtain,

$$\sec^2 x=1+\left(\frac{4}{3}\right)^2$$

$$\sec^2 x=1+\frac{16}{9}$$

$$=\frac{25}{9}$$

$$\sec x=\pm\frac{5}{3}$$

Since x lies in the 3^{rd} quadrant, the value of $\sec x$ will be negative.

$$\sec x=-\frac{5}{3}$$

Therefore, $\cos x=\frac{1}{\sec x}$

$$=\frac{1}{\left(-\frac{5}{3}\right)}$$

$$=-\frac{3}{5}$$

Now , $\tan x=\frac{\sin x}{\cos x}$

Therefore, $\sin x=\tan x\cos x$

Hence we have, $\sin x=\frac{4}{3}\times\left(-\frac{3}{5}\right)$

$$=\left(-\frac{4}{5}\right)$$

And

$$\operatorname{cosec} x=\frac{1}{\sin x}$$

$$=-\frac{5}{4}$$

4. Find the values of other five trigonometric functions if $\sec x=\frac{13}{5}$, x lies in fourth quadrant.

Ans: Here given that, $\sec x=\frac{13}{5}$

Therefore we have,

$$\cos x = \frac{1}{\sec x}$$

$$= \frac{1}{\left(\frac{13}{5}\right)}$$

$$= \frac{5}{13}$$

Now we know that , $\sec^2 x - \tan^2 x = 1$

Therefore we have, $\tan^2 x = \sec^2 x - 1$

Substituting $\sec x = \frac{13}{5}$ in the formula, we obtain,

$$\tan^2 x = \left(\frac{13}{5}\right)^2 - 1$$

$$\tan^2 x = \frac{169}{25} - 1$$

$$= \frac{144}{25}$$

$$\tan x = \pm\frac{12}{5}$$

Since x lies in the 4^{th} quadrant, the value of $\tan x$ will be negative.

$$\tan x = -\frac{12}{5}$$

Therefore, $\cot x = \frac{1}{\tan x}$

$$= -\frac{5}{12}$$

Now , $\tan x = \frac{\sin x}{\cos x}$

Therefore, $\sin x = \tan x \cos x$

Hence we have, $\sin x = \frac{5}{13} \times \left(-\frac{12}{5}\right)$

$$= \left(-\frac{12}{13}\right)$$

And

$$\operatorname{cosec}x=\frac{1}{\sin x}$$

$$=-\frac{13}{12}$$

5. Find the values of other five trigonometric functions if $\tan x=-\frac{5}{12}$, x lies in second quadrant.

Ans: Here given that, $\tan x=-\frac{5}{12}$

Therefore we have,

$$\cot x=\frac{1}{\tan x}$$

$$=\frac{1}{\left(-\frac{5}{12}\right)}$$

$$=-\frac{12}{5}$$

Now we know that, $\sec^2 x-\tan^2 x=1$

Therefore we have, $\sec^2 x=1+\tan^2 x$

Substituting $\tan x=-\frac{5}{12}$ in the formula, we obtain,

$$\sec^2 x=1+\left(-\frac{5}{12}\right)^2$$

$$\sec^2 x=1+\frac{25}{144}$$

$$=\frac{169}{144}$$

$$\sec x=\pm\frac{13}{12}$$

Since x lies in the 2^{nd} quadrant, the value of $\sec x$ will be negative.

$$\sec x=-\frac{13}{12}$$

Therefore, $\cos x=\frac{1}{\sec x}$

$=-\frac{12}{13}$

Now , $\tan x=\frac{\sin x}{\cos x}$

Therefore, $\sin x=\tan x\cos x$

Hence we have, $\sin x=\left(-\frac{5}{12}\right)\times\left(-\frac{12}{13}\right)$

$=\left(\frac{5}{13}\right)$

And

$\operatorname{cosec}x=\frac{1}{\sin x}$

$=\frac{13}{5}$

6. Find the value of the trigonometric function $\sin 765^{o}$.

Ans: We know that the values of $\sin x$ repeat after an interval of 2π or 360° .

Therefore we can write,

$\sin 765^{o}=\sin\left(2\times 360^{o}+45^{o}\right)$

$=\sin 45^{o}$

$=\frac{1}{\sqrt{2}}.$

7. Find the value of the trigonometric function $\operatorname{cosec}\left(-1410^{o}\right)$

Ans: We know that the values of $\operatorname{cosec}x$ repeat after an interval of 2π or 360° .

Therefore we can write,

$\operatorname{cosec}\left(-1410^{o}\right)=\operatorname{cosec}\left(-1410^{o}+4\times 360^{o}\right)$

$=\operatorname{cosec}\left(-1410^{o}+1440^{o}\right)$

$=\operatorname{cosec}30^{o}$

$=2$

8. Find the value of the trigonometric function $\tan \frac{19\pi}{3}$.

Ans: We know that the values of $\tan x$ repeat after an interval of π or 180°.

Therefore we can write,

$$\tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi$$
$$= \tan\left(6\pi + \frac{\pi}{3}\right)$$
$$= \tan \frac{\pi}{3}$$
$$= \sqrt{3}$$

9. Find the value of the trigonometric function $\sin\left(-\frac{11\pi}{3}\right)$

Ans: We know that the values of $\sin x$ repeat after an interval of 2π or 360°.

Therefore we can write,

$$\sin\left(-\frac{11\pi}{3}\right) = \sin\left(-\frac{11\pi}{3} + 2\times 2\pi\right)$$
$$= \sin\left(\frac{\pi}{3}\right)$$
$$= \frac{\sqrt{3}}{2}$$

10. Find the value of the trigonometric function $\cot\left(-\frac{15\pi}{4}\right)$

Ans: We know that the values of $\cot x$ repeat after an interval of π or 180°.

Therefore we can write,

$$\cot\left(-\frac{15\pi}{4}\right) = \cot\left(-\frac{15\pi}{4} + 4\pi\right)$$
$$= \cot \frac{\pi}{4}$$
$$= 1$$

Exercise 3.3

1, **Prove that** $\mathbf{\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}}$

Ans: Substituting the values of $\sin\frac{\pi}{6},\cos\frac{\pi}{6},\tan\frac{\pi}{4}$ on left hand side,

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

$$=\frac{1}{4}+\frac{1}{4}-1$$

$$=-\frac{1}{2}$$

$$=\text{R.H.S.}$$

Hence proved.

2. **Prove that** $\mathbf{2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}}$

Ans: Substituting the values of $\sin\frac{\pi}{6},\operatorname{cosec}\frac{7\pi}{6},\cos\frac{\pi}{3}$ on left hand side,

$$\text{L.H.S.}=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

$$=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$$

Since, cosecx repeat its value after interval of 2π,

We have, $\operatorname{cosec}\frac{7\pi}{6}=-\operatorname{cosec}\frac{\pi}{6}$

$$\text{L.H.S}=\frac{1}{2}+\frac{4}{4}$$

$$=\frac{3}{2}$$

$= \text{R.H.S.}$

Hence proved.

3. Prove that $\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Ans: Substituting the values of $\cot\frac{\pi}{6},\text{cosec}\frac{5\pi}{6},\tan\frac{\pi}{6}$ on left hand side,

$$\text{L.H.S.}=\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$$

$$=\left(\sqrt{3}\right)^2+\text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$=3+\text{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$$

Since $\text{cosec}x$ repeat its value after interval of 2π,

We have, $\text{cosec}\frac{5\pi}{6}=\text{cosec}\frac{\pi}{6}$

$\text{L.H.S}=3+2+1$

$=1$

$= \text{R.H.S.}$

Hence proved.

4. Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Ans: Substituting the values of $\sin\frac{3\pi}{4},\cos\frac{\pi}{4},\sec\frac{\pi}{3}$ on left hand side,

$$\text{L.H.S.}=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$$

$$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2\left(2\right)^2$$

$$=2\left\{\sin\frac{\pi}{4}\right\}^2+2\times\frac{1}{2}+8$$

Since $\sin x$ repeat its value after interval of 2π

,

We have, $\sin\frac{3\pi}{4}=\sin\frac{\pi}{4}$

L.H.S $=1+1+8$

$=10$

$=$ R.H.S.

Hence proved.

5. Find the value of :

(i) $\sin 75^o$

Ans: we have,

$\sin 75^o=\sin(45^o+30^o)$

$=\sin 45^o\cos 30^o+\cos 45^o\sin 30^o$

Since we know that, $\sin\left(x+y\right)=\sin x\cos y+\cos x\sin y$

Therefore we have,

$$\sin 75^o=\frac{1}{\sqrt{2}}\times\frac{1}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$$

$$=\frac{1}{\sqrt{2}}$$

(ii) $\tan 15^o$

Ans: We have,

$\tan 15^o=\tan\left(45^o-30^o\right)$

$$=\frac{\tan 45^o-\tan 30^o}{1+\tan 45^o\tan 30^o}$$

$$\tan\left(x-y\right)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

Since we know,

Therefore we have,

$$\tan 15^o=\frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)}$$

$$=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

Further computing we have,

$$\tan 15^{\circ}=\frac{3+1-2\sqrt{3}}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{4-2\sqrt{3}}{3-1}$$

$$=2-\sqrt{3}$$

6. **Prove that** $\mathbf{\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin\left(x+y\right)}$

Ans: We know that, $\cos\left(x+y\right)=\cos x\cos y-\sin x\sin y$

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\cos\left[\frac{\pi}{4}-x+\frac{\pi}{4}-y\right]$$

$$=\cos\left[\frac{\pi}{2}-\left(x+y\right)\right]$$

$$=\sin\left(x+y\right)$$

$$=\text{R.H.S.}$$

Hence proved.

7. **Prove that** $\mathbf{\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2}$

Ans: We know that , $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

and $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\text{L.H.S.}=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

Using the above formula,

$$\text{L.H.S}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}$$

$$=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)} \qquad \left[\text{ substituting } \tan\frac{\pi}{4}=1\ \right]$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

$$=\text{R.H.S.}$$

Hence proved.

8. Prove that $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Ans: Observe that $\cos x$ repeats same value after an interval 2π

and $\sin x$ repeats same value after an interval 2π.

$$\text{L.H.S.} = \frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$$= \cot^2 x$$

Hence proved.

9. Prove that,

$$\mathbf{Cos\left(\frac{3\pi}{2}+x\right)Cos(2\pi+x)\left[cot\left(\frac{3\pi}{2}-x\right)+cot(2\pi+x)\right]=1}$$

Ans: We know that $\cot x$ repeats same value after an interval 2π .

$$\text{L.H.S.} = \text{Cos}\left(\frac{3\pi}{2}+x\right)\text{Cos}(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$= \sin x\cos x[\tan x+\cot x]$$

Substituting $\tan x=\frac{\sin x}{\cos x}$ and

$\cot x=\frac{\cos x}{\sin x}$,

$$\text{L.H.S} = \sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$= (\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$$

$$= 1$$

$$= \text{R.H.S.}$$

Hence proved.

10: Prove that $\mathbf{\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x}$

Ans: We know that , $\cos(x-y)=\cos x\cos y+\sin x\sin y$

$$\text{L.H.S.} = \sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$$

$=\cos\left[(n+1)x-(n+2)x\right]$
$=\cos(-x)$
$=\cos x$
$=$ R.H.S.

11: **Prove that** $\mathbf{\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x}$

Ans: We know that , $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $=\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

$$=-2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$=-2\sin\left(\frac{3\pi}{4}\right)\sin x$$

Since $\sin x$ repeats the same value after an interval 2π ,

we have, $\sin\left(\frac{3\pi}{4}\right)=\sin\left(\pi-\frac{\pi}{4}\right)$

therefore,

L.H.S $=-2\sin\frac{\pi}{4}\sin x$
$=-2\times\frac{1}{\sqrt{2}}\times\sin x$
$=-\sqrt{2}\sin x$
$=$ R.H.S.

Hence proved.

12. **Prove that** $\mathbf{\sin^2 6x-\sin^2 4x=\sin 2x\sin 10x}$

Ans: We know that, $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

And $\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $=\sin^2 6x-\sin^2 4xa$

$$=(\sin 6x+\sin 4x)(\sin 6x-\sin 4x)$$

$$=\left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

$$=(2\sin 5x\cos x)(2\cos 5x\sin x)$$

Now we know that, $\sin 2x=2\sin x\cos x$,

Therefore we have,

L.H.S$=(2\sin 5x\cos 5x)(2\sin x\cos x)$

$=\sin 10x\sin 2x$

$=$ R.H.S.

13. **Prove that $\cos^2 2x-\cos^2 6x=\sin 4x\sin 8x$**

Ans: We know that,

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

And $\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

L.H.S. $=\cos^2 2x-\cos^2 6x$

$$=(\cos 2x+\cos 6x)(\cos 2x-6x)$$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]$$

Further computing, we have,

L.H.S$=\left[2\cos 4x\cos(-2x)\right]\left[-2\sin 4x\sin(-2x)\right]$

$$=\left[2\cos 4x\cos 2x\right]\left[-2\sin 4x(-\sin 2x)\right]$$

$$=(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$$

Now we know that, $\sin 2x=2\sin x\cos x$

Therefore we have,

L.H.S$=\sin 8x\sin 4x$

$=$ R.H.S.

hence proved.

14. Prove that $\sin 2x + 2\sin 4x + \sin 6 = 4\cos^2 x\sin 4x$

Ans: We know that, $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$
\begin{aligned}
\text{L.H.S.} &= \sin 2x + 2\sin 4x + \sin 6x \\
&= [\sin 2x + \sin 6x] + 2\sin 4x \\
&= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x \\
&= 2\sin 4x\cos(-2x) + 2\sin 4x
\end{aligned}
$$

Further computing,

We have, $\text{L.H.S} = 2\sin 4x\cos 2x + 2\sin 4x$

$= 2\sin 4x(\cos 2x + 1)$

Now we know that, $\cos 2x + 1 = 2\cos^2 x$

Therefore we have,

$\text{L.H.S} = 2\sin 4x(2\cos^2 x)$

$= 4\cos^2 x\sin 4x$

$=$ R.H.S.

Hence proved.

15. Prove that $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

Ans: We know that, $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$
\begin{aligned}
\text{L.H.S.} &= \cot 4x(\sin 5x + \sin 3x) \\
&= \frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x+3x}{2}\right)\right] \\
&= \left(\frac{\cos 4x}{\sin 4x}\right)[2\sin 4x\cos x] \\
&= 2\cos 4x\cos x
\end{aligned}
$$

Now also ,we know that, $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$
\begin{aligned}
\text{R.H.S.} &= \cot x(\sin 5x - \sin 3x) \\
&= \frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]
\end{aligned}
$$

$$=\frac{\cos x}{\sin x}[2\cos 4x\sin x]$$

$$=2\cos 4x\cos x$$

Therefore , we can conclude that,

L.H.S.=R.H.S.

Hence proved.

16. Prove that $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\frac{\sin 2x}{\cos 10x}$

Ans: We know that,

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

And $\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$\text{L.H.S.}=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$ [following the formula]

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

= R.H.S.

Hence proved.

17. Prove that: $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Ans: We know that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{Now , L.H.S.}=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$ [using the formula]

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x\cos x}{2\cos 4x\cos x}$$

Further computing we have,

L.H.S $=\tan 4x$

$=$ R.H.S.

18. Prove that $\dfrac{\sin x-\sin y}{\cos x+\cos y}=\tan\dfrac{x-y}{2}$

Ans: We know that,

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.}=\frac{\sin x-\sin y}{\cos x+\cos y}$$

$$=\frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$=\tan\left(\frac{x-y}{2}\right)$$

L.H.S=R.H.S

Therefore

Hence proved.

19. Prove that $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Ans: We know that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A+B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Now , L.H.S. $=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$ [using the fomula]

$$=\frac{\sin 2x}{\cos 2x}$$

$$=\tan 2x$$

Therefore L.H.S = R.H.S.

Hence proved.

20. Prove that $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Ans: We know that,

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

And $\cos^2 A - \sin^2 A = \cos 2A$

$$\text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= -2 \times (-\sin x)$$

Therefore , we have,

$$\text{L.H.S} = 2\sin x$$

$$= \text{R.H.S.}$$

Hence proved.

21. Prove that $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Ans: We know that,

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

And, $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

Now, $\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x} \quad \text{[using the formulas]}$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

Further computing, we obtain,

$$\text{L.H.S} = \frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}$$

$$= \cot 3x$$

$$= \text{R.H.S.}$$

Hence proved.

22. **Prove that $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$**

Ans: We know that, $\cot(A+B)=\dfrac{\cot A\cot B-1}{\cot A+\cot B}$

Now , L.H.S. $=\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x$

$$=\cot x\cot 2x-\cot 3x(\cot 2x+\cot x)$$

$$=\cot x\cot 2x-\cot(2x+x)(\cot 2x+\cot x)$$

$$=\cot x\cot 2x-\left[\frac{\cot 2x\cot x-1}{\cot x+\cot 2x}\right](\cot 2x+\cot x)$$

Further computing we obtain,

$$\text{L.H.S}=\cot x\cot 2x-(\cot 2x\cot x-1)$$

$$=1$$

$$= \text{R.H.S.}$$

Hence proved.

23. **Prove that $\tan 4x=\dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$**

Ans: We know that $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

L.H.S. $=\tan 4x$

$$=\tan 2(2x)$$

$$=\frac{2\tan 2x}{1-\tan^2(2x)} \quad \text{[using the formula]}$$

$$=\frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\dfrac{4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

Further computing, we obtain

$$\text{L.H.S} = \frac{\left(\dfrac{4\tan x}{1-\tan^2 x}\right)}{\left[\dfrac{(1-\tan^2 x)^2\ 4\tan^2 x}{(1-\tan^2 x)^2}\right]}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$$

$$= \text{R.H.S.}$$

Hence proved.

24. Prove that: $\cos 4x=1-8\sin^2 x\cos^2 x$

Ans: We know that, $\cos 2x=1-2\sin^2 x$

And $\sin 2x=2\sin x\cos x$

$$\text{L.H.S.} = \cos 4x$$

$$= \cos 2(2x)$$

$$= 1-2\sin^2 2x$$

$$= 1-2(2\sin x\cos x)^2$$

Further computing we get,

$$\text{L.H.S} = 1-8\sin^2 x\cos^2 x$$

$$= \text{R.H.S.}$$

Hence proved.

25. Prove that: $\cos 6x=32x\cos^6 x-48\cos^4 x+18\cos^2 x-1$

Ans: We know that, $\cos 3A=4\cos^3 A-3\cos A$

and $\cos 2x=1-2\sin^2 x$

$$\text{L.H.S.} = \cos 6x$$

$$= \cos 3(2x)$$

$$= 4\cos^3 2x-3\cos 2x$$

$$= 4\left[(2\cos^2 x-1)^3-3(2\cos^2 x-1)\right]$$

Further computing,

$$\text{L.H.S}=4\left[\left(2\cos^2 x\right)^3-(1)^3-3\left(2\cos^2 x\right)\right]-6\cos^2 x+3$$

$$=4\left[\left(2\cos^2 x\right)^3-(1)^3-3\left(2\cos^2 x\right)^2+3\left(2\cos^2 x\right)\right]-6\cos^2 x+3$$

$$=4\left[8\cos^6 x-1-12\cos^4 x+6\cos^2 x\right]-6\cos^2 x+3$$

$$=32\cos^6 x-48\cos^4 x+18\cos^2 x-1$$

Therefore we have,

L.H.S = R.H.S.

Hence proved.

Miscellaneous Exercise

1. Prove that: — $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Ans: We know that $\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

Now $\text{L.H.S.}=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$

$$=2\cos\frac{\pi}{3}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right) \qquad \text{[using the formula]}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{\pi}{13}\right)$$

Simplifying,

$$\text{L.H.S}=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

Substituting $\cos\frac{\pi}{2}=0$, we get,

$$\text{L.H.S}=2\cos\frac{\pi}{13}\times2\times0\times\cos\frac{5\pi}{26}$$
$$=0$$
$$=\text{R.H.S}$$

Hence proved.

2. **Prove that: $(\sin3x+\sin x)\sin x+(\cos3x-\cos x)\cos x=0$**

Ans: We know that, $\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

And $\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Now,

$$\text{L.H.S.}=(\sin3x+\sin x)\sin x+(\cos3x-\cos x)\cos x$$
$$=\sin3x\sin x+\sin^2x+\cos3x\cos x-\cos^2x \quad \text{[using the formula]}$$
$$=\cos3x\cos x+\sin3x\sin x-(\cos^2x-\sin^2x)$$

Simplifying we get,

$$\text{L.H.S}=\cos(3x-x)-\cos2x$$
$$=\cos2x-\cos2x$$
$$=0$$
$$=\text{R.H.S.}$$

3. **Prove that: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$**

Ans: We know that, $\cos(x+y)=\cos x\cos y-\sin x\sin y$

and $\text{L.H.S}=(\cos x+\cos y)^2+(\sin x-\sin y)^2$

$=\cos^2x+\cos^2y+2\cos x\cos y+\sin^2x+\sin^2y-2\sin x\sin y$

$=\left(\cos^2x+\sin^2x\right)+\left(\cos^2y+\sin^2y\right)+2\left(\cos x\cos y-\sin x\sin y\right)$

Simplifying and using the formula,

$\text{L.H.S}=1+1+2\cos\left(x+y\right)$

$=2\left[1+\cos\left(x+y\right)\right]$

$=2\left[1+2\cos^2\frac{\left(x+y\right)}{2}-1\right]$

[since $2\cos^2\frac{\left(x+y\right)}{2}-1=\cos\left(x+y\right)$]

$=4\cos^2\left(x+y\right)$

Therefore L.H.S = R.H.S

Hence proved.

4. **Prove that:** $\left(\cos x-\cos y\right)^2+\left(\sin x-\sin y\right)^2=4\sin^2\frac{x-y}{2}$

Ans: We know that, $\cos\left(x-y\right)=\cos x\cos y+\sin x\sin y$

$\text{L.H.S.}=\left(\cos x-\cos y\right)^2+\left(\sin x-\sin y\right)^2$

$=\cos^2x+\cos^2y-2\cos x\cos y+\sin^2x+\sin^2y-2\sin x\sin y$

$=\left(\cos^2x+\sin^2x\right)+\left(\cos^2y+\sin^2y\right)-2\left[\cos x\cos y+\sin x\sin y\right]$

Simplifying and using the formula we get,

$\text{L.H.S}=1+1-2\left[\cos\left(x-y\right)\right]$

$=2\left[1-\cos\left(x-y\right)\right]$

$=2\left[1-\left\{1-2\sin^2\left(\frac{x-y}{2}\right)\right\}\right]$

[since $1-2\sin^2\frac{\left(x-y\right)}{2}=\cos\left(x-y\right)$]

$=4\sin^2\left(\frac{x-y}{2}\right)$

Therefore L.H.S = R.H.S

Hence proved.

5. Prove that: sinx+sin3x+sin5x+sin7x=4cosxcos2xsin4x

Ans: We know that $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$

L.H.S. $=\sin x+\sin 3x+\sin 5x+\sin 7x$

$=(\sin x+\sin 5x)+(\sin 3x+\sin 7x)$

Using the formula and simplifying,

$$=2\sin\left(\frac{x+5x}{2}\right).\cos\left(\frac{x-5x}{2}\right)+2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$$

$$=2\cos 2x\left[\sin 3x+\sin 5x\right]$$

$$=2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right).\cos\left(\frac{3x-5x}{2}\right)\right]$$

$$=2\cos 2x\left[2\sin 4x.\cos(-x)\right]$$

Therefore we have,

L.H.S$=4\cos 2x\sin 4x\cos x$

$=$ R.H.S

6. Prove that: $\boldsymbol{\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x}$

Ans: We known that,

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$$

And

$$\text{L.H.S.} =\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$

Using the formula and simplifying,

$$=\frac{\left[2\sin\left(\frac{7x+5x}{2}\right).\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right).\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right).\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right).\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$=\frac{\left[2\sin6x.\cos x\right]+\left[2\sin6x.\cos3x\right]}{\left[2\cos6x.\cos x\right]+\left[2\cos6x.\cos6x\right]}$$

$$=\frac{2\sin6x\left[\cos x+\cos3x\right]}{2\cos6x\left[\cos x+\cos3x\right]}$$

$$=\tan6x$$

Therefore L.H.S = R.H.S

Hence proved.

7. Prove that: $\sin3x+\sin2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Ans: We know that,

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$$

And

$$\sin A-\sin B=2\sin\left(\frac{A-B}{2}\right).\cos\left(\frac{A+B}{2}\right)$$

L.H.S. $=\sin3x+\sin2x-\sin x$

$$=\sin3x+\left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right]$$

$$=\sin3x+\left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$

Since we know that, $\sin2x=2\sin x\cos x$

$$\text{L.H.S}=2\sin\frac{3x}{2}.\cos\frac{3x}{2}+2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$=2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{x}{2}\right)\right]$$

$$=2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right]$$

$$=2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$$

$$\text{L.H.S}=4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right)$$

$$=\text{R.H.S}$$

8. Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\tan x=-\frac{4}{3}$, x lies in 2nd quadrant.

Ans: Here, x is in 2nd quadrant.

Therefore ,

$$\frac{\pi}{2}<x<\pi$$

i.e, $\frac{\pi}{4}<\frac{x}{2}<\frac{\pi}{2}$

hence $\frac{x}{2}$ lies in 1st quadrant.

Therefore, $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are positive.

Given that $\tan x=-\frac{4}{3}$

We know that, $\sec^2 x=1+\tan^2 x$

$$\sec^2 x=1+\tan^2 x$$

$$=1+\left(-\frac{4}{3}\right)^2$$

$$=\frac{25}{9}$$

As x is in 2nd quadrant, $\sec x$ is negative.

Therefore , $\sec x=-\frac{5}{3}$

Then $\cos x=-\frac{3}{5}$

Now we know that, $2\cos^2\frac{x}{2}=\cos x+1$

Computing we get, $2\cos^2\frac{x}{2}=\frac{2}{5}$

Hence $\cos\frac{x}{2}=\frac{1}{\sqrt{5}}$

Now we know that, $\sin^2 x+\cos^2 x=1$

Therefore substituting $\cos\frac{x}{2}=\frac{1}{\sqrt{5}}$ and computing ,

$\sin\frac{x}{2}=\frac{2}{\sqrt{5}}$

Hence ,

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$=2$$

Thus, the respective values of $\sin\frac{x}{2},\cos\frac{x}{2},\tan\frac{x}{2}$

are $\frac{2\sqrt{5}}{5},\frac{\sqrt{5}}{5}$, 2

9. Find $\sin\frac{x}{2},\cos\frac{x}{2}$ and $\tan\frac{x}{2}$,if $\cos x=-\frac{1}{3}$, x lies in 3^{rd} quadrant.

Ans: Here, x is in 3^{rd} quadrant.

Therefore ,

$\pi<x<\frac{3\pi}{2}$

i.e, $\frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{4}$

hence $\frac{x}{2}$ lies in 2^{nd} quadrant.

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative and $\sin\frac{x}{2}$ is positive.

Given that $\cos x=-\frac{1}{3}$

Now we know that, $2\cos^2\frac{x}{2}=\cos x+1$

Computing we get, $2\cos^2\frac{x}{2}=\frac{2}{3}$

Hence $\cos\frac{x}{2}=-\frac{1}{\sqrt{3}}$

Now we know that, $\sin^2 x+\cos^2 x=1$

Therefore substituting $\cos\frac{x}{2}=-\frac{1}{\sqrt{3}}$ and computing ,

$\sin\frac{x}{2}=\sqrt{\frac{2}{3}}$

Hence ,

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$
$$=-\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2},\cos\frac{x}{2},\tan\frac{x}{2}$

are $\sqrt{\frac{2}{3}},-\frac{1}{\sqrt{3}},-\sqrt{2}$.

10. Find $\sin\frac{x}{2},\cos\frac{x}{2}$ and $\tan\frac{x}{2}$,if $\sin x=\frac{1}{4}$, x lies in 2^{nd} quadrant.

Ans: Here, x lies in 2^{nd} quadrant .

Therefore ,

$\frac{\pi}{2}<x<\pi$

i.e, $\frac{\pi}{4}<\frac{x}{2}<\frac{\pi}{2}$

hence $\frac{x}{2}$ lies in 1^{st} quadrant.

Therefore, $\sin\frac{x}{2},\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are positive.

Given that $\sin x=\frac{1}{4}$

Now we know that, $\sin^2 x+\cos^2 x=1$

Therefore substituting $\sin x=\frac{1}{4}$ and computing ,

$\cos x=-\frac{\sqrt{15}}{4}$

since x lies in 2^{nd} quadrant, $\cos x$ is negative.

Now we know that, $2\sin^2\frac{x}{2}=1-\cos x$

Computing we get, $2\sin^2\frac{x}{2}=1+\frac{\sqrt{15}}{4}$

Hence $\sin\frac{x}{2}=\sqrt{\frac{4+\sqrt{15}}{8}}$

Now we know that, $\sin^2 x+\cos^2 x=1$

Therefore substituting $\sin\frac{x}{2}=\sqrt{\frac{4+\sqrt{15}}{8}}$ and computing ,

$\cos\frac{x}{2}=\sqrt{\frac{4-\sqrt{15}}{8}}$

Hence ,

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$=\frac{\sqrt{4+\sqrt{15}}}{\sqrt{4-\sqrt{15}}}$$

$$=4+\sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2},\cos\frac{x}{2},\tan\frac{x}{2}$

are $\sqrt{\frac{4+\sqrt{15}}{8}},\sqrt{\frac{4-\sqrt{15}}{8}},4+\sqrt{15}$.