realtion and functions

2
Chapter

Exercise 2.1

1. **If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$, find the values of x and y.**

Ans: We are provided with the fact that $\left(\frac{x}{3}\quad 1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$

These are ordered pairs which are equal with each other, then the corresponding elements should also be equal to each other.

Thus, we will have, $\frac{}{3}+1=\frac{5}{3}$

And also $y-\frac{}{3}=\frac{1}{3}$

Now, we will try to simplify the given equations and find our needed values.

$$-+1=\frac{5}{3}$$

$$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$$

Simplifying further,

$$\Rightarrow \frac{x}{3}=\frac{5-3}{3}=\frac{2}{3}$$

$$\Rightarrow x=2$$

So, we have the value of x as 2 .

Again, for the second equation,

$$y--=\frac{1}{3}$$

$$\Rightarrow y=\frac{2}{3}+\frac{}{3}$$

And, after more simplification,

$$\Rightarrow y=\frac{1+2}{3}=\frac{3}{}$$

$$\Rightarrow y=1$$

So, we have the value of x and y as 2 and 1 respectively.

2. If the set A has 3 elements and the set $B=\{3,4,5\}$, then find the number of elements in $(A\times B)$?

Ans: We are provided with the fact that the set A has 3 elements and the set B is given as $\{3,4,5\}$.

So, the number of elements in set B is 3 .

Thus, the number of elements in $(A\times B)$ will be,

= Number of elements in $A\times$ Number of elements in B

$=3\times3=9$

So, the number of elements in $(A\times B)$ is 9 .

3. If $G=\{7,8\}$ and $H=\{5,4,2\}$, find $G\times H$ and $H\times G$.

Ans: We have the sets $G=\{7,8\}$ and $H=\{5,4,2\}$.

The Cartesian product of two non-empty sets A and B is defined as

$A\times B=\{(a,b):a\in A \text{ and } b\in B\}$

So, the value of $G\times H$ will be,

$G\times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

And similarly the value of $H\times G$ will be,

$H\times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$

4. State whether each of the following statement are true of false. If the statement is false, rewrite the given statement correctly.

(i) If $P=\{m,n\}$ and $Q=\{n,m\}$, then $P\times Q=\{(m,n),(n,m)\}$.

Ans: The statement is False.

We have the value as, $P=\{m,n\}$ and $Q=\{n,m\}$.

Thus, $P\times Q=\{(m,m),(m,n),(n,m),(n,n)\}$

The statement is True.

(ii) If A and B are non-empty sets, then $A\times B$ is a non-empty set of ordered pairs (x,y) such that $x\in A$ and $y\in B$.

Ans: The statement is False.

We have the value as, $P=\{m,n\}$ and $Q=\{n,m\}$.

Thus, $P\times Q=\{(m,m),(m,n),(n,m),(n,n)\}$

The statement is True.

(iii) If $A=\{1,2\}, B=\{3,4\}$, then $A\times\{B\cap\varnothing\}=\varnothing$.

Ans: The statement is True.

We know, $B\cap\varnothing=\varnothing$

Thus, we have, $A\times\{B\cap\varnothing\}=A\times\varnothing=\varnothing$.

5. If $A=\{-1,1\}$, find $A\times A\times A$.

Ans: For any non-empty set A, the set $A\times A\times A$ is defined by,

$A\times A\times A=\{(p,q,r):p,q,r\in A\}$

Now, we are provided with the fact that, $A=\{-1,1\}$

Thus,

$A\times A\times A=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$

6. If $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$. Find A and B.

Ans: We are provided with the fact that $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$

On the other hand, the Cartesian product of two non-empty sets A and B is defined as $A\times B=\{(a,b):a\in A \text{and } b\in B\}$

As we can see, A is the set of all the first elements and B is the set of all the second elements.

So, we will have, $A=\{a,b\}$ and $B=\{x,y\}$.

7. Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that

(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

Ans: We are provided with 3 sets and we have to prove $A\times(B\cap C)=(A\times B)\cap(A\times C)$

To start with, we will have, $B\cap C=\varnothing$, as there are no elements in common between these sets.

Thus, we have, $A\times(B\cap C)=A\times\varnothing=\varnothing$

For the right hand side, we have,

$A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

And similarly,

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

Again, we can see there are no elements in common between these sets. So, we have, $(A\times B)\cap(A\times C)=\varnothing$

So, we get, L.H.S = R.H.S.

(ii) $A \times C$ is a subset of $B \times D$

Ans: Again, we are to verify, $A \times C$ is a subset of $B \times D$

So, we have, $A \times C = \{(1,5),(1,6),(2,5),(2,6)\}$

And similarly,

$B \times D = \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),(4,5),$
$(4,6),(4,7),(4,8)\}$

We can easily see, every element of $A \times C$ is an element of $B \times D$. So, $A \times C$ is a subset of $B \times D$.

8. Let A={1,2} and B={3,4} . Write $A \times B$. How many subsets will $A \times B$ have? List them.

Ans: We are provided with the fact that $A = \{1,2\}$ and $B = \{3,4\}$

Thus, we have, $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

So, the set $A \times B$ has 4 elements.

Now, this is also known to us that, if a set A has n elements, then the number of subsets of A is 2^n .

We can thus conclude that, $A \times B$ will have $2^4 = 16$ subsets.

Now, noting down the subsets of $A \times B$ we get,

$\varnothing, \{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\}, \{(1,3),(1,4)\},$
$\{(1,3),(2,3)\}, \{(1,3),(2,4)\},$
$\{(1,4),(2,3)\}, \{(1,4),(2,4)\}, \{(2,3),(2,4)\},$
$\{(1,3),(1,4),(2,3)\}, \{(1,3),(1,4),(2,4)\},$
$\{(1,3),(2,3),(2,4)\}, \{(1,4),(2,3),(2,4)\},$
$\{(1,3),(1,4),(2,3),(2,4)\}$

9. Let A and B be two sets such that n(A)=3 and n(B)=2 . If (x,1),(y,2),(z,1) are in $A \times B$, find A and B , where x,y and z are distinct elements.

Ans: We are provided with the fact that $n(A) = 3$ and $n(B) = 2$; and $(x,1),(y,2),(z,1)$ are in $A \times B$.

We also know that, A is the set of all the first elements and B is the set of all the second elements.

So, we can conclude, A having elements x, y, z and B having elements $1, 2$.

Thus, we get, $n(A) = 3, n(B) = 2$.

So, $A = \{x, y, z\}, B = \{1, 2\}$.

10. The Cartesian product $A \times A$ has 9 elements among which are found $(-1,0)$ and $(0,1)$. Find the set A and the remaining elements of $A \times A$.

Ans: We are provided with, $n(A \times A) = 9$.

We also know that, if $n(A) = a, n(B) = b$, then $n(A \times B) = ab$

As it is given that, $n(A \times A) = 9$

It can be written as,

$n(A) \times n(A) = 9$

$\Rightarrow n(A) = 3$

And it is also given that $(-1,0),(0,1)$ are the two elements of $A \times A$.

Again, the fact is also known that, $A \times A = \{(a,a): a \in A\}$. And also $-1,0,1$ are the elements of A .

Also, $n(A) = 3$, implies $A = \{-1,0,1\}$.

So, $(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)$ are the remaining elements of $A \times A$.

Exercise 2.2

1. Let $A=\{1,2,3,......,14\}$. Define a relation R from A to A by $R=\{(x,y):3x-y=0\}$, where $x,y \in A$. Write down its domain, codomain and range.

Ans: We are given with the relation R from A to A as, $R = \{(x,y): 3x - y = 0\}$ where $x, y \in A$.

So, we can write R as, $R = \{(1,3),(2,6),(3,9),(4,12)\}$.

Thus, the domain of R is, $\{1,2,3,4\}$.

And similarly, the range of R is, $\{3,6,9,12\}$.

And also, the codomain of R is, $A = \{1,2,3,......,14\}$.

2. Define a relation R on the set N of natural numbers by $R=\{(x,y):y=x+5, x \text{ is a natural number less than } 4; x,y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

Ans: We are given with the fact that,

$R = \{(x,y): y = x+5, x \text{ is a natural number less than } 4; x, y \in N\}$.

We have the value of x as $1,2,3$ are it must be less than 4.

So, the relation R will look like, $R = \{(1,6),(2,7),(3,8)\}$

The domain of R will be, $=\{1,2,3\}$

And similarly, the range of R will be, $=\{6,7,8\}$.

3. **A={1,2,3,5} and B={4,6,9} . Define a relation R from A to B by R={(x,y): the differenece between x and y is odd; $x \in A, y \in B$}. Write R in roster form.**

Ans: We are provided with the fact that $A=\{1,2,3,5\}$ and $B=\{4,6,9\}$.

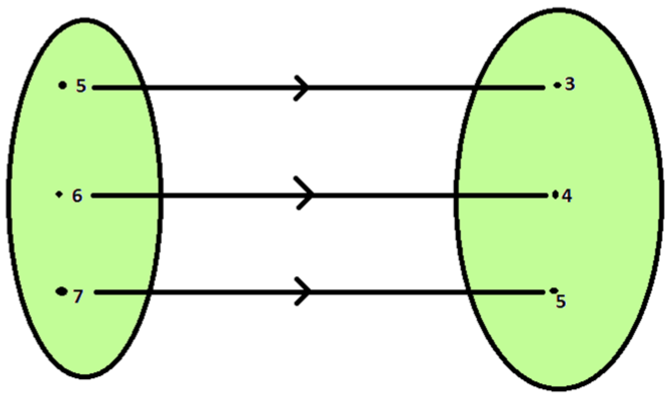
We are also given that,

$R=\{(x,y): \text{the difference between x and y is odd}; x \in A, y \in B\}$

Simply, writing down according to the given condition,

$R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$

4. **The given figure shows a relationship between the sets P and Q .**



Write this relation

(i) In set-builder form

Ans: From the given figure in the problem, we have, $P=\{5,6,7\}, Q=\{3,4,5\}$

Now, writing the relation in the set-builder form,

$(i) R=\{(x,y): y=x-2; x \in P\}$

And in another form,

$R=\{(x,y): y=x-2; x \in 5,6,7\}$

(ii) In roster form

What is its domain and range?

Ans: From the given figure in the problem, we have, . .

And again, in roster form,

$(ii) R=\{(5,3),(6,4),(7,5)\}$

Where the domain of R is $\{5,6,7\}$ and range of R is $\{3,4,5\}$.

5. **Let $A=\{1,2,3,4,6\}$. Let R be the relation on A defined by $\{(a,b):a,b \in A, b$ is exactly divisible by $a\}$.**

(i) Write R in roster form.

Ans: We are provided with the fact that,

$A=\{1,2,3,4,6\}, R=\{(a,b):a,b\in A, b\,\text{is exactly divisible by}\,a\}$

Using the conditions given in the problem, we get,

$R=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),$

$(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$

And this is the roster form of the relation.

(ii) Find the domain of R.

Ans: We are provided with the fact that,

$A=\{1,2,3,4,6\}, R=\{(a,b):a,b\in A, b\,\text{is exactly divisible by}\,a\}$

We can clearly see, the domain of R is, $\{1,2,3,4,6\}$

(iii) Find the range of R.

Ans: We are provided with the fact that,

$A=\{1,2,3,4,6\}, R=\{(a,b):a,b\in A, b\,\text{is exactly divisible by}\,a\}$

And similarly, the range of R is, $\{1,2,3,4,6\}$

6. **Determine the domain and range of the relation R defined by $R=\{(x,x+5):x\in\{0,1,2,3,4,5\}\}$**

Ans: We are provided with the fact, $R=\{(x,x+5):x\in\{0,1,2,3,4,5\}\}$

Using the condition given,

We can clearly write that, $R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

And this is our needed relation.Now, it can be clearly observed, that the domain of R is, $\{(x:x\in(0,1,2,3,4,5)\}$. And similarly, the range of R is, $\{(y:y\in(5,6,7,8,9,10)\}$.

7. **Write the relation $R=\{(x,x^3):x\,\text{is a prime number less than}\,10\}$ in roster form.**

Ans: We are provided with the fact that,

$R=\{(x,x^3):x\,\text{is a prime number less than}\,10\}$.

We know, the prime numbers less than 10 are $2,3,5,7$.

Thus, the relation can be written as,

$R=\{(2,8),(3,27),(5,125),(7,343)\}$

8. Let $A=\{x,y,z\}$ and $B=\{1,2\}$. Find the number of relations from A to B.

Ans: The facts provided to us are, $A=\{x,y,z\}$ and $B=\{1,2\}$.

Now, we will try to find out the Cartesian product of these to sets, $A\times B=\{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Thus, we see, the number of elements in $A\times B$ is 6.

So, the number of subsets, 2^6.

Then, the number of relations from A to B is 2^6.

9. Let R be the relation on Z defined by $R=\{(a,b):a,b\in Z, a-b \text{ is an integer}\}$. Find the domain and range of R.

Ans: The relation is given as, $R=\{(a,b):a,b\in Z, a-b \text{ is an integer}\}$.

And, we know the fact that, the difference of two given integers in always an integer. Thus, it can be concluded that, Domain of R is Z and similarly, the range of R is also Z.

Exercise 2.3

1. Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Ans: We have the given relation as, $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$.

Thus, we can see, the domain of the relation consists of $\{2,5,8,11,14,17\}$ and range is $\{1\}$.

And we also have, every element of the domain is having their unique images, then it is a function.

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Ans: We have our given relation, $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$.

Thus, we have our domain as, $\{2,4,6,8,10,12,14\}$ and range as, $\{1,2,3,4,5,6,7\}$.

Every element of the domain is having their unique images, so this is a function.

(iii) $\{(1,3),(1,5),(2,5)\}$

Ans: Our given relation is, $\{(1,3),(1,5),(2,5)\}$.

From the domain of the relation the element 1 is having two different images $3,5$.

So,every element of the domain is not having their unique images. So, this is not a function.

2. Find the domain and range of the following real function:

(i) $f(x)=-|x|$

Ans: We have the given function as, $f(x)=-|x|$.

It is also know that, $|x|=\begin{cases} x, \text{if } x\geq 0 \\ -x, \text{if } x<0 \end{cases}$

Thus, $f(x)=-|x|=\begin{cases} -x, \text{if } x\geq 0 \\ x, \text{if } x<0 \end{cases}$

As the function is a real function, the domain of the function is R .

And again, we can see that the function is giving values of all real numbers except positive ones.

So, the range of the function is, $(-\infty,0]$.

(ii) $f(x)=\sqrt{9-x^2}$

Ans: The function is given as,

$f(x)=\sqrt{9-x^2}$

We can clearly see that the function is well defined for all the real numbers which are greater than or equal to -3 and less than or equal to 3 , thus, the domain of the function is, $\{x:-3\leq x\leq 3\}$ or $[-3,3]$.

And for such value of x , the value of the function will always be between 0 and 3

.

Thus, the range is, $\{x:0\leq x\leq 3\}$ or $[0,3]$.

3. A function is defined by $f(x)=2x-5$

(i) $f(0)$

Ans: We have the given function as, $f(x)=2x-5$

So, the value of,

$f(0)=2\times 0-5=-5$

(ii) $f(7)$

Ans: We have the given function as, $f(x)=2x-5$

So, the value of,

$f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) f(-3)

Ans: We have the given function as, $f(x) = 2x - 5$

So, the value of,

$f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

4. The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find

(i) t(0)

Ans: We have our given function as, $t(C) = \frac{9C}{5} + 32$.

Thus, to find the values of the function we just have to put the values in the given function and simplify it.

So, we get now,

$t(0) = \frac{9 \times 0}{5} + 32 = 32$

(ii) t(28)

Ans: We have our given function as, $t(C) = \frac{9C}{5} + 32$.

Thus, to find the values of the function we just have to put the values in the given function and simplify it.

So, we get now,

$$t(28) = \frac{9 \times 28}{5} + 32$$
$$= \frac{252 + 160}{5}$$
$$= \frac{412}{5}$$
$$= 82.4$$

(iii) t(-10)

Ans: We have our given function as, $t(C) = \frac{9C}{5} + 32$.

Thus, to find the values of the function we just have to put the values in the given function and simplify it.
We are now getting,

$$t(-10)=\frac{9\times(-10)}{5}+32$$
$$=-18+32$$
$$=14$$

(iv) The value of C, when t(C)=212 .

Ans: We have our given function as, $t(C)=\frac{9C}{5}+32$.

For this problem, we are given that, $t(C)=212$.

So, it can be written as, $\frac{9C}{5}+32=212$

Simplifying further,

$$\frac{9C}{5}=212-32$$
$$\Rightarrow \frac{9C}{5}=180$$
$$\Rightarrow C=\frac{900}{9}=100$$

Thus, it can be said that, for $t(C)=212$, the value of t is 100 .

5: Find the range of each of the following functions:

(i) f(x)=2-3x,x $\in$ R,x>0 .

Ans: We have the given function as, $f(x)=2-3x, x\in R, x>0$

Let us try to write the value of the given function in a tabular form as,

x	0.01	0.1	0.9	1	2	2.5	4	5	…
f(x)	1.97	1.7	–0.7	–1	–4	–5.5	–10	–13	…

We can now see, it can be seen that the elements of the range is less than 2.
So, the range will be, $f=(-\infty,2)$
Alternative solution:
Let us take, $x>0$
We can again go forward by writing,

$3x > 0$
$\Rightarrow 2 - 3x < 2$
$\Rightarrow f(x) < 2$
So, the range of f is $(-\infty, 2)$

(ii) $f(x)=x^2+2$, x is a real number.

Ans: We have our given function that, $f(x) = x^2 + 2$

Let us try to write the value of the given function in a tabular form as,

x	0	± 0.3	± 0.8	± 1	± 2	± 3	$\ldots$
$f(x)$	. .	2.09	2.64	3	6	11	$\ldots$

So, we see that the range of the function f is the set of all numbers which are greater than or equal to 2.

Thus, we can conclude that the range of the function is, $[2, \infty)$.

Alternative Method:

Let x be any real number. So,

$x^2 > 0$

After further simplification,

$x^2 + 2 \geq 2$
$\Rightarrow f(x) \geq 2$

Thus, the range of the function is $= [2, \infty)$

(iii) f(x) = x, x is a real number

Ans: We have our given function as, $f(x) = x$, x is a real number .

Now, we can clearly have, that the range of the function is the set of all the numbers.

So, the range of the function will be, R .

Miscellaneous Exercise

1. **The relation f is defined by $f(x)=\begin{cases} x^2, 0 \leq x \leq 3 \\ 3x, 3 \leq x \leq 10 \end{cases}$**

And the relation g is defined by $g(x)=\begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$

Show that f is a function and g is not a function.

Ans: According to the problem, we have the function f as,

$$f(x)=\begin{cases} x^2, 0\le x\le 3 \\ 3x, 3\le x\le 10 \end{cases}$$

We can see that, for $x=3$,

$f(x)=3^2=9$ from the first given condition.

And again, $f(x)=3\times 3=9$ from the second condition.

But now,

$$g(x)=\begin{cases} x^2, 0\le x\le 2 \\ 3x, 2\le x\le 10 \end{cases}$$

We can see that, for $x=2$,

$f(x)=2^2=4$ from the first given condition.

And again, $f(x)=3\times 2=6$ from the second condition.

Thus, the domain of the relation g is having two different images from a single element.

So, it can be concluded that the relation is not a function.

2. If $f(x)=x^2$, find $\frac{f(1.1)-f(1)}{(1.1-1)}$.

Ans: We have the function, $f(x)=x^2$.

So, we will have,

$\frac{f(1.1)-f(1)}{(1.1-1)}$ equaling to,

$\frac{(1.1)^2-1^2}{1.1-1}$, putting the values.

After further simplification,

$$\frac{1.21-1}{0.1}$$

$$=\frac{0.21}{0.1}$$

$$=2.1$$

3. Find the domain of the function $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

Ans: According to the problem, we have the given function as $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$.

Let us try to simplify the given function and bring it to a form where we can analyze the problem.

The denominator can be factorized as,

$x^2-8x+12$

$=x^2-6x-2x+12$

$=x(x-6)-2(x-6)$

$=(x-2)(x-6)$

So, we see that the function is defined for every real numbers except $6,2$.

Thus, the domain of the function will be, $R-\{2,6\}$

4. Find the domain and the range of the real function f defined by $f(x)=\sqrt{(x-1)}$

Ans: We have the given function as, $f(x)=\sqrt{(x-1)}$.

Clearly, the term inside the root sign must be non-negative.

So, the function is valid for all values of $x\geq 1$.

Thus, the domain of the function will be, $[1,\infty)$.

Now, again, for $x\geq 1$, the value of the function will always be greater than or equal to zero.

So, the range of the function is, $[0,\infty)$.

5. Find the domain and the range of the real function f defined by $f(x)=|x-1|$.

Ans: The function which is given is, $f(x)=|x-1|$.

We can clearly see that, the function is well defined for all the real numbers.

Thus, it can be concluded that, the domain of the function is R .

And for every $x\in R$, the function gives all non-negative real numbers.

So, the range of the function is the set of all non-negative real numbers. i.e, $[0,\infty)$.

6: Let $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$ be a function from R to R. Determine the range of f.

Ans: We have our given function as, $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$.

Expressing it by term to term, we are getting,

$$f=\left\{(0,0),\left(\pm0.5,\frac{1}{5}\right),\left(\pm1,\frac{1}{2}\right),\left(\pm1.5,\frac{9}{13}\right),\left(\pm2,\frac{4}{5}\right),\left(3,\frac{9}{10}\right),\left(4,\frac{16}{17}\right),....\right\}$$

And we also know, the range of f is the set of all the second elements. We can also see that the terms are greater than or equal to 0 but less than 1.

So, the range of the function is, $[0,1)$.

7. Let $f,g:R\to R$ be defined, respectively by $f(x)=x+1, g(x)=2x-3$. Find $f+g, f-g$ and $\frac{f}{g}$.

Ans: We have the functions defined as, $f,g:R\to R$ is defined as,

$f(x)=x+1, g(x)=2x-3$.

Thus, the function

$$(f+g)(x)=f(x)+g(x)$$
$$=(x+1)+(2x-3)$$
$$=3x-2$$

So, the function $(f+g)(x)=3x-2$.

Again, the function,

$$(f-g)(x)=f(x)-g(x)$$
$$=(x+1)-(2x-3)$$
$$=-x+4$$

So, the function $(f-g)(x)=-x+4$.

Similarly,

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ where $g(x)\neq0$ and also $x\in R$.

Now, putting the values,

$$\left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3}$$

where,

$$2x-3\neq0$$
$$\Rightarrow x\neq\frac{3}{2}$$

8. **Let $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function from Z to Z defined by $f(x)=ax+b$, for some integers a,b. Determine a,b**

Ans: We have the given function as, $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ and also $f(x)=ax+b$.

As, $(1,1)\in f$, we get,

$a\times 1+b=1$

$\Rightarrow a+b=1$

And again, $(0,-1)\in f$, from this we can get,

$a\times 0+b=-1$

$\Rightarrow b=-1$

Putting this value in the first equation, we have,

$a-1=1$

$\Rightarrow a=2$

So, the value of a and b are respectively, $2,-1$.

9. **Let R be a relation from N to N defined by $R=\{(a,b):a,b\in N \text{ and } a=b^2\}$.**

Are the following true? Justify your answer in each case.

(i) $(a,a)\in R$, for all $a\in N$

Ans: We are given our relation as, $R=\{(a,b):a,b\in N \text{ and } a=b^2\}$

Let us take, $2\in N$.

But we have, $2\neq 2^2=4$

So, the statement that $(a,a)\in R$, for all $a\in N$ is not true.

(ii) $(a,b)\in R$, implies $(b,a)\in R$

Ans: We are given our relation as, $R=\{(a,b):a,b\in N \text{ and } a=b^2\}$

Let us take, $(9,3)\in N$.

We have to check if, $(3,9)\in N$ or not.

But, the condition of the relation says, $R=\{(a,b):a,b\in N \text{ and } a=b^2\}$ and $9^2\neq 3$.

(iii) $(a,b)\in R,(b,c)\in R$ implies $(a,c)\in R$.

Ans: We are given our relation as, $R=\left\{(a,b):a,b\in N \text{ and } a=b^2\right\}$

So, the statement $(a,b)\in R$, implies $(b,a)\in R$ is not true.

Now, let us take, $(9,3)\in R,(16,4)\in R$.

We have to check if, $(9,4)\in N$ or not.

Thus can also easily see, $9\neq 4^2=16$.

So, the given statement $(a,b)\in R,(b,c)\in R$ implies $(a,c)\in R$.

10. Let $A=\{1,2,3,4\},B=\{1,5,9,11,15,16\}$ and $f=\left\{(1,5),(2,9),(3,1),(4,5),(2,11)\right\}$.

Are the following true? Justify your answer in each case.

(i) f is a relation from A to B .

Ans: We are provided with two sets, $A=\{1,2,3,4\},B=\{1,5,9,11,15,16\}$

Thus, the Cartesian product of these two sets will be,

$$A\times B=\left\{\begin{matrix}(1,1),(1,5),(1,9),(1,11),(1,15),(1,16),\\(2,1),(2,5),(2,9),(2,11),(2,15),(2,16),\\(3,1),(3,5),(3,9),(3,11),(3,15),(3,16),\\(4,1),(4,5),(4,9),(4,11),(4,15),(4,16)\end{matrix}\right\}$$

And it is also given that,

$f=\left\{(1,5),(2,9),(3,1),(4,5),(2,11)\right\}$

A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A\times B$.

Thus, it can be easily checked that f is a relation from A to B.

(ii) f is a function from A to B

Ans: We are provided with two sets, $A=\{1,2,3,4\},B=\{1,5,9,11,15,16\}$

Thus, the Cartesian product of these two sets will be,

$$A\times B=\left\{\begin{matrix}(1,1),(1,5),(1,9),(1,11),(1,15),(1,16),\\(2,1),(2,5),(2,9),(2,11),(2,15),(2,16),\\(3,1),(3,5),(3,9),(3,11),(3,15),(3,16),\\(4,1),(4,5),(4,9),(4,11),(4,15),(4,16)\end{matrix}\right\}$$

And it is also given that,

$f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$

If we check carefully, we see that the first element 2 is providing us two different value of the image $9,11$.

So, it can be concluded that f is not a function from A to B.

11. Let f be the subset of $Z\times Z$ defined by $f=\{(ab,a+b):a,b\in Z\}$. If f a function from Z to Z. Justify your answer.

Ans: Our given relation f is defined as $f=\{(ab,a+b):a,b\in Z\}$.

We also know that a relation will be called a function from A to B if every element of the set A has unique images in set B.

Let us take 4 elements, $2,6,-2,-6\in Z$.

So, for the first two elements,

$(2\times 6,2+6)\in f$

$\Rightarrow (12,8)\in f$

And for the last two elements,

$(-2\times -6,-2+-6)\in f$

$\Rightarrow (12,-8)\in f$

So, it is clearly visible that one single element 12 having two different images $8,-8$. Thus, the relation is not a function.

12. Let $A=\{9,10,11,12,13\}$ and let $f:A\rightarrow N$ be defined by $f(n)=$ the highest prime factor of n. Find the range of f.

Ans: We have our given set as, $A=\{9,10,11,12,13\}$ and the relation is given as $f(n)=$ the highest prime factor of n.

The prime factor of 9 is 3.

The prime factors of 10 is $2,5$.

The prime factor of 11 is 11.

The prime factor of 12 is $2,3$.

The prime factor of 13 is 13.

Thus, it can be said,

$f(9)=$ the highest prime factor of $9=3$.

$f(10)=$ the highest prime factor of $10=5$.

$f(11)=$ the highest prime factor of $11=11$.

$f(12)=$ the highest prime factor of $12=3$.

$f(13)=$ the highest prime factor of $13=13$.

Now, the range of the function will be, $\{3,5,11,13\}$.