sets

1
Chapter

Exercise 1.1

1. Which of the following are sets? Justify your answer.

i. The collection of all months of a year beginning with the letter J.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

We can definitely identify the collection of months beginning with a letter J.

Thus, the collection of all months of a year beginning with the letter J is the set.

ii. The collection of ten most talented writers of India

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

The criteria for identifying the collection of ten most talented writers of India may vary from person to person. So it is not a well-defined object.

Thus, the collection of ten most talented writers of India is not a set.

iii. A team of eleven best cricket batsmen of the world.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

The criteria for determining the eleven best cricket batsmen may vary from person to person. So it is not a well-defined object.

Thus, a team of eleven best cricket batsmen in the world is not a set.

iv. The collection of all boys in your class.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

We can definitely identify the boys who are all studying in the class. So it is a well-defined object.

Thus, the collection of all boys in your class is a set.

v. The collection of all natural numbers less than 100.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

We can identify the natural numbers less than 100 can easily be identified. So it is a well-defined object.

Thus, the collection of all natural numbers less than 100 is a set.

vi. A collection of novels written by the writer Munshi Prem Chand.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

We can identify the books that belong to the writer Munshi Prem Chand. So it is a well-defined object.

Thus, a collection of novels written by the writer Munshi Prem Chand is a set.

vii. The collection of all even integers.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

We can identify integers that are all the collection of even integers. So it is not a

well-defined object.

Thus, the collection of all even integers is a set.

viii. The collection of questions in this chapter.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

We can easily identify the questions that are in this chapter. So it is a well-defined object.

Thus, the collection of questions in this chapter is a set.

ix. A collection of the most dangerous animals in the world.

Ans: To determine if the given statement is a set

A set is a collection of well-defined objects.

The criteria for determining the most dangerous animals may vary according to the person. So it is not a well-defined object.

Thus, the collection of the most dangerous animals in the world is a set.

2. Let $\mathbf{A=\{1,2,3,4,5,6\}}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

i. 5...A

Ans: Given that,

$A=\{1,2,3,4,5,6,\}$

To insert the appropriate symbol $\in$ or $\notin$

The number 5 is in the set.

$\therefore 5 \in A$

ii. 8...A

Ans: Given that,

$A=\{1,2,3,4,5,6,\}$

To insert the appropriate symbol $\in$ or $\notin$

The number 8 is not in the set.

$\therefore 8 \notin A$

iii. 0...A

Ans: Given that,

$A = \{1, 2, 3, 4, 5, 6,\}$

To insert the appropriate symbol $\in$ or $\notin$

The number 0 is not in the set.

$\therefore 0 \notin A$

iv. 4...A

Ans: Given that,

$A = \{1, 2, 3, 4, 5, 6,\}$

To insert the appropriate symbol $\in$ or $\notin$

The number 4 is in the set.

$\therefore 4 \in A$

v. 2...A

Ans: Given that,

$A = \{1, 2, 3, 4, 5, 6,\}$

To insert the appropriate symbol $\in$ or $\notin$

The number 2 is in the set.

$\therefore 2 \in A$

vi. 10...A

Ans: Given that,

$A = \{1, 2, 3, 4, 5, 6,\}$

To insert the appropriate symbol $\in$ or $\notin$

The number 10 is not in the set.

$\therefore 10 \notin A$

3. Write the following sets in roster form:

i. $\mathbf{A=\{x:x \text{ is an integer and } -3<x<7\}}$

Ans: Given that,

$A=\{x:x \text{ is an integer and } -3<x<7\}$

To write the above expression in its roaster form

In roaster form, the order in which the elements are listed is immaterial.

The elements of the set are $-2,-1,0,1,2,3,4,5,6$.

$\therefore$The roaster form of the set $A=\{x:x \text{ is an integer and } -3<x<7\}$ is $A=\{-2,-1,0,1,2,3,4,5,6\}$.

ii. $\mathbf{B=\{x:x \text{ is a natural number less than } 6\}}$

Ans: Given that,

$B=\{x:x \text{ is a natural number less than } 6\}$

To write the above expression in its roaster form

In roaster form, the order in which the elements are listed is immaterial.

The elements of the set are $1,2,3,4,5$.

$\therefore$The roaster form of the set $B=\{x:x \text{ is a natural number less than } 6\}$ is $B=\{1,2,3,4,5\}$.

iii. $\mathbf{C=\{x:x \text{ is a two-digit natural number such that sum of its digits is } 8\}}$

Ans: Given that,

$C=\{x:x \text{ is a two-digit natural number such that sum of its digits is } 8\}$

To write the above expression in its roaster form

In roaster form, the order in which the elements are listed is immaterial.

The elements of the set are $17,26,35,44,53,62,71,80$ such that their sum is 8

$\therefore$The roaster form of the set
$C=\{x:x \text{ is a two-digit natural number such that sum of its digits is } 8\}$ is
$\{17,26,35,44,53,62,71,80\}$.

iv. $D=\{x : x$ is a prime number which is divisor of 60$\}$

Ans: Given that,

$D=\{x : x$ is a prime number which is divisor of 60$\}$

To write the above expression in its roaster form

In roaster form, the order in which the elements are listed is immaterial.

The divisors of 60 are $2,3,4,5,6$. Among these the prime numbers are $2,3,5$

The elements of the set are $2,3,5$.

$\therefore$ The roaster form of the set $D=\{x : x$ is a prime number which is divisor of 60$\}$ is $D=\{2,3,5\}$.

v. E = The set of all letters in the word TRIGONOMETRY

Ans: Given that,

$E=$ The set of all letters in the word TRIGONOMETRY

To write the above expression in its roaster form

In roaster form, the order in which the elements are listed is immaterial.

There are 12 letters in the word TRIGONOMETRY out of which T, R and O gets repeated.

The elements of the set are T, R, I G, O, N, M, E, Y.

$\therefore$ The roaster form of the set $E=$ The set of all letters in the word TRIGONOMETRY is $E=\{T,R,I,G,O,N,M,E,Y\}$.

vi. F = The set of all letters in the word BETTER

Ans: Given that,

$F=$ The set of all letters in the word BETTER

To write the above expression in its roaster form

In roaster form, the order in which the elements are listed is immaterial.

There are 6 letters in the word BETTER out of which E and T are repeated.

The elements of the set are B, E, T, R**.**

$\therefore$ The roaster form of the set $\mathrm{F}=$ The set of all letters in the word BETTER is $\mathrm{F}=\{\mathrm{B}, \mathrm{E}, \mathrm{T}, \mathrm{R}\}$.

4. Write the following sets in the set builder form:

i. $\mathbf{(3,6,9,12)}$

Ans: Given that,

$\{3,6,9,12\}$

To represent the given set in the set builder form

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set**.**

From the given set, we observe that the numbers in the set are multiple of 3 from 1 to 4 such that $\{x : x=3n, n \in \mathrm{N} \text{ and } 1 \leq n \leq 4\}$

$\therefore \{3,6,9,12\}=\{x : x=3n, n \in \mathrm{N} \text{ and } 1 \leq n \leq 4\}$

ii. $\mathbf{\{2,4,8,16,32\}}$

Ans: Given that,

$\{2,4,8,16,32\}$

To represent the given set in the set builder form

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set**.**

From the given set, we observe that the numbers in the set are powers of 2 from 1 to 5 such that $\{x : x=2^n, n \in \mathrm{N} \text{ and } 1 \leq n \leq 5\}$

$\therefore \{2,4,8,16,32\}=\{x : x=2^n, n \in \mathrm{N} \text{ and } 1 \leq n \leq 5\}$

iii. $\mathbf{\{5,25,125,625\}}$

Ans: Given that,

$\{5, 25, 125, 625\}$

To represent the given set in the set builder form

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

From the given set, we observe that the numbers in the set are powers of 5 from 1 to 4 such that $\{x : x = 5^n, n \in N \text{ and } 1 \le n \le 4\}$

$\therefore \{5, 25, 125, 625\} = \{x : x = 5^n, n \in N \text{ and } 1 \le n \le 4\}$

iv. $\{\mathbf{2, 4, 6, ...}\}$

Ans: Given that,

$\{2, 4, 6, ...\}$

To represent the given set in the set builder form

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

From the given set, we observe that the numbers are the set of all even natural numbers.

$\therefore \{2, 4, 6, ...\} = \{x : x \text{ is an even natural number}\}$

v. $\{\mathbf{1, 4, 9, ...100}\}$

Ans: Given that,

$\{1, 4, 9, ...100\}$

To represent the given set in the set builder form

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

From the given set, we observe that the numbers in the set squares of numbers form 1 to 10 such that $\left\{x : x = n^2, n \in N \text{ and } 1 \le n \le 10\right\}$

$\therefore \left\{1, 4, 9, ...100\right\} = \left\{x : x = n^2, n \in N \text{ and } 1 \le n \le 10\right\}$

5. List all the elements of the following sets:

i. $\mathbf{A = \left\{x : x \text{ is an odd natural number}\right\}}$

Ans: Given that,

$A = \left\{x : x \text{ is an odd natural number}\right\}$

To list the elements of the given set

The odd natural numbers are $1, 3, 5, ...$

$\therefore$ The set $A = \left\{x : x \text{ is an odd natural number}\right\}$ has the odd natural numbers that are $\left\{1, 3, 5, ...\right\}$

ii. $\mathbf{B = \left\{x : x \text{ is an integer;} -\frac{1}{2} < x < \frac{9}{2}\right\}}$

Ans: Given that,

$B = \left\{x : x \text{ is an integer;} -\frac{1}{2} < x < \frac{1}{2}\right\}$

To list the elements of the given set

$-\frac{1}{2} = -0.5$ and $\frac{9}{2} = 4.5$

So the integers between -0.5 and 4.5 are $0, 1, 2, 3, 4$

$\therefore$ The set $B = \left\{ x : x \text{ is an integer;} -\frac{1}{2} < x < \frac{1}{2} \right\}$ has an integers that are between

$\{0, 1, 2, 3, 4\}$

iii. $\mathbf{C = \{x : x \text{ is an integer;} x^2 \leq 4\}}$

Ans: Given that,

$C = \{x : x \text{ is an integer;} x^2 \leq 4\}$

To list the elements of the given set

It is observed that,

$x^2 \leq 4$

$(-2)^2 = 4 \leq 4$

$(-1)^2 = 1 \leq 4$

$(0)^2 = 0 \leq 4$

$(1)^2 = 1 \leq 4$

$(2)^2 = 4 \leq 4$

$\therefore$ The set $C = \{x : x \text{ is an integer;} x^2 \leq 4\}$ contains elements such as $\{-2, -1, 0, 1, 2\}$

iv. $\mathbf{D = \{x : x \text{ is a letter in the word "LOYAL"}\}}$

Ans: Given that,

$D = \{x : x \text{ is a letter in the word "LOYAL"}\}$

To list the elements of the given set

There are 5 total letters in the given word in which L gets repeated two times.

So the elements in the set are $\{L, O, Y, A\}$

$\therefore$ The set $D = \{x : x \text{ is a letter in the word "LOYAL"}\}$ consists the elements $\{L, O, Y, A\}$.

v. $E = \{x : x$ is a month of a year not having 31 days$\}$

Ans: Given that,

$E = \{x : x \text{ is a month of a year not having 31 days}\}$

To list the elements of the given set

The months that don't have 31 are as follows:

February, April, June, September, November

$\therefore$ The set $E = \{x : x \text{ is a month of a year not having 31 days}\}$ consist of the elements such that $\{\text{February, April, June, September, November}\}$

vi. $F = \{x : x$ is a consonant in the English alphabet which precedes k$\}$

Ans: Given that,

$F = \{x : x \text{ is a consonant in the English alphabet which precedes k}\}$

To list the elements of the given set

The consonants are letters in English alphabet other than vowels such as a, e, i, o, u and the consonants that precedes k include b, c, d, f, g, h, j

$\therefore$ The set $F = \{x : x \text{ is a consonant in the English alphabet which precedes k}\}$ consists of the set $\{b, c, d, f, g, h, j\}$

6. Match each of the sets on the left in the roaster form with the same set on the right described in set-builder form.

1. $\{\mathbf{1,2,3,6}\}$

Ans: Given that,

$\{1,2,3,6\}$

To match the roaster form in the left with the set builder form in the right

In roaster form, the order in which the elements are listed is immaterial.

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set**.**

It has been observed from the set that these set of numbers are the set of natural numbers which are also the divisors of 6

Thus, $\{1,2,3,6\} = \{x : x \text{ is a natural number and is a divisor of } 6\}$ is the correct option which is option (c)

2. $\{\mathbf{2,3}\}$

Ans: Given that,

$\{2,3\}$

To match the roaster form in the left with the set builder form in the right

In roaster form, the order in which the elements are listed is immaterial.

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set**.**

It has been observed from the set that these set of numbers are the set of prime numbers which are also the divisors of 6

Thus, $\{2,3\} = \{x : x \text{ is a prime number and is a divisor of } 6\}$ is the correct option which is option (a)

3. $\{\mathbf{M, A, T, H, E, I, C, S}\}$

Ans: Given that,

$\{M, A, T, H, E, I, C, S\}$

To match the roaster form in the left with the set builder form in the right

In roaster form, the order in which the elements are listed is immaterial.

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set**.**

It has been observed from the set of these letters of word MATHEMATICS.

Thus, $\{M, A, T, H, E, I, C, S\} = \{x : x \text{ is aletter of the word MATHEMATICS}\}$ is the correct option which is option (d)

4. $\{\mathbf{1, 3, 5, 7, 9}\}$

Ans: Given that,

$\{1, 3, 5, 7, 9\}$

To match the roaster form in the left with the set builder form in the right

In roaster form, the order in which the elements are listed is immaterial.

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set**.**

It has been observed from the set that these set of numbers are the set of odd numbers that are less than 10.

Thus, $\{1, 3, 5, 7, 9\} = \{x : x \text{ is a odd number less than } 10\}$ is the correct option which is option (b)

Exercise 1.2

1. **Which of the following are examples of the null set**

i. Set of odd natural numbers divisible by 2

Ans: Given that,

Set of odd natural numbers divisible by 2

To find if the given statement is an example of null set

A set which does not contain any element is called the empty set or the null set or the void set.

There was no odd number that will be divisible by 2 and so this set is a null set.

$\therefore$ The set of odd natural number divisible by 2 is a null set.

ii. Set of even prime numbers

Ans: Given that,

Set of even prime numbers.

To find if the given statement is an example of null set

A set which does not contain any element is called the empty set or the null set or the void set.

There was an even number 2, will be the one and only even prime number. So the set contains an element. So it is not a null set.

$\therefore$ The set of even prime numbers is not a null set.

iii. $\{x : x$ is a natural numbers, $x<5$ and $x>7\}$

Ans: Given that,

$\{x : x$ is a natural numbers, $x<5$ and $x>7\}$

To find if the given statement is an example of null set

A set which does not contain any element is called the empty set or the null set or the void set.

There was no number that will be less than 5 and greater than 7 simultaneously.

So it is a null set

$\therefore \{x : x \text{ is a natural numbers, } x<5 \text{ and } x>7\}$ is a null set

iv. $\{\mathbf{y : y}$ is a point common to any two parallel lines$\}$

Ans: Given that,

$\{y : y \text{ is a point common to any two parallel lines}\}$

To find if the given statement is an example of null set

A set which does not contain any element is called the empty set or the null set or the void set.

The parallel lines do not intersect each other. So it does not have a common point of intersection. So it is a null set.

$\therefore \{y : y \text{ is a point common to any two parallel lines}\}$ is a null set.

2. Which of the following sets are finite or infinite.

i. The sets of months of a year

Ans: Given that,

The sets of months of a year

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

A year has twelve months which has defined number of elements

$\therefore$ The set of months of a year is finite.

ii. $\{1,2,3...\}$

Ans: Given that,

$\{1,2,3...\}$

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

The set consists of an infinite number of natural numbers.

$\therefore$ The set $\{1,2,3...\}$ is infinite since it contains an infinite number of elements.

iii. $\{1,2,3,...,99,100\}$

Ans: Given that,

$\{1,2,3,...,99,100\}$

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

This set contains the elements from 1 to 100 which are finite in number.

$\therefore$ The set $\{1,2,3,...,99,100\}$ is finite.

iv. The set of positive integers greater than 100

Ans: Given that,

The set of positive integers greater than 100

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

The positive integers which are greater than 100 are infinite in number.

$\therefore$ The set of positive integers greater than 100 is infinite.

v. The set of prime numbers less than 99

Ans: Given that,

The set of prime numbers less than 99

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

The prime numbers less than 99 are finite in number.

$\therefore$ The set of prime numbers less than 99 is finite.

3. State whether each of the following set is finite or infinite:

i. The sets of lines which are parallel to x axis

Ans: Given that,

The set of lines which are parallel to x axis

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

The lines parallel to x axes are infinite in number.

$\therefore$ The set of line parallel to x axis is infinite.

ii. The set of letters in English alphabet

Ans: Given that,

The set of letter sin English alphabet

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

English alphabet consist of 26 elements which is finite in number

$\therefore$ The set of letters in the English alphabet is finite.

iii. The set of numbers which are multiple of 5

Ans: Given that,

The set of numbers which are multiple of 5

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

The numbers which are all multiple of 5 are infinite in number.

$\therefore$ The set of numbers which are multiple of 5 is infinite.

iv. The set of animals living on the earth

Ans: Given that,

The set of animals living on the earth

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

Although the number of animals on the earth is quite a big number, it is finite.

$\therefore$ The set of animals living on the earth is finite.

v. The set of circles passing through the origin $(0,0)$

Ans: Given that,

The set of circles passing through the origin $(0,0)$

To find if the set is finite or infinite

A set which is empty or consists of a definite number of elements is called finite otherwise the set is called infinite.

The number of circles passing through the origin may be infinite in number.

$\therefore$ The set of circles passing through origin $(0,0)$ is infinite.

4. In the following, state whether $\mathbf{A=B}$ or not

i. $\mathbf{A=\{a,b,c,d\};B=\{d,c,b,a\}}$

Ans: Given that,

$A=\{a,b,c,d\};B=\{d,c,b,a\}$

To state whether $A=B$

We know that the order in which the elements are listed are insignificant. So $A=B$

$\therefore A=B$

ii. $\mathbf{A=\{4,8,12,16\}:B=\{8,4,16,18\}}$

Ans: Given that,

$A=\{4,8,12,16\}:B=\{8,4,16,18\}$

To state whether $A=B$

We know that $12\in A$ but $12\notin B$

$\therefore A\neq B$

iii. $\mathbf{A = \{2,4,6,8,10\}; B = \{x : x \text{ is a positive integer and } x \le 10\}}$

Ans: Given that,

$A = \{2,4,6,8,10\}; B = \{x : x \text{ is a positive integer and } x \le 10\}$

To state whether $A = B$

$A = \{2,4,6,8,10\}$

The positive integers less than 10 are $B = \{2,4,6,8,10\}$ So $A = B$

$\therefore A = B$

iv. $\mathbf{A = \{x : x \text{ is a multiple of } 10\}; B = \{10,15,20,25,30,...\}}$

Ans: Given that,

$A = \{x : x \text{ is a multiple of } 10\}; B = \{10,15,20,25,30,...\}$

To state whether $A = B$

$A = \{10,20,30,40,...\}$

$B = \{10,15,20,25,30,...\}$

The elements of A consists only the multiples of 10 and not of 5. So $A \ne B$

$\therefore A \ne B$

5. Are the following pair of sets equal? Give reasons.

i. $\mathbf{A = \{2,3\}; B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}}$

Ans: Given that,

$A = \{2,3\}; B = \{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\}$

To state whether $A = B$

Solving $x^2+5x+6=0$,

$x^2+3x+2x+6=0$

$(x+2)(x+3)=0$

$x=-2,-3$

$B=\{-2,-3\}$ and $A=\{2,3\}$

So $A\neq B$

$\therefore A\neq B$

ii. $A=\{x : x$ is a letter in the word FOLLOW$\}$;
$B=\{y : y$ is a letter in the word WOLF$\}$

Ans: Given that,

$A=\{x : x$ is a letter in the word FOLLOW$\}$; $B=\{y : y$ is a letter in the word WOLF$\}$

To state whether $A=B$

$A=\{x : x$ is a letter in the word FOLLOW$\}=\{F,O,L,W\}$

$B=\{y : y$ is a letter in the word WOLF$\}=\{W,O,L,F\}$

We know that the order in which the elements are listed are insignificant. So $A=B$

$\therefore A=B$

6. From the sets given below, select equal sets:

$A=\{2,4,8,12\}, B=\{1,2,3,4\}, C=\{4,8,12,14\}, D=\{3,1,4,2\}$
$E=\{-1,1\}, F=\{0,a\}, G=\{1,-1\}, H=\{0,1\}$

Ans: Given that,

$A=\{2,4,8,12\}, B=\{1,2,3,4\}, C=\{4,8,12,14\}, D=\{3,1,4,2\}$

$E=\{-1,1\}, F=\{0,a\}, G=\{1,-1\}, H=\{0,1\}$

To select equal sets from the given set

Two sets A and B are said to be equal if they have exactly the same elements and we write A = B

We can observe from the sets that,

$8\in A, 8\notin B, 8\notin D, 8\notin E, 8\notin F, 8\notin G, 8\notin H$

And thus

$A\neq B, A\neq D, A\neq E, A\neq F, A\neq G, A\neq H$

But $8\in C$

And checking other elements,

$2\in A, 2\notin C$

So $A\neq C$

$3\in B, 3\notin C, 3\notin E, 3\notin F, 3\notin G, 3\notin H$

And thus,

$B\neq C, B\neq E, B\neq F, B\neq G, B\neq H$

$12\in C, 12\notin D, 12\notin E, 12\notin F, 12\notin G, 12\notin H$

And thus

$C\neq D, C\neq E, C\neq F, C\neq G, C\neq H$

$4\in D, 4\notin E, 4\notin F, 4\notin G, 4\notin H$

And thus,

$D\neq E, D\neq F, D\neq G, D\neq H$

Similarly $E\neq F, E\neq G, E\neq H$

$F\neq G, F\neq H$

$G \neq H$

We know that the order of the elements I which they are listed in insignificant.

So $B = D, E = G$

$\therefore$ He equal sets are $B = D$ and $E = G$

Exercise 1.3

1. **Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces.**

i. $\{2,3,4\} ... \{1,2,3,4,5\}$

Ans: Given that,

$\{2,3,4\} ... \{1,2,3,4,5\}$
To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

The element in the set $\{2,3,4\}$ is also in the set $\{1,2,3,4,5\}$

$\therefore \{2,3,4\} \subset \{1,2,3,4,5\}$

ii. $\{a,b,c\} ... \{b,c,d\}$

Ans: Given that,

$\{a,b,c\} ... \{b,c,d\}$
To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

The element in the set $\{a,b,c\}$ is not in the set $\{b,c,d\}$

$\therefore \{a,b,c\} \not\subset \{b,c,d\}$

iii. $\{\mathbf{x : x \text{ is a student of class XI of your school}}\}$**...**

$\{\mathbf{x : x \text{ is a student of your school}}\}$

Ans: Given that,

$\{x : x \text{ is a student of class XI of your school}\}...$

$\{x : x \text{ is a student of your school}\}$

To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

The set of students of class XI would also be inside the set of students in school

$\ni \{x : x \text{ is a student of class XI of your school}\} \subset \{x : x \text{ is a student of your school}\}$

iv. $\{\mathbf{x : x \text{ is a circle in the plane }}\}$**...**

$\{\mathbf{x : x \text{ is a circle in the same plane with radius 1 unit}}\}$

Ans: Given that,

$\{x : x \text{ is a circle in the plane }\}...$

$\{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

The set of circles in the plane with a unit radius will be in the set of the circles in the same plane. So the set of circles in the plane is not in the set of circles with unit radius in the same plane.

$\therefore \{x : x \text{ is a circle in the plane }\} \not\subset$
$\{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

v. $\{x : x$ is a triangle in the plane$\}$... $\{x : x$ is a rectangle in the plane$\}$

Ans: Given that,

$\{x : x \text{ is a triangle in the plane}\}...$

$\{x : x \text{ is a rectangle in the plane}\}$

To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

From the given expression itself, we know that the set of triangles in the plane are not in the set of rectangles in the plane.

$\therefore \{x : x \text{ is a triangle in the plane}\} \not\subset \{x : x \text{ is a rectangle in the plane}\}$

vi. $\{x : x$ is an equilateral triangle in the plane$\}$...
$\{x : x$ is a triangle in the plane$\}$

Ans: Given that,

$\{x : x \text{ is an equilateral triangle in the plane}\}...\{x : x \text{ is a triangle in the plane}\}$

To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

From the above expression, we know that the set of equilateral triangles in the plane is in the set of triangles in the same plane

$\therefore \{x : x \text{ is an equilateral triangle in the plane}\} \subset \{x : x \text{ is a triangle in the plane}\}$

vii. $\{\mathbf{x : x \text{ is an even natural number}}\}\mathbf{...}\{\mathbf{x : x \text{ is an integer}}\}$

Ans: Given that,

$\{x : x \text{ is an even natural number}\}...\{x : x \text{ is an integer}\}$

To fill in the correct symbols $\subset$ or $\not\subset$ inn the blank spaces

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

The set of even natural numbers are in the set of integers.

$\therefore \{x : x \text{ is an even natural number}\} \subset \{x : x \text{ is an integer}\}$

2. Examine whether the following statements are true or false

i. $\{\mathbf{a,b}\} \not\subset \{\mathbf{b,c,a}\}$

Ans: Given that,

$\{a,b\} \not\subset \{b,c,a\}$

To examine whether the above statement is true or false

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

The element in the set $\{a,b\}$ is also in the set $\{b,c,a\}$

$\therefore \{a,b\} \subset \{b,c,a\}$

$\therefore$ The given statement is false

ii. $\{\mathbf{a,e}\} \subset \{\mathbf{x : x \text{ is an vowel in English alpahbet}}\}$

Ans: Given that,

$\{a, e\} \subset \{x : x \text{ is an vowel in English alpahbet}\}$

To examine whether the above statement is true or false

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

The element in the set $\{a, e\}$ is also in the set $\{a, e, i, o, u\}$

$\therefore \{a, e\} \subset \{x : x \text{ is an vowel in English alpahbet}\}$

$\therefore$ The given statement is true.

iii. $\mathbf{\{1, 2, 3\} \subset \{1, 3, 5\}}$

Ans: Given that,

$\{1, 2, 3\} \subset \{1, 3, 5\}$

To examine whether the above statement is true or false

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

The element in the set $\{1, 2, 3\}$ is not in the set $\{1, 3, 5\}$ since $2 \in \{1, 2, 3\}$ and $2 \notin \{1, 3, 5\}$

$\{1, 2, 3\} \not\subset \{1, 3, 5\}$

$\therefore$ The given statement is false.

iv. $\mathbf{\{a\} \subset \{a, b, c\}}$

Ans: Given that,

$\{a\} \subset \{a, b, c\}$

To examine whether the above statement is true or false

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

The element in the set $\{a\}$ is also in the set $\{a,b,c\}$

$\therefore \{a\} \subset \{a,b,c\}$

$\therefore$ The given statement is true.

v. $\{a\} \in \{a,b,c\}$

Ans: Given that,

$\{a\} \in \{a,b,c\}$

To examine whether the above statement is true or false

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

The element in the set $\{a\}$ and the elements in the set $\{a,b,c\}$ are a,b,c

$\{\therefore\ a\} \subset \{a,b,c\}$

$\therefore$ The given statement is false.

vi. $\{x : x \text{ is an even natural less than } 6\} \subset$
$\{x : x \text{ is a natural number which divide } 36\}$

Ans: Given that,

$\{x : x \text{ is an even natural less than } 6\} \subset \{x : x \text{ is a natural number which divide } 36\}$

To examine whether the above statement is true or false

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

$\{x : x \text{ is an even natural less than } 6\} = \{2,4\}$

$\{x : x \text{ is a natural number which divide } 36\} = \{1,2,3,4,6,9,12,18,36\}$
$\therefore \{x : x \text{ is an even natural less than } 6\} \subset$
$\{x : x \text{ is a natural number which divide } 36\}$

$\therefore$ The given statement is true.

3. Let $A = \{1,2,\{3,4\},5\}$. Which of the following statements are incorrect and why?
i. $\{3,4\} \subset A$

Ans: Given that,

$A = \{1,2,\{3,4\},5\}$

To find if $\{3,4\} \subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

From the above statement,

$3 \in \{3,4\}$, however $3 \notin A$

$\therefore$ The given statement $\{3,4\} \subset A$ is incorrect

ii. $\{3,4\} \in A$

Ans: Given that,

$A = \{1,2,\{3,4\},5\}$

To find if $\{3,4\} \in A$ is correct or incorrect.

From the above statement,

$\{3,4\}$ is an element of A.

$\therefore \{3,4\} \in A$

$\therefore$ The given statement is correct.

iii. $\mathbf{\{\{3,4\}\} \subset A}$

Ans: Given that,

$A = \{1, 2, \{3,4\}, 5\}$

To find if $\{\{3,4\}\} \subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

From the above statement,

$\{3,4\} \in \{\{3,4\}\}$ so that $\{\{3,4\}\} \in A$

$\therefore \{\{3,4\}\} \subset A$

$\therefore$ The given statement $\{\{3,4\}\} \subset A$ is correct.

iv. $\mathbf{1 \in A}$

Ans: Given that,

$A = \{1, 2, \{3,4\}, 5\}$

To find if $1 \in A$ is correct or incorrect.

From the above statement,

1 is an element of A.

$\therefore$ The statement $1 \in A$ is a correct statement.

v. $1 \subset A$

Ans: Given that,

$A = \{1, 2, \{3, 4\}, 5\}$

To find if $1 \subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

From the above statement,

An element of a set can never be a subset of itself. So $1 \not\subset A$

$\therefore$ The given statement $1 \subset A$ is incorrect statement.

vi. $\{1, 2, 5\} \subset A$

Ans: Given that,

$A = \{1, 2, \{3, 4\}, 5\}$

To find if $\{1, 2, 5\} \subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

From the above statement,

The each element of $\{1, 2, 5\}$ is also an element of A, So $\{1, 2, 5\} \subset A$

$\therefore$ The given statement $\{1, 2, 5\} \subset A$ is a correct statement

vii. $\{1, 2, 5\} \in A$

Ans: Given that,

$A=\{1,2,\{3,4\},5\}$

To find if $\{1,2,5\}\subset A$ is correct or incorrect.

From the above statement,

Element of $\{1,2,5\}$ is not an element of A, So $\{1,2,5\}\notin A$

So the given statement $\{1,2,5\}\in A$ is an incorrect statement.

viii. $\{1,2,3\}\subset A$

Ans: Given that,

$A=\{1,2,\{3,4\},5\}$

To find if $\{1,2,3\}\subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A\subset B$ if $a\in A, a\in B$

From the above statement, we notice that,

$3\in\{1,2,3\}$ but $3\notin A$

$\{1,2,3\}\not\subset A$

$\therefore$ The given statement $\{1,2,3\}\subset A$ is an incorrect statement.

ix. $\varnothing\in A$

Ans: Given that,

$A=\{1,2,\{3,4\},5\}$

To find if $\varnothing\in A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

From the above statement,

$\varnothing$ is not an element of A. So, $\varnothing \notin A$

$\therefore$ The given statement $\varnothing \in A$ is an incorrect statement.

x. $\varnothing \subset A$

Ans: Given that,

$A = \{1, 2, \{3, 4\}, 5\}$

To find if $\varnothing \subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

From the above statement,

Since $\varnothing$ is a subset of every set, $\varnothing \subset A$

$\therefore$ The given statement $\varnothing \subset A$ is a correct statement.

xi. $\{\varnothing\} \subset A$

Ans: Given that,

$A = \{1, 2, \{3, 4\}, 5\}$

To find if $\{\varnothing\} \subset A$ is correct or incorrect.

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B \text{ if } a \in A, a \in B$

From the above statement,

$\varnothing$ is an element of A and it is not a subset of A.

$\therefore$ The given statement $\{\varnothing\} \subset A$ is an incorrect statement.

4. Write down all the subsets of the following sets:

i. $\{a\}$

Ans: Given that,

$\{a\}$

To write the subset of the given sets

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

Subsets of $\{a\}$ are $\varnothing$ and $\{a\}$

ii. $\{a, b\}$

Ans: Given that,

$\{a, b\}$

To write the subset of the given sets

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

Subsets of $\{a, b\}$ are $\varnothing$ and $\{a\}, \{b\}, \{a, b\}$

iii. $\{1, 2, 3\}$

Ans: Given that,

$\{1, 2, 3\}$

To write the subset of the given sets

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

Subsets of $\{1,2,3\}$ are $\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}$

iv. $\boldsymbol{\varnothing}$

Ans: Given that,

$\varnothing$

To write the subset of the given sets

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

Subsets of $\varnothing$ is $\varnothing$.

5. Write the following as intervals

i. $\boldsymbol{\{x : x \in R, -4 < x \le 6\}}$

Ans: Given that,

$\{x : x \in R, -4 < x \le 6\}$

To write the above expression as intervals

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a,b). The interval which contains the end points also is called close interval and is denoted by $[a,b]$

$\therefore \{x : x \in R, -4 < x \le 6\} = (-4, 6]$

ii. $\{\mathbf{x : x \in R, -12 < x < -10}\}$

Ans: Given that,

$\{x : x \in R, -12 < x < -10\}$

To write the above expression as intervals

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a, b). The interval which contains the end points also is called close interval and is denoted by $[a, b]$

$\therefore \{x : x \in R, -12 < x < -10\} = (-12, -10)$

iii. $\{\mathbf{x : x \in R, 0 \le x < 7}\}$

Ans: Given that,

$\{x : x \in R, 0 \le x < 7\}$

To write the above expression as intervals

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a, b). The interval which contains the end points also is called close interval and is denoted by $[a, b]$

$\because \{x : x \in R, 0 \le x < 7\} = [0, 7)$

iv. $\{\mathbf{x : x \in R, 3 \le x \le 4}\}$

Ans: Given that,

$\{x : x \in R, 3 \le x \le 4\}$

To write the above expression as intervals

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a, b). The interval which contains the end points also is called close interval and is denoted by $[a, b]$

$\therefore \{x : x \in R, 3 \le x \le 4\} = [3, 4]$

6. Write the following intervals in set builder form.

i. $\mathbf{(-3, 0)}$

Ans: Given that,

$(-3, 0)$

To write the above interval in set builder form

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a, b). The interval which contains the end points also is called close interval and is denoted by $[a, b]$

$\therefore (-3, 0) = \{x : x \in R, -3 < x < 0\}$

ii. $[6,12]$

Ans: Given that,

$[6,12]$

To write the above interval in set builder form

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a,b). The interval which contains the end points also is called close interval and is denoted by $[a,b]$

$\therefore [6,12] = \{x : x \in R, 6 \le x \le 12\}$

iii. (6,12]

Ans: Given that,

$(6,12]$

To write the above interval in set builder form

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a,b). The interval which contains the end points also is called close interval and is denoted by $[a,b]$

$\therefore (6,12] = \{x : x \in R, 6 < x \le 12\}$

iv. $[-23,5)$

Ans: Given that,

$[-23,5)$

To write the above interval in set builder form

The set of real numbers $\{y : a < y < b\}$ is called an open interval and is denoted by (a, b). The interval which contains the end points also is called close interval and is denoted by $[a, b]$

$\therefore [-23,5) = \{x : x \in R, -23 \le x < 5\}$

7. **What universal set(s) would you propose for each of the following:**

i. The set of right triangles

Ans: To propose the universal set for the set of right triangles

For the set of right triangles, the universal set can be the set of all kinds of triangles or the set of polygons.

ii. The set of isosceles triangles

Ans: To propose the universal set for the set of right triangles

For the set of isosceles triangles, the universal set can be the set of all kinds of triangles or the set of polygons or the set of two dimensional figures.

8. **Given the sets $A = \{1,3,5\}, B = \{2,4,6\}$ and $C = \{0,2,4,6,8\}$ Which of the following may be considered as a universal set(s) for all the three sets A, B and C?**

i. $\{0,1,2,3,4,5,6\}$

Ans: Given that,

$A = \{1,3,5\}, B = \{2,4,6\}, C = \{0,2,4,6,8\}$

To find if the given set $\{0,1,2,3,4,5,6\}$ is the universal set of A, B and C

It can be observed that,

$A \subset \{0,1,2,3,4,5,6\}$

$B \subset \{0,1,2,3,4,5,6\}$

$C \not\subset \{0,1,2,3,4,5,6\}$

$\therefore$ The set $\{0,1,2,3,4,5,6\}$ cannot be the universal set for the sets A, B and C

ii. $\varnothing$

Ans: Given that,

$A = \{1,3,5\}, B = \{2,4,6\}, C = \{0,2,4,6,8\}$

To find if the given set $\varnothing$ is the universal set of A, B and C

It can be observed that,

$A \not\subset \varnothing$

$B \not\subset \varnothing$

$C \not\subset \varnothing$

$\therefore$ The set $\varnothing$ cannot be an universal set for A, B and C.

iii. $\mathbf{\{0,1,2,3,4,5,6,7,8,9,10\}}$

Ans: Given that,

$A = \{1,3,5\}, B = \{2,4,6\}, C = \{0,2,4,6,8\}$

To find if the given set $\{0,1,2,3,4,5,6,7,8,9,10\}$ is the universal set of A, B and C

It can be observe that,

$A \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$B \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$C \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$\therefore$ The set $\{0,1,2,3,4,5,6,7,8,9,10\}$ is the universal set of A, B and C

iv. $\{1,2,3,4,5,6,7,8\}$

Ans: Given that,

$A = \{1,3,5\}, B = \{2,4,6\}, C = \{0,2,4,6,8\}$

To find if the given set $\{0,1,2,3,4,5,6\}$ is the universal set of A, B and C

It can be observed that,

$A \subset \{1,2,3,4,5,6,7,8\}$

$B \subset \{1,2,3,4,5,6,7,8\}$

$C \not\subset \{1,2,3,4,5,6,7,8\}$

$\therefore$ The set $\{1,2,3,4,5,6,7,8\}$ is not the universal set of A, B and C

Exercise 1.4

1. Find the union of each of following pair of sets

i. $X = \{1,3,5\}, Y = \{1,2,3\}$

Ans: Given that,

$X = \{1,3,5\}, Y = \{1,2,3\}$

To find the union of two sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$X \cup Y = \{1,3,5\} \cup \{1,2,3\}$

$\therefore X \cup Y = \{1,2,3,5\}$

ii. $\mathbf{A = \{a,e,i,o,u\}, B = \{a,b,c\}}$

Ans: Given that,

$A = \{a,e,i,o,u\}, B = \{a,b,c\}$

To find the union of two sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A \cup B = \{a,e,i,o,u\} \cup \{a,b,c\}$

$\therefore A \cup B = \{a,b,c,e,i,o,u\}$

iii. $\mathbf{A = \{x : x \text{ is a natural number an multiple of } 3\},}$
$\mathbf{B = \{x : x \text{ is a natural number less than } 6\}}$

Ans: Given that,

$A = \{x : x \text{ is a natural number an multiple of } 3\},$

$B = \{x : x \text{ is a natural number less than } 6\}$

To find the union of two sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A = \{x : x \text{ is a natural number an multiple of } 3\},$

$= \{3,6,9,...\}$

$B = \{x : x \text{ is a natural number less than } 6\}$

$=\{1,2,3,4,5,6\}$

$A\cup B=\{3,6,9,...\}\cup\{1,2,3,4,5,6\}$

$=\{1,2,3,4,5,6,9,12,15...\}$

$\therefore A\cup B=\{1,2,3,4,5,6,9,12,15...\}$

iv. $\mathbf{A=\{x:x \text{ is a natural number } 1<x\le 6\},}$

$\mathbf{B=\{x:x \text{ is anatural number } 6<x<10\}}$

Ans: Given that,

$A=\{x:x \text{ is a natural number } 1<x\le 6\}, B=\{x:x \text{ is a natural number } 6<x<10\}$

To find the union of two sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A=\{x:x \text{ is a natural number } 1<x\le 6\}=\{2,3,4,5,6\}$

$B=\{x:x \text{ is a natural number } 6<x<10\}=\{7,8,9\}$

$A\cup B=\{2,3,4,5,6\}\cup\{7,8,9\}$

$\therefore A\cup B=\{2,3,4,5,6,7,8,9\}$

v. $\mathbf{A=\{1,2,3\},B=\varnothing}$

Ans: Given that,

$A=\{1,2,3\},B=\varnothing$

To find the union of two sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A \cup B = \{1,2,3\} \cup \varnothing$

$\therefore A \cup B = \{1,2,3\}$

2. Let $A = \{a,b\}$ and $B = \{a,b,c\}$. Is $A \subset B$? What is $A \cup B$?

Ans: Given that,

$A = \{a,b\}$ and $B = \{a,b,c\}$

To find if $A \subset B$ and $A \cup B$

A set A is said to be a subset of B if every element of A is also an element of B

$A \subset B$ if $a \in A, a \in B$

It can be observed that $A \subset B$

$A \cup B = \{a,b\} \cup \{a,b,c\}$

$\therefore A \cup B = \{a,b,c\}$

3. If A and B are two sets such that $A \subset B$ then what is $A \cup B$

Ans: Given that,

A and B are two sts

To find $A \cup B$ when $A \subset B$

If A and B are two sets such that $A \subset B$, then $A \cup B = B$

4. If $A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{5,6,7,8\}$ and $D = \{7,8,9,10\}$; find

i. $A \cup B$

Ans: Given that,

$A=\{1,2,3,4\}, B=\{3,4,5,6\}, C=\{5,6,7,8\}$, $D=\{7,8,9,10\}$

To find,

$A \cup B$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A \cup B=\{1,2,3,4\} \cup\{3,4,5,6\}$

$\therefore A \cup B=\{1,2,3,4,5,6\}$

ii. $A \cup C$

Ans: Given that,

$A=\{1,2,3,4\}, B=\{3,4,5,6\}, C=\{5,6,7,8\}$, $D=\{7,8,9,10\}$

To find,

$A \cup C$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A \cup C=\{1,2,3,4\} \cup\{5,6,7,8\}$

$\therefore A \cup C=\{1,2,3,4,5,6,7,8\}$

iii. $B \cup C$

Ans: Given that,

$A=\{1,2,3,4\}, B=\{3,4,5,6\}, C=\{5,6,7,8\}$, $D=\{7,8,9,10\}$

To find,

$B \cup C$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$B \cup C = \{3,4,5,6\} \cup \{5,6,7,8\}$

$\therefore B \cup C = \{3,4,5,6,7,8\}$

iv. $B \cup D$

Ans: Given that,

$A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{5,6,7,8\}$, $D = \{7,8,9,10\}$

To find,

$B \cup D$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$B \cup D = \{3,4,5,6\} \cup \{7,8,9,10\}$

$\therefore B \cup D = \{3,4,5,6,7,8,9,10\}$

v. $A \cup B \cup C$

Ans: Given that,

$A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{5,6,7,8\}$, $D = \{7,8,9,10\}$

To find,

$A \cup B \cup C$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$A \cup B \cup C = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{5,6,7,8\}$

$\therefore A \cup B \cup C = \{1,2,3,4,5,6,7,8\}$

vi. $A \cup B \cup D$

Ans: Given that,

$$A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{5,6,7,8\}, D = \{7,8,9,10\}$$

To find,

$$A \cup B \cup D$$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$$A \cup B \cup D = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{7,8,9,10\}$$

$$\therefore A \cup B \cup D = \{1,2,3,4,5,6,7,8,9,10\}$$

vii. $B \cup C \cup D$

Ans: Given that,

$$A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{5,6,7,8\}, D = \{7,8,9,10\}$$

To find,

$$B \cup C \cup D$$

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and B.

$$B \cup C \cup D = \{3,4,5,6\} \cup \{5,6,7,8\} \cup \{7,8,9,10\}$$

$$\therefore B \cup C \cup D = \{3,4,5,6,7,8,9,10\}$$

5. Find the intersection of each pair of sets of question 1 above.

Ans: (i) $X \cap Y = \{1,3\}$
(ii) $A \cap B = \{a\}$
(iii) $A \cap B = \{3\}$
(iv) $A \cap B = \{\phi\}$
(v) $A \cap B = \{\phi\}$

6. **If $A=\{3,5,7,9,11\}, B=\{7,9,11,13\}, C=\{11,13,15\}$ and $D=\{15,17\}$; find**

i. $\mathbf{A \cap B}$

Ans: Given that,

$$A=\{3,5,7,9,11\}, B=\{7,9,11,13\}, C=\{11,13,15\}, D=\{15,17\}$$

To find,

$$A \cap B$$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$A \cap B=\{3,5,7,9,11\} \cap \{7,9,11,13\}$$

$$\therefore A \cap B=\{7,9,11\}$$

ii. $\mathbf{B \cap C}$

Ans: Given that,

$$A=\{3,5,7,9,11\}, B=\{7,9,11,13\}, C=\{11,13,15\}, D=\{15,17\}$$

To find,

$$B \cap C$$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$B \cap C=\{7,9,11,13\} \cap \{11,13,15\}$$

$$\therefore B \cap C=\{11,13\}$$

iii. $A \cap C \cap D$

Ans: Given that,

$$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$$

To find,

$$A \cap C \cap D$$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$A \cap C \cap D = \{3,5,7,9,11\} \cap \{11,13,15\} \cap \{15,17\}$$

$$\therefore A \cap C \cap D = \varnothing$$

iv. $A \cap C$

Ans: Given that,

$$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$$

To find,

$$A \cap C$$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$A \cap C = \{3,5,7,9,11\} \cap \{11,13,15\}$$

$\therefore A \cap C = \{11\}$

v. $\mathbf{B \cap D}$

Ans: Given that,

$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$

To find,

$B \cap D$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$B \cap D = \{7,9,11,13\} \cap \{15,17\}$

$\therefore B \cap D = \varnothing$

vi. $\mathbf{A \cap (B \cup C)}$

Ans: Given that,

$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$

To find,

$A \cap (B \cup C)$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\}$

$A \cap B = \{7,9,11\}$

$A \cap D = \{11\}$

$$A \cap (B \cup C) = \{7,9,11\} \cup \{11\}$$

$$= \{11\}$$

$$\therefore A \cap (B \cup C) = \{11\}$$

vii. $A \cap D$

Ans: Given that,

$$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$$

To find,

$$A \cap D$$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$A \cap D = \{3,5,7,9,11\} \cap \{15,17\}$$

$$\therefore A \cap D = \varnothing$$

viii. $A \cap (B \cup D)$

Ans: Given that,

$$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$$

To find,

$$A \cap (B \cup D)$$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$$

$$A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\}$$

$A \cap D = \varnothing$

$\therefore A \cap (B \cup D) = \{7,9,11\} \cup \varnothing$

$= \{7,9,11\}$

ix. $\mathbf{(A \cap B) \cap (B \cup C)}$

Ans: Given that,

$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$

To find,

$(A \cap B) \cap (B \cup C)$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\}$

$A \cap B = \{7,9,11\}$

$B \cup C = \{7,9,11,13\} \cup \{11,13,15\}$

$= \{7,9,11,13,15\}$

$(A \cap B) \cap (B \cup C) = \{7,9,11\} \cap \{7,9,11,13,15\}$

$= \{7,9,11\}$

x. $\mathbf{(A \cup D) \cap (B \cup C)}$

Ans: Given that,

$A = \{3,5,7,9,11\}, B = \{7,9,11,13\}, C = \{11,13,15\}, D = \{15,17\}$

To find,

$(A \cup D) \cap (B \cup C)$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$A \cap D = \{3,5,7,9,11\} \cap \{15,17\}$

$A \cap D = \{3,5,7,9,11,15,17\}$

$B \cup C = \{7,9,11,13\} \cup \{11,13,15\}$

$= \{7,9,11,13,15\}$

$(A \cup D) \cap (B \cup C) = \{3,5,7,9,11,15,17\} \cap \{7,9,11,13,15\}$

$\therefore (A \cup D) \cap (B \cup C) = \{7,9,11,15\}$

7. If $A = \{x : x \text{ is a natural number}\}, B = \{x : x \text{ is an even natural number}\}$

$C = \{x : x \text{ is an odd natural number}\}, D = \{x : x \text{ is a prime number}\}$, find

i. $A \cap B$

Ans: Given that,

$A = \{x : x \text{ is a natural number}\} = \{1,2,3,4,...\}$

$B = \{x : x \text{ is an even natural number}\} = \{2,4,6,8...\}$

$C = \{x : x \text{ is an odd natural number}\} = \{1,3,5,7,...\}$

$D = \{x : x \text{ is a prime number}\} = \{2,3,5,7,...\}$

To find,

$A \cap B$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$A \cap B = \{1, 2, 3, 4, ...\} \cap \{2, 4, 6, 8...\}$

$\therefore A \cap B = B = \{x : x \text{ is an even natural number}\}$

ii. $A \cap C$

Ans: Given that,

$A = \{x : x \text{ is a natural number}\} = \{1, 2, 3, 4, ...\}$

$B = \{x : x \text{ is an even natural number}\} = \{2, 4, 6, 8...\}$

$C = \{x : x \text{ is an odd natural number}\} = \{1, 3, 5, 7, ...\}$

$D = \{x : x \text{ is a prime number}\} = \{2, 3, 5, 7, ...\}$

To find,

$A \cap C$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$A \cap C = \{1, 2, 3, 4, ...\} \cap \{1, 3, 5, 7...\}$

$\therefore A \cap C = C \{x : x \text{ is an odd natural number}\}$

iii. $A \cap D$

Ans: Given that,

$A = \{x : x \text{ is a natural number}\} = \{1, 2, 3, 4, ...\}$

$B = \{x : x \text{ is an even natural number}\} = \{2, 4, 6, 8...\}$

$C = \{x : x \text{ is an odd natural number}\} = \{1, 3, 5, 7, ...\}$

$D = \{x : x \text{ is a prime number}\} = \{2, 3, 5, 7, ...\}$

To find,

$A \cap D$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$A \cap D = \{1,2,3,4,...\} \cap \{2,3,5,7,...\}$

$\therefore A \cap D = D\,\{x : x \text{ is a prime number}\}$

iv. $\mathbf{B \cap C}$

Ans: Given that,

$A = \{x : x \text{ is a natural number}\} = \{1,2,3,4,...\}$

$B = \{x : x \text{ is an even natural number}\} = \{2,4,6,8...\}$

$C = \{x : x \text{ is an odd natural number}\} = \{1,3,5,7,...\}$

$D = \{x : x \text{ is a prime number}\} = \{2,3,5,7,...\}$

To find,

$B \cap C$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$B \cap C = \{2,4,6,8...\} \cap \{1,3,5,7...\}$

$\therefore B \cap C = \varnothing$

v. $\mathbf{B \cap D}$

Ans: Given that,

$A = \{x : x \text{ is a natural number}\} = \{1,2,3,4,...\}$

$B=\{x : x \text{ is an even natural number}\}=\{2,4,6,8 \ldots\}$

$C=\{x : x \text{ is an odd natural number}\}=\{1,3,5,7, \ldots\}$

$D=\{x : x \text{ is a prime number}\}=\{2,3,5,7, \ldots\}$

To find,

$B \cap D$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$B \cap D=\{2,4,6,8, \ldots\} \cap\{2,3,5,7, \ldots\}$

$\therefore A \cap D=\{2\}$

vi. $C \cap D$

Ans: Given that,

$A=\{x : x \text{ is a natural number}\}=\{1,2,3,4, \ldots\}$

$B=\{x : x \text{ is an even natural number}\}=\{2,4,6,8 \ldots\}$

$C=\{x : x \text{ is an odd natural number}\}=\{1,3,5,7, \ldots\}$

$D=\{x : x \text{ is a prime number}\}=\{2,3,5,7, \ldots\}$

To find,

$C \cap D$

The intersection of sets A and B is the set of all elements which are common to both A and B.

$C \cap D=\{1,3,5,7, \ldots\} \cap\{2,3,5,7, \ldots\}$

$\therefore C \cap D=\{x : x \text{ is a odd prime number}\}$

8. Which of the following pairs of sets are disjoint

i. $\{1,2,3,4\}$ and $\{x : x \text{ is a antural number and } 4 \le x \le 6\}$

Ans: Given that,

$\{1,2,3,4\}$ and

$\{x : x \text{ is a antural number and } 4 \le x \le 6\} = \{4,5,6\}$

To find if the given sets are disjoint

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\{1,2,3,4\} \cap \{4,5,6\} = \{4\}$

Thus the element exists.

$\therefore$ The given pair of sets is not a disjoint set

ii. $\{a,e,i,o,u\}$ and $\{c,d,e,f\}$

Ans: Given that,

$\{a,e,i,o,u\}$ and $\{c,d,e,f\}$

To find if the given sets are disjoint

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\{a,e,i,o,u\} \cap \{c,d,e,f\} = \{e\}$

Thus the element exists.

$\therefore$ The given pair of sets is not a disjoint set

iii. $\{x : x \text{ is an even integer}\}$ and $\{x : x \text{ is an odd integer}\}$

Ans: Given that,

$\{x : x \text{ is an even integer}\}$ and

$\{x : x \text{ is an odd integer}\}$

To find if the given sets are disjoint

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\{x : x \text{ is an even integer}\} \cap \{x : x \text{ is an odd integer}\} = \varnothing$

Thus the element does not exist.

$\therefore$ The given pair of sets is a disjoint set

9. **If $A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$,**
$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$

i. $A - B$

Ans: Given that,

$A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$
$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$

To find,

$A - B$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$A - B = \{3,6,9,12,15,18,21\} - \{4,8,12,16,20\}$

$\therefore A - B = \{3,6,9,15,18,21\}$

ii. $A - C$

Ans: Given that,

$$A=\{3,6,9,12,15,18,21\}, B=\{4,8,12,16,20\}$$
$$C=\{2,4,6,8,10,12,14,16\}, D=\{5,10,15,20\}$$

To find,

$$A-C$$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$$A-C=\{3,6,9,12,15,18,21\}-\{2,4,6,8,10,12,14,16\}$$

$$\therefore A-C=\{3,9,15,18,21\}$$

iii. $\mathbf{A-D}$

Ans: Given that,

$$A=\{3,6,9,12,15,18,21\}, B=\{4,8,12,16,20\}$$
$$C=\{2,4,6,8,10,12,14,16\}, D=\{5,10,15,20\}$$

To find,

$$A-D$$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$$A-D=\{3,6,9,12,15,18,21\}-\{5,10,15,20\}$$

$$\therefore A-D=\{3,6,9,15,18,21\}$$

iv. $\mathbf{B-A}$

Ans: Given that,

$$A=\{3,6,9,12,15,18,21\}, B=\{4,8,12,16,20\}$$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, D = \{5, 10, 15, 20\}$

To find,

$B - A$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$B - A = \{4, 8, 12, 16, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$

$\therefore B - A = \{4, 8, 16, 20\}$

v. C – A

Ans: Given that,

$A = \{3, 6, 9, 12, 15, 18, 21\}, B = \{4, 8, 12, 16, 20\}$
$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, D = \{5, 10, 15, 20\}$

To find,

$C - A$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$C - A = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{3, 6, 9, 12, 15, 18, 21\}$

$\therefore C - A = \{2, 4, 8, 10, 14, 16\}$

vi. D – A

Ans: Given that,

$A = \{3, 6, 9, 12, 15, 18, 21\}, B = \{4, 8, 12, 16, 20\}$
$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, D = \{5, 10, 15, 20\}$

To find,

$D - A$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$D - A = \{5,10,15,20\} - \{3,6,9,12,15,18,21\}$

$\therefore D - A = \{5,10,20\}$

vii. $B - C$

Ans: Given that,

$A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$
$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$

To find,

$B - C$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$B - C = \{4,8,12,16,20\} - \{2,4,6,8,10,12,14,16\}$

$\therefore B - C = \{20\}$

viii. $B - D$

Ans: Given that,

$A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$
$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$

To find,

$B - D$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$B - D = \{4, 8, 12, 16, 20\} - \{5, 10, 15, 20\}$

$\therefore B - D = \{4, 8, 12, 16\}$

ix. C – B

Ans: Given that,

$A = \{3, 6, 9, 12, 15, 18, 21\}, B = \{4, 8, 12, 16, 20\}$
$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, D = \{5, 10, 15, 20\}$

To find,

$C - B$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$C - B = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{4, 8, 12, 16, 20\}$

$\therefore C - B = \{2, 6, 10, 14\}$

x. D – B

Ans: Given that,

$A = \{3, 6, 9, 12, 15, 18, 21\}, B = \{4, 8, 12, 16, 20\}$
$C = \{2, 4, 6, 8, 10, 12, 14, 16\}, D = \{5, 10, 15, 20\}$

To find,

$D - B$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$D - B = \{5, 10, 15, 20\} - \{4, 8, 12, 16, 20\}$

$\therefore D - B = \{5, 10, 15\}$

xi. $C - D$

Ans: Given that,

$$A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$$
$$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$$

To find,

$C - D$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$$C - D = \{2,4,6,8,10,12,14,16\} - \{5,10,15,20\}$$

$$\therefore C - D = \{2,4,6,8,12,14,16\}$$

xii. $D - C$

Ans: Given that,

$$A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$$
$$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$$

To find,

$D - C$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$$D - C = \{5,10,15,20\} - \{2,4,6,8,10,12,14,16\}$$

$$\therefore D - C = \{5,10,15\}$$

10. If $X = \{a,b,c,d\}, Y = \{f,b,d,g\}$, find

i. $X - Y$

Ans: Given that,

$$X = \{a, b, c, d\}, Y = \{f, b, d, g\}$$

To find,

$$X - Y$$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$$X - Y = \{a, b, c, d\} - \{f, b, d, g\}$$

$$\therefore X - Y = \{a, c\}$$

ii. $\mathbf{Y - X}$

Ans: Given that,

$$X = \{a, b, c, d\}, Y = \{f, b, d, g\}$$

To find,

$$Y - X$$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$$Y - X = \{f, b, d, g\} - \{a, b, c, d\}$$

$$\therefore Y - X = \{f, g\}$$

iii. $\mathbf{X \cap Y}$

Ans: Given that,

$$X = \{a, b, c, d\}, Y = \{f, b, d, g\}$$

To find,

$X \cap Y$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$X \cap Y = \{a, b, c, d\} \cap \{f, b, d, g\}$

$\therefore X \cap Y = \{b, d\}$

11. **If R is the real numbers and Q is the set of rational numbers, then what is $\mathbf{R - Q}$?**

Ans: Given that,

R is the real numbers

Q is the set of rational numbers

To find,

$R - Q$

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\therefore R - Q$ is the set of irrational number.

12. **State whether each of the following statement is true or false. Justify your answer.**

i. $\mathbf{\{2,3,4,5\}}$ and $\mathbf{\{3,6\}}$ are disjoint sets

Ans: Given that,

$\{2,3,4,5\}, \{3,6\}$

To state whether the given statement is true

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\{2,3,4,5\} \cap \{3,6\} = \{3\}$

$\therefore$ The given statement is false.

ii. $\{a,e,i,o,u\}$ and $\{a,b,c,d\}$ are disjoint sets

Ans: Given that,

$\{a,e,i,o,u\},\{a,b,c,d\}$

To state whether the given statement is true

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\{a,e,i,o,u\} \cap \{a,b,c,d\} = \{a\}$

$\therefore$ The given statement is false.

iii. $\{2,6,10,14\}$ and $\{3,7,11,15\}$ are disjoint sets

Ans: Given that,

$\{2,6,10,14\},\{3,7,11,15\}$

To state whether the given statement is true

The difference of the sets A and B in this order is the set of elements which belong to A but not to B.

$\{2,6,10,14\} \cap \{3,7,11,15\} = \varnothing$

$\therefore$ The given statement is true.

iv. $\{2,6,10\}$ and $\{3,7,11\}$ are disjoint sets

Ans: Given that,

$\{2,6,10\},\{3,7,11\}$

To state whether the given statement is true

$\{2,6,10\} \cap \{3,7,11\} = \varnothing$

$\therefore$ The given statement is true.

Exercise 1.5

1. Let $\mathbf{U=\{1,2,3,4,5,6,7,8,9\}, A=\{1,2,3,4\}, B=\{2,4,6,8\}}$ and $\mathbf{C=\{3,4,5,6\}}$, find

i. $\mathbf{A'}$

Ans: Given that,

$U=\{1,2,3,4,5,6,7,8,9\}, A=\{1,2,3,4\}, B=\{2,4,6,8\}$, $C=\{3,4,5,6\}$

To find,

A'

The complement of set A is the set of all elements of U which are not the elements of A.

$A' = U - A$

$=\{1,2,3,4,5,6,7,8,9\}-\{1,2,3,4\}$

$=\{5,6,7,8,9\}$

$\therefore A' = \{5,6,7,8,9\}$

ii. $\mathbf{B'}$

Ans: Given that,

$U=\{1,2,3,4,5,6,7,8,9\}, A=\{1,2,3,4\}, B=\{2,4,6,8\}$, $C=\{3,4,5,6\}$

To find,

B'

The complement of set A is the set of all elements of U which are not the elements of A.

$B'=U-B$

$=\{1,2,3,4,5,6,7,8,9\}-\{2,4,6,8\}$

$=\{1,3,5,7,9\}$

$\therefore B'=\{1,3,5,7,9\}$

iii. $\mathbf{(A\cup C)'}$

Ans: Given that,

$U=\{1,2,3,4,5,6,7,8,9\}, A=\{1,2,3,4\}, B=\{2,4,6,8\}$, $C=\{3,4,5,6\}$

To find,

$(A\cup C)'$

The complement of set A is the set of all elements of U which are not the elements of A.

$A\cup C=\{1,2,3,4,5,6\}$

$(A\cup C)'=U-(A\cup C)$

$=\{1,2,3,4,5,6,7,8,9\}-\{1,2,3,4,5,6\}$

$=\{7,8,9\}$

$\therefore (A\cup C)'=\{7,8,9\}$

iv. $\mathbf{(A \cup B)'}$

Ans: Given that,

$U = \{1,2,3,4,5,6,7,8,9\}, A = \{1,2,3,4\}, B = \{2,4,6,8\}$, $C = \{3,4,5,6\}$

To find,

$(A \cup B)'$

The complement of set A is the set of all elements of U which are not the elements of A.

$A \cup B = \{1,2,3,4,5,6,8\}$

$(A \cup B)' = U - A \cup B$

$= \{1,2,3,4,5,6,7,8,9\} - \{1,2,3,4,5,6,8\}$

$= \{5,7,9\}$

$\therefore (A \cup B)' = \{5,7,9\}$

v. $\mathbf{(A')'}$

Ans: Given that,

$U = \{1,2,3,4,5,6,7,8,9\}, A = \{1,2,3,4\}, B = \{2,4,6,8\}$, $C = \{3,4,5,6\}$

To find,

$(A')'$

The complement of set A is the set of all elements of U which are not the elements of A.

$(A')' = A$

$= \{1,2,3,4\}$

$\therefore (A')' = \{1,2,3,4\}$

vi. $\mathbf{(B - C)'}$

Ans: Given that,

$U = \{1,2,3,4,5,6,7,8,9\}, A = \{1,2,3,4\}, B = \{2,4,6,8\}$, $C = \{3,4,5,6\}$

To find,

$(B - C)'$

The complement of set A is the set of all elements of U which are not the elements of A.

$B - C = \{2,8\}$

$(B - C)' = U - (B - C)$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,8\}$

$= \{1,3,4,5,6,7,9\}$

$\therefore (B - C)' = \{1,3,4,5,6,7,9\}$

2. If $\mathbf{U = \{a,b,c,d,e,f,g,h\}}$**, then find the complements of the following sets:**

i. $\mathbf{A = \{a,b,c\}}$

Ans: Given that,

$U = \{a,b,c,d,e,f,g,h\}$

$A = \{a,b,c\}$

To find the complement of A

The complement of set A is the set of all elements of U which are not the elements of A.

$A' = U - A$

$= \{a, b, c, d, e, f, g, h\} - \{a, b, c\}$

$= \{d, e, f, g, h\}$

$\therefore$ The complement of A is $A' = \{d, e, f, g, h\}$

ii. $\mathbf{B = \{d, e, f, g\}}$

Ans: Given that,

$U = \{a, b, c, d, e, f, g, h\}$

$b = \{d, e, f, g\}$

To find the complement of B

The complement of set A is the set of all elements of U which are not the elements of A.

$B' = U - B$

$= \{a, b, c, d, e, f, g, h\} - \{d, e, f, g\}$

$= \{a, b, c, h\}$

$\therefore$ The complement of B is $B' = \{b, e, c, h\}$

iii. $\mathbf{C = \{a, c, e, g\}}$

Ans: Given that,

$U = \{a, b, c, d, e, f, g, h\}$

$C = \{a, c, e, g\}$

To find the complement of A

The complement of set A is the set of all elements of U which are not the elements of A.

$C' = U - C$

$= \{a, b, c, d, e, f, g, h\} - \{a, c, e, g\}$

$= \{b, d, f, h\}$

$\therefore$ The complement of C is $C' = \{b, d, f, h\}$

iv. $\mathbf{D = \{f, g, h, a\}}$

Ans: Given that,

$U = \{a, b, c, d, e, f, g, h\}$

$A = \{f, g, h, a\}$

To find the complement of A

The complement of set A is the set of all elements of U which are not the elements of A.

$D' = U - D$

$= \{a, b, c, d, e, f, g, h\} - \{f, g, h, a\}$

$= \{b, c, d, e\}$

$\therefore$ The complement of D is $D' = \{b, c, d, e\}$

3. Taking the set of natural numbers as the universal set, write down the complements of the following sets:

i. $\{x : x \text{ is an even natural number}\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of even natural number

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is an even natural number}\}' = \{x : x \text{ is an odd natural number}\}$

ii. $\{x : x \text{ is an odd natural number}\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of odd natural number

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is an odd natural number}\}' = \{x : x \text{ is an even natural number}\}$

iii. $\{x : x \text{ is a positive multiple of } 3\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of positive multiples of 3

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is a positive multiple of } 3\}' = \{x : x \in N \text{ and } x \text{ is not a posiitve multiple of } 3\}$

iv. $\{x : x \text{ is a prime number}\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of prime number

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is a prime number}\}' = \{x : x \text{ is a positive composite number and } x=1\}$

v. $\{x : x \text{ is a natural number divisible by 3 and 5}\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of natural number divisible by 3 and 5

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is a number divisible by 3 and 5}\}' =$

$\{x : x \text{ is a natural number that is not divisible by 3 or 5}\}$

vi. $\{x : x \text{ is a perfect square}\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of perfect squares.

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is a perfect squares}\}' = \{x : x \in N \text{ and x is not a perfect square}\}$

vii. $\{x : x \text{ is a perfect cube}\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the set of perfect cube

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \text{ is a perfect cube}\}' = \{x : x \in N \text{ and } x \text{ is not a perfect cube}\}$

viii. $\{x : x + 5 = 8\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of $\{x : x + 5 = 8\}$

$x + 5 = 8$

$x = 3$

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x + 5 = 8\}' = \{x : x \in N \text{ and } x \neq 3\}$

ix. $\{x : 2x + 5 = 9\}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the

$\{x : 2x + 5 = 9\}$

The complement of set A is the set of all elements of U which are not the elements of A.

$2x+5=9$

$2x=4$

$x=2$

$\therefore \{x : 2x+5=9\}' = \{x : x \in N \text{ and } x \neq 2\}$

x. $\boldsymbol{\{x : x \geq 7\}}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of

$\{x : x \geq 7\}$

The complement of set A is the set of all elements of U which are not the elements of A.

$\therefore \{x : x \geq 7\}' = \{x : x \in N \text{ and } x<7\}$

xi. $\boldsymbol{\{x : x \in N \text{ and } 2x+1>10\}}$

Ans: Given that,

The set of natural number is the universal set

To find the complement of the

$\{x : x \in N \text{ and } 2x+1>10\}$

The complement of set A is the set of all elements of U which are not the elements of A.

$2x+1>10$

$2x > 9$

$x > \frac{9}{2}$

$\therefore \{x : x \in N \text{ and } 2x+1>10\}' = \left\{x : x \in N \text{ and } x \le \frac{9}{2}\right\}$

4. **If $U = \{1,2,3,4,5,6,7,8,9\}, A = \{2,4,6,8\}$ and $B = \{2,3,5,7\}$. Verify that,**

i. $(A \cup B)' = A' \cap B'$

Ans: Given that.

$U = \{1,2,3,4,5,6,7,8,9\}$

$A = \{2,4,6,8\}$

$B = \{2,3,5,7\}$

To prove that $(A \cup B)' = A' \cap B'$

$A \cup B = \{2,4,6,8\} \cup \{2,3,5,7\}$

$= \{2,3,4,5,6,7,8\}$

$(A \cup B)' = U = A \cup B$

$= \{1,9\}$

$A' = U - A$

$= \{1,3,5,7,9\}$

$B' = U - B$

$= \{1,4,6,8,9\}$

$A' \cap B' = \{1,3,5,7,9\} \cap \{1,4,6,8,9\}$

$=\{1,9\}$

Hence it has been proved that $(A\cup B)'=A'\cap B'$

ii. $(A\cap B)'=A'\cup B'$

Ans: Given that.

$U=\{1,2,3,4,5,6,7,8,9\}$

$A=\{2,4,6,8\}$

$B=\{2,3,5,7\}$

To prove that $(A\cap B)'=A'\cup B'$

$A\cap B=\{2,4,6,8\}\cap\{2,3,5,7\}$

$=\{2\}$

$(A\cap B)'=U-A\cap B$

$=\{1,3,4,5,6,7,8,9\}$

$A'=U-A$

$=\{1,3,5,7,9\}$

$B'=U-B$

$=\{1,4,6,8,9\}$

$A'\cup B'=\{1,3,5,7,9\}\cup\{1,4,6,8,9\}$

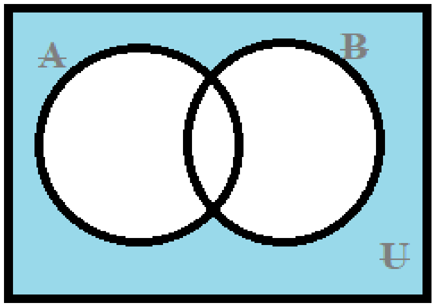
$=\{1,3,4,5,6,7,8,9\}$

Hence it has been proved that $(A\cap B)'=A'\cup B'$

5. Draw appropriate Venn diagrams for each of the following:

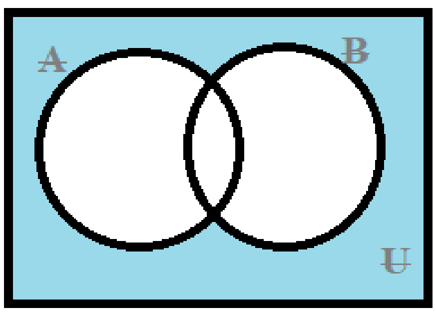
i. $(A \cup B)'$

Ans: To draw the Venn diagram for $(A \cup B)'$



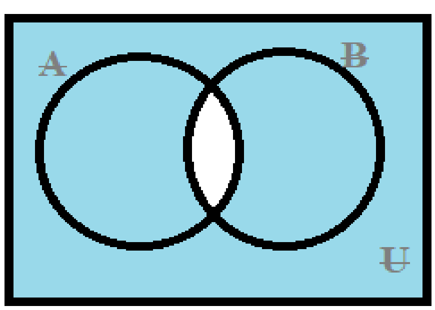
ii. $A' \cap B'$

Ans: To draw the Venn diagram for $A' \cap B'$



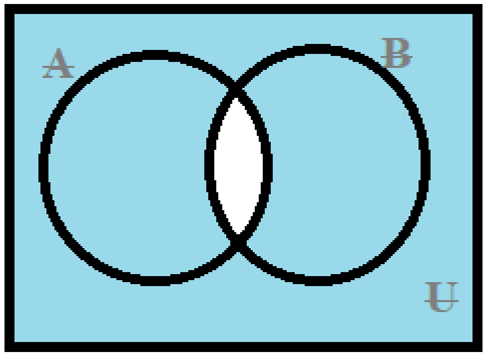
iii. $(A \cap B)'$

Ans: To draw the Venn diagram for $(A \cap B)'$



iv. $\mathbf{A' \cup B'}$

Ans: To draw the Venn diagram for $A' \cup B'$



6. **Let $\mathbf{U}$ be the set of all triangles in a plane. If $\mathbf{A}$ is the set of all triangles with at least one angle different from $\mathbf{60°}$, what is $\mathbf{A'}$?**

Ans: Given that,

U is the set of all triangles in the plane

$A =$ Set of triangles different form $60°$

To find A'

The complement of set A is the set of all elements of U which are not the elements of A.

$A' = U - A$

$=$ Set of all equilateral triangles

$\therefore A'$ is the set of all equilateral triangles

7. **Fill in the blanks to make each of the following a true statement:**

i. $\mathbf{A \cup A' = ...}$

Ans: To fill the blanks given in the statement

The union of the set and its complement is the universal set

$\therefore A \cup A' = U$

ii. $\boldsymbol{\varnothing' \cap A = \ldots}$

Ans: To fill the blanks given in the statement

We know that,

$\varnothing' \cap A = U \cap A = A$

$\therefore \varnothing' \cap A = A$

iii. $\boldsymbol{A \cap A' = \ldots}$

Ans: To fill the blanks given in the statement

The intersection of the set and its complement is an empty set.

$\therefore A \cap A' = \varnothing$

iv. $\boldsymbol{U' \cap A = \ldots}$

Ans: To fill the blanks given in the statement

We know that,

$\varnothing \cap A = U' \cap A = \varnothing$

$\therefore U' \cap A = \varnothing$

Miscellaneous Exercise 1

1. Decide among the following sets, which sets are the subsets of one and another:

$\mathbf{A = \{x : x \in R \text{ satisfy } x^2 - 8x + 12 = 0\}, B = \{2, 4, 6\}}$

$\mathbf{C = \{2, 4, 6, 8, \ldots\}, D = \{6\}}$

Ans: Given that,

$A=\{x:x\in R \text{ satisfy } x^2-8x+12=0\}, B=\{2,4,6\}$

$C=\{2,4,6,8,...\}, D=\{6\}$

$A=\{x:x\in R \text{ satisfy } x^2-8x+12=0\}$

$x^2-8x+12=0$

$(x-2)(x-6)=0$

$x=2,6$

$A=\{2,6\}, B=\{2,4,6\}, C=\{2,4,6,8,...\}, D=\{6\}$

A set A is said to be a subset of B if every element of A is also an element of B

$A\subset B$ if $a\in A, a\in B$

We can observe that,

$D\subset A\subset B\subset C$

$\therefore D\subset A,B,C$ and $A\subset B,C$ and $B\subset C$

2. In each of the following statement, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

i. If $x\in A$ and $A\in B$, then $x\in B$

Ans: $x\in A$ and $A\in B$, then $x\in B$

To determine whether the statement is true

The given statement is false

For example,

Let $A=\{1,2\}, B=\{1,\{1,2\},\{3\}\}$

Now $2\in\{1,2\}, \{1,2\}\in\{1,\{1,2\},\{3\}\}$

But $2 \notin \{1,\{1,2\},\{3\}\}$

Hence the given statement is false.

ii. If $A \subset B$ and $B \in C$, then $A \in C$

Ans: $A \subset B$ and $B \in C$, then $A \in C$

To determine whether the statement is true

The statement is false

For example,

Let $A = \{2\}, B = \{0,2\}, C = \{1,\{0,2\},3\}$

As $A \subset B, B \in C$ but $A \notin C$

iii. If $A \subset B$ and $B \subset C$, then $A \subset C$

Ans: $A \subset B$ and $B \subset C$, then $A \subset C$

To determine whether the statement is true

The given statement is true.

Let $A \subset B, B \subset C$

Let $x \in A$

$x \in B$ so $A \subset B$

$x \in C$ so $B \subset C$

iv. If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$

Ans: $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$

To determine whether the statement is true

The given statement is false

Let $A=\{1,2\}, B=\{0,6,8\}, C=\{0,1,2,6,9\}$

Now by the statement,

$A \not\subset B$ and $B \not\subset C$

But $A \subset C$

v. If $x \in A$ and $A \not\subset B$, then $x \in B$

Ans: $x \in A$ and $A \not\subset B$, then $x \in B$

To determine whether the statement is true

The given statement is false

Let $A=\{3,5,7\}, B=\{3,4,6\}$

Now, $5 \in A$ and $A \not\subset B$

But $5 \notin B$

vi. If $A \subset B$ and $x \notin B$, then $x \notin A$

Ans: $A \subset B$ and $x \notin B$, then $x \notin A$

To determine whether the statement is true

The given statement is true

Let $A \subset B$ and $x \notin B$

To show that $x \notin A$

Suppose $x \in A$

Then $x \in B$ which is a contradiction

$\therefore x \notin A$

3. **Let A,B and C be the set such that $A\cup B=A\cup C$ and $A\cap B=A\cap C$.**

Show that $B=C$

Ans: To show that $B=C$

Let $x\in B$

$x\in A\cup B$ since $\left[B\subset A\cup B\right]$

$x\in A\cup C$ since $\left[A\cup B=A\cup C\right]$

$x\in A$ or $\in C$

Case I:

Also $x\in B$

$x\in A\cap B$ since $\left[B\subset A\cap B\right]$

$x\in A\cap C$ since $\left[A\cap B=A\cap C\right]$

$x\in A$ or $x\in C$

$\therefore x\in C$

$\therefore B\subset C$

Similarly, we can show that $C\subset B$

$\therefore$ It has been proved that $B=C$

4. **Show that the following four conditions are equivalent:**

i. $\mathbf{A\subset B}$

ii. $\mathbf{A-B=\varnothing}$

iii. $\mathbf{A\cup B=B}$

iv. $\mathbf{A\cap B=A}$

Ans: To show that the above four conditions are equivalent

First showing that,

(i) (ii),

Let $A \subset B$

To show,

$A - B = \varnothing$

If possible, suppose,

$A - B \neq \varnothing$

This means that there exist $x \in A, x \notin B$, which is impossible as $A \subset B$

$\therefore A - B = \varnothing$

$A \subset B \Rightarrow A - B = \varnothing$

Let $A - B = \varnothing$

To show that,

$A \subset B$

Let $x \in B$ since if $x \notin B$, then $A - B \neq \varnothing$

$\therefore A - B = \varnothing \Rightarrow A \subset B$

$\therefore$(i) (iii)

Let $A \subset B$

To show $A \cup B = B$

Clearly $B \subset A \cup B$

Let $x \in A \cup B$

$x \in A$ or $x \in B$

Case I:

$x \in A$

So that $x \in B$

$\therefore A \cup B \subset B$

Case II:

$x \in B$

Then $A \cup B = B$

Conversely let $A \cup B = B$

Let $x \in A$

$x \in A \cup B$ since $[A \subset A \cup B]$

$x \in B$ since $A \cup B = B$

$\therefore A \subset B$

Hence (i)   (iii)

Now we have to show that (i)   (iv)

Let $A \subset B$

$A \cap B \subset A$

Let $x \in A$

We have to show that $x \in A \cap B$

As $A \subset B$, $x \in B$

$x \in A \cap B$

$A \subset A \cap B$

Hence $A = A \cap B$

Let $x \in A$

$x \in A \cap B$

$x \in A$ and $x \in B$

So that $x \in B$

$\therefore A \subset B$

Hence (i) (iv)

5. Show that if $A \subset B$, then $C - B \subset C - A$

Ans: Given that,

$A \subset B$

To show that,

$C - B \subset C - A$

Let $x \in C - B$

$x \in C$ and $x \in B$

$x \notin A[A \subset B]$ and $x \in C$

$x \in C - A$

$\therefore C - B \subset C - A$

Hence it has been showed that $C - B \subset C - A$

6. Show that for any sets A and B,

$$\mathbf{A = (A \cap B) \cup (A - B) \text{ and } A \cup (B - A) = A \cup B}$$

Ans: To show that,

$A = (A \cap B) \cup (A - B)$

Let $x \in A$

We have to show that $A = (A \cap B) \cup (A - B)$

Case I:

$x \in A \cap B$

Then $x \in A \cap B \subset (A \cup B) \cup (A - B)$

Case II:

$x \notin A \cap B$

$x \notin A$ or $x \notin B$

$x \notin B[x \notin A]$

$x \notin A - B \subset (A \cup B) \cup (A - B)$

$\therefore A \subset (A \cap B) \cup (A - B)$ ……(1)

It is clear that,

$A \cap B \subset A$ and $A - B \subset A$

$\therefore (A \cap B) \cup (A - B) \subset A$ ……(2)

From (1) and (2) we obtain that,

$A = (A \cap B) \cup (A - B)$

To prove that,

$A \cup (B - A) \subset A \cup B$

Let $x \in A \cup (B - A)$

$x \in A$ or ($x \in B$ and $x \notin A$)

($x \in A$ or $x \in B$) and ($x \in A$ or $x \notin A$)

$x \in A \cup B$

$\therefore A \cup (B - A) \subset A \cup B$ ……(3)

Next we show that $A \cup (B - A) \subset A \cup B$

Let $y \in A \cup B$

$y \in A$ or $y \in B$

$(y \in A$ or $y \in B)$ and $(y \in A$ or $y \notin A)$

$y \in A \cup (B - A)$

$\therefore A \cup (B - A) \subset A \cup B$

Hence from (3) and (4) we obtain that $A \cup (B - A) = A \cup B$

Hence proved the statement

7. **Using properties of sets, show that**

i. $\mathbf{A \cup (A \cap B) = A}$

Ans: To show that, $A \cup (A \cap B) = A$ by using the property offsets

We know that,

$A \subset A$

$A \cap B \subset A$

$\therefore A \cup (A \cap B) \subset A$ ……(1)

And $A \subset A \cup (A \cap B)$ ……(2)

From (1) and (2) we get that

$A \cup (A \cap B) = A$

Hence it has been showed that $A \cup (A \cap B) = A$

ii. $A \cap (A \cup B) = A$

Ans: To show that $A \cap (A \cup B) = A$ by using the property of sets

$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$

$= A \cup (A \cap B)$

$= A$ (from (1))

Hence it has been showed that $A \cap (A \cup B) = A$

8. Show that $A \cap B = A \cap C$ need not imply $B = C$

Ans: Given that,

$A \cap B = A \cap C$

To show that the above does not imply $B = C$

Let $A = \{0,1\}, B = \{0,2,3\}, C = \{0,4,5\}$

Now,

$A \cap B = \{0\} = A \cap C$

But $B \neq C$ since $2 \in B$ and not to C

$\therefore$ It has been shown that $A \cap B = A \cap C$ does not necessarily imply $B = C$

9. Let A and B be sets. If $A \cap X = B \cap X = \varnothing$ and $A \cup X = B \cup X$ for some text X, show that $A = B$

(Hints: $A = A \cap (A \cup X), B = B \cap (B \cup X)$ and use distributive law)

Ans: Given that,

$A \cap X = B \cap X = \varnothing$

$A \cup X = B \cup X$

To show that,

$A = B$

We know that,

$A = A \cap (A \cup X)$

$= A \cap (B \cup X)$ since $A \cup X = B \cup X$

$= (A \cap B) \cup (A \cap X)$ [By distributive law]

$= (A \cap B) \cup \varnothing$ since $A \cap X = \varnothing$

$= A \cap B$ ......(1)

Now consider $B = B \cap (B \cup X)$

$= B \cap (A \cup X)$ since $B \cup X = A \cup X$

$= (B \cap A) \cup (B \cap X)$ [By distributive law]

$= (B \cap A) \cup \varnothing$ since $B \cap X = \varnothing$

$= B \cap A$

$= A \cap B$ ......(2)

It is possible only when $A = B$

So from (1) and (2), we get,

$A = B$

10. Find sets A, B and C such that $A \cap B, B \cap C$ and $A \cap C$ are non-empty subsets and $A \cap B \cap C = \varnothing$

Ans: Given that,

$A \cap B, B \cap C, A \cap C$ are non-empty subsets and

$A \cap B \cap C = \varnothing$

Let $A = \{0,2\}, B = \{1,2\}, C = \{2,0\}$

Now,

$A \cap B = \{1\}$

$B \cap C = \{1,2\}$

$A \cap C = \{0\}$

And $A \cap B \cap C = \varnothing$

$\therefore$ It has been showed that $A \cap B, B \cap C$ and $A \cap C$ are non-empty subsets but however $A \cap B \cap C = \varnothing$