probability

14 Chapter

Exercise 14.1

1. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Ans: When a dice is rolled, the sample space is $S=\{1,2,3,4,5,6\}$.

Further, $E=\{4\}$ and $F=\{2,4,6\}$.

It is observed that $E\cap F=\{4\}$.

Hence, E and F are not mutually exclusive events.

2. A die is thrown. Describe the following events:

(i) A: a number less than 7

Ans: When a die is rolled, the sample space is obtained as $S=\{1,2,3,4,5,6\}$.

$A=\{1,2,3,4,5,6\}$

(ii) B: a number greater than 7

Ans: When a die is rolled, the sample space is obtained as $S=\{1,2,3,4,5,6\}$.

$B=\{\Phi\}$

(iii) C: a multiple of 3

Ans: When a die is rolled, the sample space is obtained as $S=\{1,2,3,4,5,6\}$.

$C=\{3,6\}$

(iv) D: a number less than 4

Ans: When a die is rolled, the sample space is obtained as $S=\{1,2,3,4,5,6\}$.

$D=\{1,2,3\}$

(v) E: an even number greater than 4

Ans: When a die is rolled, the sample space is obtained as $S=\{1,2,3,4,5,6\}$.

$E=\{6\}$

(vi) F: a number not less than 3

Ans: When a die is rolled, the sample space is obtained as $S=\{1,2,3,4,5,6\}$.

$F=\{3,4,5,6\}$

(vii) Also find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F'$ and F' .

Ans: We know that $A = \{1,2,3,4,5,6\}$, $B = \{\Phi\}$, $C = \{3,6\}$, $D = \{1,2,3\}$, $E = \{6\}$,

$F = \{3,4,5,6\}$. Further, $F' = \{1,2\}$.

So, $A \cup B = \{1,2,3,4,5,6\}$

$A \cap B = \{\Phi\}$

$B \cup C = \{3,6\}$

$E \cap F = \{6\}$

$D \cap E = \{\Phi\}$

$A - C = \{1,2,4,5\}$

$D - E = \{1,2,3\}$

$E \cap F' = \{\Phi\}$

3. An experiment involves rolling a pair of dice and recording the number that comes up. Describe the following events:
A: the sum is greater than 8 ,
B: 2 occurs on either die
C: The sum is at least 7 and multiple of 3 .
Which pairs of these events are mutually exclusive?

Ans: On rolling a pair of dice, the sample space is obtained as: $S = \{(x,y) : x,y = 1,2,3,4,5,6\}$

$$= \left\{ \begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{matrix} \right\}$$

Subsequently,

$A = \{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6)(6,3),(6,4),(6,5),(6,6)\}$

$B = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)(1,2),(3,2),(4,2),(5,2),(6,2)\}$

$C = \{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

It is further observed that:

$A \cap B = \{\Phi\}$

$B \cap C = \{\Phi\}$

$A \cap C = \{(3,6),(4,5),(5,4),(6,3),(6,6)\}$

Thus, events A and B and events B and C are mutually exclusive.

4. Three coins are tossed once. Let A denote the event "three heads show", B denote the event "two heads and one tail show", C denote the event "three tails show" and D denote the event "a head shows on the first coin". Which events are (i) mutually exclusive?

Ans: On tossing three coins, the sample space is obtained as:

$S=\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Further, $A=\{HHH\}$, $B=\{HHT, HTH, THH\}$, $C=\{TTT\}$, $D=\{HHH, HHT, HTH, HTT\}$.

We conclude that $A\cap B=\{\Phi\}$, $A\cap C=\{\Phi\}$, $A\cap D=\{HHH\}$, $B\cap C=\{\Phi\}$, $B\cap D=\{HHT, HTH\}$, $C\cap D=\{\Phi\}$.

Hence, events A and B; A and C; B and C; and C and D are all mutually exclusive.

(ii) simple?

Ans: A simple event is an event that has only one sample point of a sample space. Therefore, A and C are simple events.

(iii) compound?

Ans: A compound event is an event that has more than one sample point of a sample space. Therefore, B and D are compound events.

5. Three coins are tossed. Describe:

(i) Two events which are mutually exclusive.

Ans: On tossing three coins, the sample space is obtained as:

$S=\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

Two mutually exclusive events can be:

A: Getting no heads, and B: Getting no tails.

Here, $A=\{TTT\}$ and $B=\{HHH\}$ are disjoint.

(ii) Three events which are mutually exclusive and exhaustive.

Ans: Three mutually exclusive events are:

A: Getting no heads, B: Getting exactly one head, and C: Getting at least two heads.

Here, $A=\{TTT\}$, $B=\{HTT, THT, TTH\}$, $C=\{HHH, HHT, HTH, THH\}$ are disjoint but $A\cup B\cup C=S$.

(iii) Two events, which are not mutually exclusive.

Ans: Two events that are not mutually exclusive are:

A: Getting three heads, and B: Getting at least two heads.

Here, $A=\{HHH\}$, $B=\{HHH, HHT, HTH, THH\}$ and $A\cap B=\{HHH\}$.

(iv) Two events which are mutually exclusive but not exhaustive.

Ans: Two mutually exclusive but not exhaustive events are:

A: Getting exactly one head, and B: Getting exactly one tail.

Here, $A=\{HTT, THT, TTH\}$, $B=\{HHT, HTH, THH\}$ are disjoint and $A\cup B\neq S$.

(v) Three events which are mutually exclusive but not exhaustive.

Ans: Three mutually exclusive but not exhaustive events are:

A: Getting exactly three heads, B: Getting one tail and two heads, C: Getting one head and two tails.

Here, $A=\{HHH\}$, $B=\{HTH, HHT, THH\}$, $C=\{HTT, TTH, THT\}$ are disjoint and $A\cup B\cup C\neq S$.

6. Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the number on the dice ≤ 5 .

Describe the events:

(i) A'

Ans: When two dice are thrown, the sample space is obtained as:

$S=\{(x, y): x, y=1,2,3,4,5,6\}$

$$\Rightarrow\left\{\begin{array}{l}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\\(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{array}\right\}$$

Subsequently,

$A=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$B=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$C=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$

Further, $A'=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$\Rightarrow A'=B$

(ii) not B

Ans: Not B can be rewritten as B' .

$B'=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$\Rightarrow B'=A$

(iii) A or B

Ans: A or B can be written as $A\cup B$.

$$A \cup B = \begin{Bmatrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{Bmatrix}$$

$A \cup B = S$

(iv) A and B

Ans: A and B, or $A \cap B$ is Φ .

(v) A but not C

Ans: A but not C, or $A - C$,

$A - C = \{(2,4),(2,5),(2,6),(4,2),(4,3),(4,4),(4,5),$

$(4,6),(6,1),(6,2),(6,3),(6,4),(6,5)(6,6)\}$

(vi) B or C

Ans: B or C, or $B \cup C$,

$B \cup C = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),$

$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

(vii) B and C

Ans: B and C, or $B \cap C$,

$B \cap C = \{(1,1),(1,2),(1,3),(1,4),(3,1),(3,2)\}$

(viii) $A \cap B' \cap C'$

Ans: We comprehend C' as follows:

$C' = \{(1,5),(1,6),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),(3,6),(4,2),(4,3),(4,4),$

$(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$ Now,

$A \cap B' \cap C' = A \cap A \cap C'$

$\Rightarrow A \cap C'$

$\Rightarrow A \cap C' = \{(2,4),(2,5),(2,6)(4,3),(4,4),(4,5),(4,6),$

$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

7. Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the number on the dice ≤ 5 .

State true or false (give reason for your answer) :

(i) A and B are mutually exclusive.

Ans: When two dice are thrown, the sample space is obtained as:

$S=\{(x,y): x,y=1,2,3,4,5,6\}$

$$\Rightarrow \left\{\begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$

Accordingly,

$A=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$B=\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$

$C=\{(1,1), (1,2), (1,3), (1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$

So, $A \cap B=\{\Phi\}$, that is, the sets are mutually exclusive.

Hence the statement is true.

(ii) A and B are mutually exclusive and exhaustive.

Ans: We observe that $A \cap B=\{\Phi\}$, that is, the sets are mutually exclusive. Further, $A \cup B=S$, so the sets are also exhaustive.

Hence the statement is true.

(iii) $A=B'$

Ans: We observe $\begin{matrix} B'=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2), \\ (4,3),(4,4),(4,5),(4,6).(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \end{matrix}$, that is, $A=B'$.

Hence the statement is true.

(iv) A and C are mutually exclusive.

Ans: We observe that $A \cap B=\{(2,1),(2,2),(2,3),(4,1)\}$.

Thus, A and C are not mutually exclusive.

Hence the statement is false.

(v) A and B' are mutually exclusive.

Ans: We observe $\begin{matrix} B'=\{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(4,1),(4,2), \\ (4,3),(4,4),(4,5),(4,6).(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\} \end{matrix}$, that is, $A=B'$.

So, A and B' are not mutually exhaustive.

Hence the given statement is false.

(vi) A' , B' , C are mutually exclusive and exhaustive.

Ans: We observe that $A'\cup B'\cup C=S$, which means they are exhaustive.
However, $B'\cap C\neq\Phi$, so they are not mutually exhaustive.
Hence the given statement is false.

Exercise 14.2

1. Which of the following cannot be valid assignment of probabilities for outcomes of sample space $S=\{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6,\omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	**0.1**	**0.01**	**0.05**	**0.03**	**0.01**	**0.2**	**0.6**

Ans: Here, each of the given probabilities is less than 1 and positive.
Further, sum of possibilities:

$\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6+\omega_7$

$\Rightarrow 0.1+0.01+0.05+0.03+0.01+0.2+0.6$

$\Rightarrow 1$

Hence, the given assignment is valid.

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Ans: Here, each of the given probabilities is less than 1 and positive.
Further, sum of possibilities:

$\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6+\omega_7$

$\Rightarrow \frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$

$\Rightarrow 1$

Hence, the given assignment is valid.

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(c)	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**

Ans: Here, each of the given probabilities is less than 1 and positive.
Further, sum of possibilities:

$\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6+\omega_7$

$\Rightarrow 0.1+0.2+0.3+0.4+0.5+0.6+0.7$

$\Rightarrow 2.8$

Since $2.8>1$, the given assignment is not valid.

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(d)	**–0.1**	**0.2**	**0.3**	**0.4**	**–0.2**	**0.1**	**0.3**

Ans: Here, ω_1 and ω_5 are less than 1 and negative.
Hence, the given assignment is not valid.

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(e)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Ans: Here, ω_7 is more than 1.
Hence, the given assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Ans: When a coin is tossed twice, the sample space is obtained as:

$S = \{HH, HT, TH, TT\}$

Let A be the event that tail occurs at least once.

Accordingly, $A = \{HT, TH, TT\}$

So, the probability of A occurring is the number of favorable outcomes over the total possible outcomes.

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{3}{4}$$

3. A die is thrown, find the probability of following events:

(i) A prime number will appear,

Ans: When a die is thrown, the sample space is obtained as:

$S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event of a prime number occurring. So, $A = \{2, 3, 5\}$.

$$P(A) = \frac{n(A)}{n(S)}$$

$$\Rightarrow P(A) = \frac{3}{6}$$

$$\Rightarrow P(A) = \frac{1}{2}$$

(ii) A number greater than or equal to 3 will appear,

Ans: When a die is thrown, the sample space is obtained as:

$S=\{1,2,3,4,5,6\}$

Let B be the event a number greater than or equal to 3 occurs. So, $B=\{3,4,5,6\}$.

$$P(B)=\frac{n(B)}{n(S)}$$

$$\Rightarrow P(B)=\frac{4}{6}$$

$$\Rightarrow P(B)=\frac{2}{3}$$

(iii) A number less than or equal to 1 will appear,

Ans: When a die is thrown, the sample space is obtained as:

$S=\{1,2,3,4,5,6\}$

Let C be the event a number less than or equal to 1 occurs. So, $C=\{1\}$.

$$P(C)=\frac{n(C)}{n(S)}$$

$$\Rightarrow P(C)=\frac{1}{6}$$

(iv) A number more than 6 will appear,

Ans: Let D be the event that a number greater than six appears.

So, $D=\Phi$.

$$P(D)=\frac{n(D)}{n(S)}$$

$$\Rightarrow P(D)=\frac{0}{6}$$

$$\Rightarrow P(D)=0$$

(v) A number less than 6 will appear.

Ans: Let E be the event that a number less than six appears.

So, $E=\{1,2,3,4,5\}$.

$$P(E)=\frac{n(E)}{n(S)}$$

$$\Rightarrow P(E)=\frac{5}{6}$$

4. A card is selected from a pack of 52 cards.

(i) How many points are there in the sample space?

Ans: When a card is selected from a pack 52 cards, the number of possible outcomes is 52 , that is, the sample space contains 52 elements.

Hence, there are 52 points in the sample space.

(ii) Calculate the probability that the card is an ace of spades.

Ans: Let A be the event in which the card drawn is an ace of spades. Subsequently, $n(A)=1$.

$$P(A)=\frac{n(A)}{n(S)}$$

$$\Rightarrow P(A)=\frac{1}{52}$$

(iii) Calculate the probability that the card is:

(a) an ace

Ans: Let E be the event in which the card drawn is an ace. Subsequently, $n(E)=4$.

$$P(E)=\frac{n(E)}{n(S)}$$

$$\Rightarrow P(E)=\frac{4}{52}$$

$$\Rightarrow P(E)=\frac{1}{13}$$

(b) black card.

Ans: Let F be the event in which the card drawn is black. Subsequently, $n(F)=26$.

$$P(F)=\frac{n(F)}{n(S)}$$

$$\Rightarrow P(F)=\frac{26}{52}$$

$$\Rightarrow P(F)=\frac{1}{2}$$

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is:

(i) 3

Ans: Since the fair coin has 1 marked on one face and 6 on the other, and the die has six faces that are numbered $1,2,3,4,5,6$, the sample space is obtained as:

$$S=\{(1,1),(1,2),(1,3)(1,4),(1,5),(1,6),(6,1),(6,2),(6,3)(6,4),(6,5),(6,6)\}$$

Let A be the event in which the sum of numbers that turn up is 3 . So, $A=\{(1,2)\}$.

$$P(A)=\frac{n(A)}{n(S)}$$

$$\Rightarrow P(A)=\frac{1}{12}$$

(ii) 12

Ans: Let B be the event in which the sum of numbers that turn up is 12. So, $B=\{(6,6)\}$.

$$P(B)=\frac{n(B)}{n(S)}$$

$$\Rightarrow P(B)=\frac{1}{12}$$

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Ans: There are four men and six women on the city council.

Since one council member has to be selected for the committee randomly, the sample space will contain 10 elements.

Let A be the event in which the council member selected is a woman.

Accordingly, $n(A)=6$

$$P(A)=\frac{n(A)}{n(S)}$$

$$\Rightarrow P(A)=\frac{6}{10}$$

$$\Rightarrow P(A)=\frac{3}{5}$$

7. A fair coin is tossed four times, and a person win Re 1 for each head and loss Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Ans: As the coin is tossed four times, there can be a maximum of 4 heads and 4 tails:

(1) When 4 heads turn up, Rs. $(1+1+1+1)=$ Rs. 4 gain.

(2) When 3 heads and 1 tail turn up, Rs. $(1+1+1-1.50)=$ Rs. 1.50 gain.

(3) When 2 heads and 2 tails turn up, Rs. $(1+1-1.50-1.50)=$ Rs. 1 loss.

(4) When 1 head and 3 tails turn up, Rs. $(1-1.50-1.50-1.50)=$ Rs. 3.50 loss.

(5) When 4 tails turn up, Rs. $(-1.50-1.50-1.50-1.50)=$ Rs. 6 loss.

There are $2^4=16$ elements in the sample space S, which is obtained as:

$$S = \left\{ \begin{array}{l} HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, \\ HTHT, THTH, THHT, HTTT, THTT, TTHT, TTHT, TTTH, TTTT \end{array} \right\}$$

$n(S) = 16$

The person wins Rs. 4 when 4 heads turn up, that is, when the event {HHHH} occurs.

Probability of winning Rs. 4 is $\frac{1}{16}$.

The person wins Rs. 1.50 when 3 heads and 1 tail turn up, that is, when the events $\{HHHT, HHTH, HTHH, THHH\}$ occur.

Probability of winning Rs. 1.50 is $\frac{4}{16} = \frac{1}{4}$.

The person loses Rs. 1 when 2 heads and 2 tails turn up, that is, when the events $\{HHTT, HTTH, TTHH, HTHT, THTH, THHT\}$ occur.

Probability of losing Rs. 1 is $\frac{6}{16} = \frac{3}{8}$.

The person loses Rs. 3.50 when 1 head and 3 tails turn up, that is, when the events $\{HTTT, THTT, TTHT, TTTH\}$ occur.

Probability of losing Rs. 3.50 is $\frac{4}{16} = \frac{1}{4}$.

The person loses Rs. 6 when 4 tails turn up, that is, when the event $\{TTTT\}$ occurs.

Probability of losing Rs. 6 is $\frac{1}{16}$.

8. Three coins are tossed once. Find the probability of getting:
(i) 3 heads

Ans: On tossing three coins, the sample space is obtained as:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

Hence, $n(S) = 8$

Let B be the event that 3 heads occur.

So, $B = \{HHH\}$

$$P(B) = \frac{n(B)}{n(S)}$$

$$\Rightarrow P(B) = \frac{1}{8}$$

(ii) 2 heads

Ans: Let C be the event that 2 heads occur.

So, $C=\{HHT,\ HTH, THH\}$

$$P(C)=\frac{n(C)}{n(S)}$$

$$\Rightarrow P(C)=\frac{3}{8}$$

(iii) At least 2 heads

Ans: Let D be the event that at least 2 heads occur .

So, $D=\{HHH, HHT, HTH, THH\}$

$$P(D)=\frac{n(D)}{n(S)}$$

$$\Rightarrow P(D)=\frac{4}{8}$$

$$\Rightarrow P(D)=\frac{1}{2}$$

(iv) At most 2 heads

Ans: Let E be the event that at most 2 heads occur.

So, $E=\{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$$P(E)=\frac{n(E)}{n(S)}$$

$$\Rightarrow P(E)=\frac{7}{8}$$

(v) No heads

Ans: Let F be the event that no heads occur.

So, $F=\{TTT\}$

$$P(F)=\frac{n(F)}{n(S)}$$

$$\Rightarrow P(F)=\frac{1}{8}$$

(vi) 3 tails

Ans: Let G be the event that 3 tails occur.

So, $G=\{TTT\}$

$$P(G)=\frac{n(G)}{n(S)}$$

$$\Rightarrow P(G)=\frac{1}{8}$$

(vii) Exactly 2 tails

Ans: Let H be the event that exactly 2 tails occur.

So, $H=\{HTT, THT, TTH\}$

$$P(H)=\frac{n(H)}{n(S)}$$

$$\Rightarrow P(H)=\frac{3}{8}$$

(viii) No tail

Ans: Let I be the event that no tails occur.

So, $I=\{HHH\}$

$$P(I)=\frac{n(I)}{n(S)}$$

$$\Rightarrow P(I)=\frac{1}{8}$$

(ix) At most 2 tails

Ans: Let J be the event that at most 2 tails occur.

So, $J=\{HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$$P(J)=\frac{n(J)}{n(S)}$$

$$\Rightarrow P(J)=\frac{7}{8}$$

9. If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'?

Ans: We are given that $P(A)=\frac{2}{11}$.

Consequently, $P(A')=1-P(A)$

$$\Rightarrow P(A')=1-\frac{2}{11}$$

$\Rightarrow P(A') = \frac{9}{11}$

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that the letter is:
(i) A vowel

Ans: There are a total of 13 letters in the word 'ASSASSINATION'. Hence $n(S) = 13$

.

There are 6 vowels in the word give. Let A be the event that a vowel is chosen.

$$P(A) = \frac{n(A)}{n(S)}$$

$$\Rightarrow P(A) = \frac{6}{13}$$

(ii) A consonant

Ans: There are 7 consonants in the word given. Let B be the event that a vowel is chosen.

$$P(B) = \frac{n(B)}{n(S)}$$

$$\Rightarrow P(B) = \frac{7}{13}$$

11. In a lottery, person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by lottery committee, he wins the price. What is the probability of winning the prize in the games?
[Hint: order of the numbers is not important.]

Ans: The total number of ways in which a person can choose six different numbers from 1 to 20 are:

$${}^{20}C_6 = \frac{20!}{6!(20-6)!}$$

$$\Rightarrow {}^{20}C_6 = \frac{20!}{6!14!}$$

$$\Rightarrow {}^{20}C_6 = \frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$\Rightarrow 38760$$

Hence, there are 38760 possible combinations of six numbers.
Out of all these combinations, one combination is fixed by the lottery committee.
Hence, the required probability of winning the lottery prize is $\frac{1}{38760}$.

12. Check whether the following probabilities P(A) and P(B) are consistently defined:

(i) $P(A)=0.5$, $P(B)=0.7$, $P(A\cap B)=0.6$

Ans: We know that $P(A)=0.5$, $P(B)=0.7$, $P(A\cap B)=0.6$.

We know that if E and F are two events such that $E\subset F$, then $P(E)\le P(F)$.

In the given case, however, $P(A\cap B)>P(A)$. Hence, P(A) and P(B) are not consistently defined.

(ii) $P(A)=0.5$, $P(B)=0.4$, $P(A\cap B)=0.8$

Ans: We know that $P(A)=0.5$, $P(B)=0.4$, $P(A\cap B)=0.8$.

We know that if E and F are two events such that $E\subset F$, then $P(E)\le P(F)$.

In the given case, $P(A\cap B)>P(A)$ and $P(A\cap B)>P(B)$. Hence, P(A) and P(B) are consistently defined.

13. Fill in the blanks in the following table:

	P(A)	**P(B)**	**$P(A\cap B)$**	**$P(A\cup B)$**
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	**…**

Ans: We know that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{5}$, $P(A\cap B)=\frac{1}{15}$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{1}{3}+\frac{1}{5}-\frac{1}{15}$

$\Rightarrow P(A\cup B)=\frac{7}{15}$

	P(A)	**P(B)**	**$P(A\cap B)$**	**$P(A\cup B)$**
(ii)	**0.35**	**…**	**0.25**	**0.6**

Ans: We know that $P(A)=0.35$, $P(A\cup B)=0.6$, $P(A\cap B)=0.25$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow 0.6=0.35+P(B)-0.25$

$\Rightarrow P(B)=0.5$

	P(A)	**P(B)**	**$P(A\cap B)$**	**$P(A\cup B)$**
(iii)	**0.5**	**0.35**	**…**	**0.7**

Ans: We know that $P(A)=0.5$, $P(B)=0.35$, $P(A\cup B)=0.7$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow 0.7 = 0.5 + 0.35 - P(A \cap B)$
$\Rightarrow P(A \cap B) = 0.15$

14. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

Ans: We know that $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$.

For mutually exclusive events A and B,
$P(A \cup B) = P(A) + P(B)$

$\Rightarrow P(A \cup B) = \frac{3}{5} + \frac{1}{5}$

$\Rightarrow P(A \cup B) = \frac{4}{5}$

15. If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \cap F) = \frac{1}{2}$, find:

(i) P(E or F)

Ans: Here, $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$, and $P(E \cap F) = \frac{1}{8}$.

We know that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

So, $P(E \cup F) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$

$\Rightarrow P(E \cup F) = \frac{5}{8}$

(ii) P(not E and not F)

Ans: We know that $P(E \cup F) = \frac{5}{8}$.

Further, $P(E \cup F)' = P(E' \cap F')$

So, $P(E' \cap F') = 1 - P(E \cup F)$

$\Rightarrow P(E' \cap F') = 1 - \frac{5}{8}$

$\Rightarrow P(E' \cap F') = \frac{3}{8}$

16. Events E and F are such that P(Not E Not F) $= 0.25$. State whether E and F are mutually exclusive.

Ans: We are given that P (not E or not F) $=0.25$, that is, $P(E'\cup F')=0.25$

Also, $P(E'\cup F')=P(E\cap F)'$

$\Rightarrow 0.25=1-P(E\cap F)$

$\Rightarrow P(E\cap F)=0.75$

We note that $P(E\cap F)\neq 0$.

Therefore, E and F are not mutually exclusive.

17. A and B are events such that $P(A)=0.42$, $P(B)=0.48$ and $P(A\cap B)=0.16$. Determine:

(i) P(not A)

Ans: It is given that $P(A)=0.42$, $P(B)=0.48$ and $P(A\cap B)=0.16$.

$P(A')=1-P(A)$

$\Rightarrow P(A')=1-0.42$

$\Rightarrow P(A')=0.58$

(ii) P(not B)

Ans: It is given that $P(A)=0.42$, $P(B)=0.48$ and $P(A\cap B)=0.16$.

$P(B')=1-P(B)$

$\Rightarrow P(B')=1-0.48$

$\Rightarrow P(B')=0.52$

(iii) P(A or B)

Ans: It is given that $P(A)=0.42$, $P(B)=0.48$ and $P(A\cap B)=0.16$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=0.42+0.48-0.16$

$\Rightarrow P(A\cup B)=0.74$

18. In class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Ans: Let A be the event that the selected student studies Mathematics and B be the event that the selected student studies Biology.

Subsequently, $P(A)=40\%$

$$\Rightarrow \frac{40}{100}=\frac{2}{5}$$

Similarly, $P(B)=30\%$

$\Rightarrow \frac{30}{100} = \frac{3}{10}$

Further, $P(A \cap B) = 10\%$

$\Rightarrow \frac{10}{100} = \frac{1}{10}$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10}$

$\Rightarrow P(A \cup B) = \frac{6}{10}$

$\Rightarrow P(A \cup B) = 0.6$

Thus, the probability that selected student studies Mathematics or Biology is 0.6.

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Ans: Let A and B be the events of passing the first and the second examinations respectively. Accordingly, $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cup B) = 0.95$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.95 = 0.8 + 0.7 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 0.8 + 0.7 - 0.95$

$\Rightarrow P(A \cap B) = 0.55$

Thus, the probability of passing both these examinations is 0.55.

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Ans: Let A be the event of passing the English examination and B be the event of passing the Hindi examination.

Accordingly, $P(A \cap B) = 0.5$, $P(A' \cap B') = 0.1$ and $P(A) = 0.75$.

Now, $P(A' \cap B') = P(A \cup B)'$

$\Rightarrow P(A \cup B)' = 0.1$

Further, $P(A \cup B)' = 1 - P(A \cup B)$.

$\Rightarrow 1 - P(A \cup B) = 0.1$

$\Rightarrow P(A \cup B) = 0.9$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$\Rightarrow 0.9 = 0.75 + P(B) - 0.5$

$\Rightarrow P(B) = 0.9 - 0.75 + 0.5$

$\Rightarrow P(B) = 0.65$

Hence, probability of passing the Hindi examination is 0.65 .

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that:

(i) The student opted for NCC or NSS.

Ans: Let A and B be the events that the selected student has opted for NCC and NSS respectively.

Total number of students $= 60$

Number of students who have opted for NCC $= 30$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\Rightarrow \frac{30}{60} = \frac{1}{2}$$

Number of students who have opted for NSS $= 32$

$$P(B) = \frac{n(B)}{n(S)}$$

$$\Rightarrow \frac{32}{60} = \frac{8}{15}$$

Number of students who have opted for both NCC and NSS $= 24$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

$$\Rightarrow \frac{24}{60} = \frac{2}{5}$$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$$\Rightarrow P(A \cup B) = \frac{1}{2} + \frac{8}{15} - \frac{2}{5}$$

$$\Rightarrow P(A \cup B) = \frac{19}{30}$$

Thus, the probability that the selected student has opted for NCC or NSS is $\frac{19}{30}$.

(ii) The student has opted neither NCC nor NSS.

Ans: We know that $P(A' \cap B') = P(A \cup B)'$.

$\Rightarrow P(A \cup B)' = 1 - P(A \cup B)$

$\Rightarrow 1 - P(A \cup B) = 1 - \frac{19}{30}$

$\Rightarrow P(A \cup B)' = \frac{11}{30}$

Thus, the probability that the selected student has opted for neither NCC nor NSS is $\frac{11}{30}$.

(iii) The student has opted NSS but not NCC.

Ans: We know that $n(B - A) = n(B) - n(A \cap B)$.

$\Rightarrow n(B - A) = 32 - 24$

$\Rightarrow n(B - A) = 8$

The number of students that have opted for NSS but not NCC is 8.

$P(B - A) = \frac{n(B - A)}{n(S)}$

$\Rightarrow P(B - A) = \frac{8}{60}$

$\Rightarrow P(B - A) = \frac{2}{15}$

Thus, the probability that the selected student has opted for NSS but not for NCC is $\frac{2}{15}$.

Miscellaneous Exercise

1. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that:

(i) all will be blue?

Ans: Total number of marbles $= 60$

The number of ways of drawing 5 marbles from 60 marbles is ${}^{60}C_5$.

All the marbles drawn will be blue if we draw 5 marbles out of the 20 blue marbles.

5 blue marbles can be drawn out of the 20 blue marbles in ${}^{20}C_5$ ways.

Probability that all marbles will be blue is $\frac{{}^{20}C_5}{{}^{60}C_5}$.

(ii) at least one will be green?

Ans: Total number of marbles $= 60$

Number of ways of drawing 5 marbles from 60 marbles is ${}^{60}C_5$.

Number of ways in which the drawn marbles is not green, that is, when red or blue marbles are drawn, is ${}^{20+10}C_5 = {}^{30}C_5$.

So, probability that no marble is green is $\frac{{}^{30}C_5}{{}^{60}C_5}$.

Therefore, probability that at least one marble will be green is $1-\frac{{}^{30}C_5}{{}^{60}C_5}$.

2. 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining three diamonds and one spade?

Ans: Number of ways of drawing 4 cards from a pack of 52 cards are ${}^{52}C_4$.

In a deck, there are 13 diamonds and 13 spades.
So, the number of ways of drawing three diamonds and one spade is ${}^{13}C_3 \times {}^{13}C_1$.

Hence, probability of drawing three diamonds and one spade is $\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$.

3. A die has two faces each with number ' 1 ', three faces each with number ' 2 ' and one face with number ' 3 '. If die is rolled once, determine:
(i) P(2)

Ans: Total number of faces of the die is 6 .
Number of faces with the number ' 2 ' is 3 .

Therefore, probability of obtaining the number 2 is $P(2)=\frac{n(2)}{n(S)}$.

$$\Rightarrow P(2)=\frac{3}{6}$$

$$\Rightarrow P(2)=\frac{1}{2}$$

(ii) $P(1\cup 3)$

Ans: Total number of faces of the die is 6 .
Number of faces with the number ' 1 ' is 2 and with the number ' 3 ' is 1 .

Therefore, probability of obtaining the number 1 or 3 is $P(1\cup 3)=\frac{n(1\cup 3)}{n(S)}$.

$$\Rightarrow P(1\cup 3)=\frac{1+2}{6}$$

$$\Rightarrow P(1\cup 3)=\frac{1}{2}$$

(iii) P(3')

Ans: Total number of faces of the die is 6.

Number of faces with the number '3' is 1.

Therefore, probability of obtaining the number 3 is $P(3)=\frac{1}{6}$.

$\Rightarrow P(3')=1-P(3)$

$\Rightarrow P(3')=1-\frac{1}{6}$

$\Rightarrow P(3')=\frac{5}{6}$

4. In a certain lottery, 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize if you buy:

(i) one ticket?

Ans: Total number of tickets sold is $10,000$.

Number of prizes awarded is 10.

If A is the event we buy 1 ticket, the probability of winning will be $P(A)=\frac{n(A)}{n(S)}$.

$\Rightarrow P(A)=\frac{10}{10,000}$

$\Rightarrow P(A)=\frac{1}{1,000}$

So, probability of not getting a prize is $P(A')=1-P(A)$.

$\Rightarrow P(A')=1-\frac{1}{1,000}$

$\Rightarrow P(A')=\frac{999}{1,000}$

(ii) two tickets?

Ans: Total number of tickets sold is $10,000$.

Number of prizes awarded is 10.

Number of tickets on which a prize is not awarded is $10,000-10=9,990$.

If B is the event we buy 2 tickets, the probability of not winning will be $P(B)=\frac{{}^{9,990}C_2}{{}^{10,000}C_2}$.

(iii) ten tickets?

Ans: Total number of tickets sold is $10,000$.

Number of prizes awarded is 10.

Number of tickets on which a prize is not awarded is $10,000 - 10 = 9,990$.

If C is the event we buy 10 tickets, the probability of not winning will be $P(C) = \frac{^{9,990}C_{10}}{^{10,000}C_{10}}$.

5. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(i) You both enter the same section?

Ans: You and your friend are among the 100 students.

Total number of ways of selecting 2 students out of 100 students is $^{100}C_2$.

If both of you enter the same section, you are either among the section having 40 students or the one having 60 students.

Number of ways in which you both can be in the same section is $^{40}C_2 + ^{60}C_2$.

Probability that you both can be in the same section is $\frac{^{40}C_2 + ^{60}C_2}{^{100}C_2}$

$$\Rightarrow \frac{\frac{40!}{2!38!} + \frac{60!}{2!58!}}{\frac{100!}{2!98!}}$$

$$\Rightarrow \frac{(40 \times 39) + (60 \times 59)}{(100 \times 99)}$$

$$\Rightarrow \frac{17}{33}$$

(ii) You both enter different sections?

Ans: Probability that you both can be in the same section is $\frac{17}{33}$.

Probability that you both are not in the same section will be $1 - \frac{17}{33}$.

$$\Rightarrow \frac{16}{33}$$

6. Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelop contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Ans: Let L_1, L_2, L_3 be the three letters and E_1, E_2, E_3 be their corresponding

envelopes.
There are 6 ways of inserting 3 letters in 3 envelopes. These are as follows:

L_1E_1, L_2E_3, L_3E_2

L_2E_2, L_1E_3, L_3E_1

L_3E_3, L_1E_2, L_2E_1

L_1E_1, L_2E_2, L_3E_3

L_1E_2, L_2E_3, L_3E_1

L_1E_3, L_2E_1, L_3E_2

We observe that there are 4 ways in which at least one letter is inserted in its proper envelope.

Hence, the required probability is $\frac{4}{6}=\frac{1}{2}$.

7. A and B are two events such that $P(A)=0.54$, $P(B)=0.69$ and $P(A\cap B)=0.35$. Find:

(i) $P(A\cup B)$

Ans: It is given to us that $P(A)=0.54$, $P(B)=0.69$, $P(A\cap B)=0.35$.

We know that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=0.54+0.69-0.35$

$\Rightarrow P(A\cup B)=0.88$

(ii) $P(A'\cap B')$

Ans: It is given to us that $P(A)=0.54$, $P(B)=0.69$, $P(A\cap B)=0.35$.

We know that $P(A'\cap B')=P(A\cup B)'$

$\Rightarrow P(A'\cap B')=1-P(A\cup B)P(A'\cap B')$

$\Rightarrow P(A'\cap B')=1-0.88$

$\Rightarrow P(A'\cap B')=0.12$

(iii) $P(A\cap B')$

Ans: It is given to us that $P(A)=0.54$, $P(B)=0.69$, $P(A\cap B)=0.35$.

We know that $P(A\cap B')=P(A)-P(A\cap B)$

$\Rightarrow P(A\cap B')=0.54-0.35$

$\Rightarrow P(A\cap B')=0.19$

(iv) $P(B \cap A')$

Ans: It is given to us that $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$.

We know that $P(B \cap A') = P(B) - P(A \cap B)$

$\Rightarrow P(B \cap A') = 0.69 - 0.35$

$\Rightarrow P(B \cap A') = 0.34$

8. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S. No.	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Ans: Let E be the event that the spokesperson will be a male and F be the event that the spokesperson will be over 35 years of age.

Subsequently, $P(E) = \frac{3}{5}$ and $P(F) = \frac{2}{5}$.

Since there is only one male who is above 35 years of age, $P(E \cap F) = \frac{1}{5}$.

We know that $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow P(E \cup F) = \frac{3}{5} + \frac{2}{5} - \frac{1}{5}$

$\Rightarrow P(E \cup F) = \frac{4}{5}$

Hence, the probability that the spokesperson will either be a male or over 35 years of age is $\frac{4}{5}$.

9. If four digits numbers greater than 5,000 are randomly formed from the digits 0 , 1 , 3 , 5 , and 7 , what is the probability of forming a number divisible by 5 when:

(i) the digits are repeated?

Ans: Since four-digit numbers greater than 5,000 are formed, the thousand's digit is either 5 or 7 . Remaining three digits can be filled by any of the given digits with repetition.

So, total number of ways of forming four-digit numbers are $2\times5\times5\times5$, and subtract 1 from the obtained answer, as we cannot count $5,000$ in the numbers.
$\Rightarrow 250-1$
$\Rightarrow 249$ ways

A number is divisible by 5 if the digit at its unit's place is either 0 or 5 .
Total number of four-digit numbers greater than $5,000$ that are divisible by 5 are $(2\times5\times5\times2)-1$.
$\Rightarrow 100-1$
$\Rightarrow 99$
Therefore, the probability of forming a number divisible by 5 when the digits are repeated is $\frac{99}{249}=\frac{33}{83}$.

(ii) the repetition of digits is not allowed?

Ans: Since four-digit numbers greater than $5,000$ are formed, the thousand's digit is either 5 or 7 . Remaining three digits can be filled by any of the given digits without repetition.
So, total number of ways of forming four-digit numbers are $2\times4\times3\times2=48$.
A number is divisible by 5 if the digit at its unit's place is either 0 or 5 .
When the digit at the thousands place is 5 , the units place can be filled only with 0 , so the number of possible ways are $1\times3\times2\times1=6$.
When the digit at the thousands place is 7 , the units place can be filled only with 0 or 5 , so the number of possible ways are $1\times3\times2\times2=12$.
Total number of four-digit numbers greater than $5,000$ that are divisible by 5 are $12+6=18$.
Therefore, the probability of forming a number divisible by 5 when the digits are repeated is $\frac{18}{48}=\frac{3}{8}$.

10. The number lock of a suitcase has 4 wheels, each labelled with ten digits i.e., from 0 to 9 . The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Ans: The number lock of suitcase has 4 wheels, each of which is labelled with ten digits, that is, from 0 to 9 .
Number of ways of selecting 4 different digits out of 10 digits is ${}^{10}C_4$.
Further, each combination of 4 different digits can be arranged in 4 ways.
Number of four digits with no repetitions $={}^{10}C_4\times4!$

$\Rightarrow \frac{10!}{4!6!}\times4!$

$\Rightarrow 10\times9\times8\times7$
$\Rightarrow 5040$
There is only one number that can open the suitcase.
Thus, the required probability is $\frac{1}{5040}$.