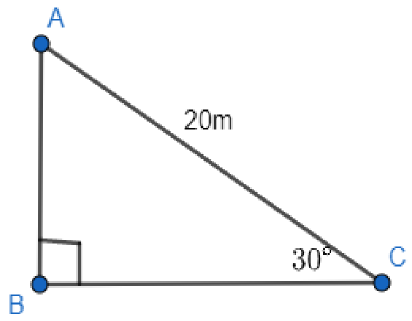
some applications of trigonometry

9
Chapter

1. A circus artist is climbing a 20m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is $30°$.



Ans: By observing the figure, AB is the pole.

In ΔABC,

$$\frac{AB}{AC} = \sin 30^\circ$$

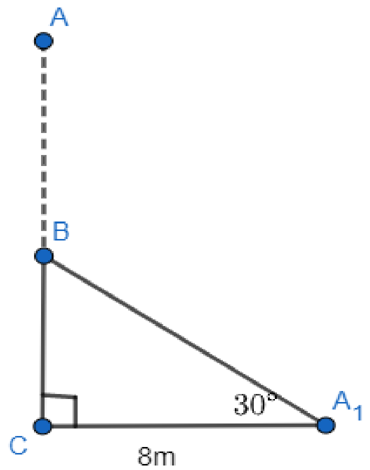
$$\Rightarrow \frac{AB}{20} = \frac{1}{2}$$

$$\Rightarrow AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is $10m$.

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle $30°$ with it. The distance between the foot of the tree to the point where the top touches the ground is $8m$. Find the height of the tree.

Ans: Let AC was the original tree. Due to the storm, it was broken into two parts. The broken part A'B is making 30° with the ground.



Let AC was the original tree. Due to the storm, it was broken into two parts. The broken part $A'B$ is making 30° with the ground. In triangle $A'BC$,

$$\Rightarrow \frac{BC}{A'C} = \tan 30^\circ$$

$$\Rightarrow \frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BC = \left(\frac{8}{\sqrt{3}}\right)m$$

$$\Rightarrow \frac{A'C}{A'B} = \cos 30$$

$$\Rightarrow \frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow A'B = \left(\frac{16}{\sqrt{3}}\right)m$$

Height of tree $= A'B + BC$

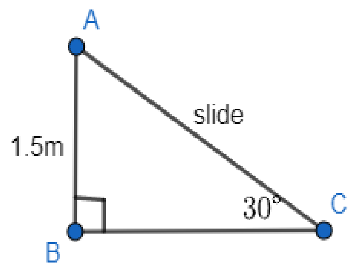
$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right)m = \frac{24}{\sqrt{3}}m$$

$$= 8\sqrt{3}\,m$$

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5m, and is inclined at an angle of 30° to the ground, whereas for the elder children she wants to have a steep slide at a height of 3m

, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Ans: It can be observed that AC and PR are the slides for younger and elder children respectively.



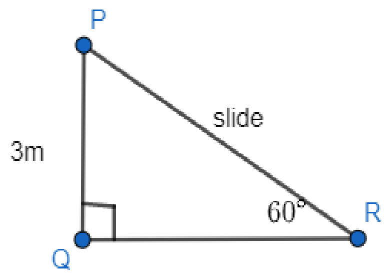
For younger children

In $\triangle ABC$

$$\Rightarrow \frac{AB}{AC} = \sin 30$$

$$\Rightarrow \frac{1.5}{AC} = \frac{1}{2}$$

$$\Rightarrow AC = 3m$$



For elder children

In $\triangle PQR$,

$$\Rightarrow \frac{PQ}{PR} = \sin 60^{\circ}$$

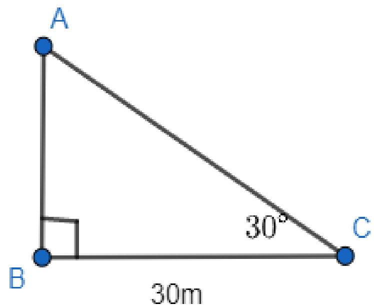
$$\Rightarrow \frac{3}{PR} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}\,m$$

Therefore, the lengths of these slides are $3m$ and $2\sqrt{3}\,m$.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is $30°$. Find the height of the tower.

Ans: Let AB be the tower and the angle of elevation from point C (on ground) is 30°



In 'ABC,

$$\Rightarrow \frac{AB}{BC} = \tan 30^{\circ}$$

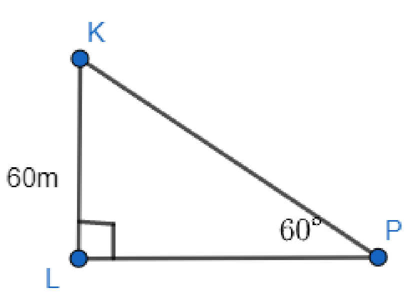
$$\Rightarrow \frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}\,m$$

Therefore, the height of the tower is $10\sqrt{3}\,m$.

5. A kite is flying at a height of 60m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60°$. Find the length of the string, assuming that there is no slack in the string.

Ans: Let K be the kite and the string is tied to point P on the ground.



In $\triangle KLP$,

$$\Rightarrow \frac{KL}{KP} = \sin 60^\circ$$

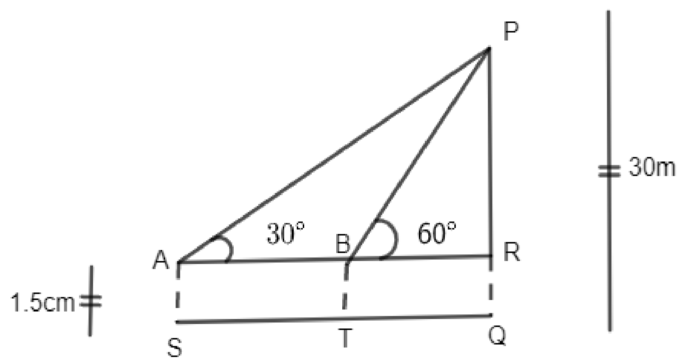
$$\Rightarrow \frac{60}{KP} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow KP = \frac{120}{\sqrt{3}} = 40\sqrt{3}\,\text{m}$$

Hence, the length of the string is $40\sqrt{3}\,\text{m}$.

6. A 1.5m tall boy is standing at some distance from a 30m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Ans : Let the boy stand at point S initially. He walked towards the building and reached point T.



$$PR = PQ - RQ$$

$$= (30 - 1.5) = 28.5 = \frac{57}{2}$$

In $\triangle PAR$,

$$\Rightarrow \frac{PR}{AR} = \tan 30^\circ$$

$$\Rightarrow \frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AR = \left(\frac{57}{2}\sqrt{3}\right)\text{m}$$

In $\triangle PRB$,

$$\Rightarrow \frac{PR}{BR} = \tan 60^{\circ}$$

$$\Rightarrow \frac{57}{2BR} = \sqrt{3}$$

$$\Rightarrow BR = \frac{57}{2\sqrt{3}} = \frac{19\sqrt{3}}{2}\text{m}$$

By observing the figure,

$ST = AB$

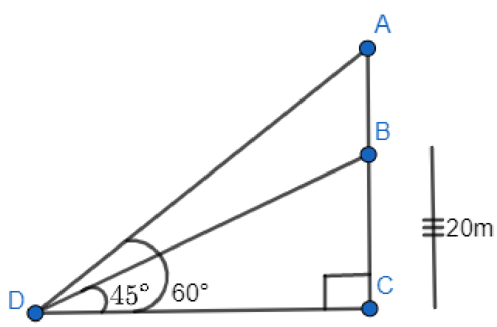
$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right)$$

$$= \left(\frac{38\sqrt{3}}{2}\right) = 19\sqrt{3}\text{m}$$

Hence, he walked $19\sqrt{3}\text{m}$ towards the building.

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20m high building are $45°$ and $60°$ respectively. Find the height of the tower.

Ans: Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.



In ΔBCD,

$$\Rightarrow \frac{BC}{CD} = \tan 45^{\circ}$$

$$\Rightarrow \frac{BC}{CD} = 1$$

$$\Rightarrow BC = CD$$

In ΔACD,

$$\Rightarrow \frac{AB+BC}{CD} = \tan 60^\circ$$

$$\Rightarrow \frac{AB+BC}{CD} = \sqrt{3}$$

As $CD = BC$,

$$\Rightarrow 1.6 + BC = BC\sqrt{3}$$

$$\Rightarrow BC\left(\sqrt{3}-1\right) = 1.6$$

By rationalization,

$$BC = \frac{1.6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$

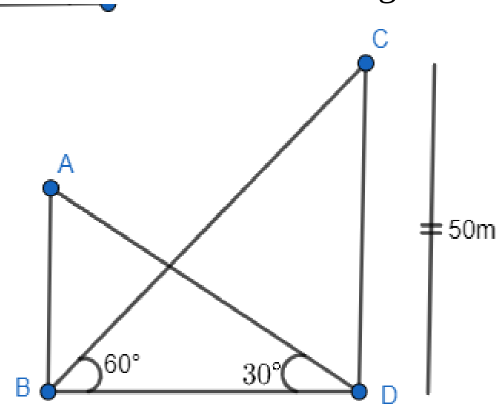
$$= \frac{1.6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2 - \left(1\right)^2}$$

$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\left(\sqrt{3}+1\right)$$

Therefore, the height of the pedestal is $0.8\left(\sqrt{3}+1\right)$ m.

8. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is $50m$ high, find the height of the building.

Ans: Let **AB** be the building and **CD** be the tower.



In $\triangle CDB$,

$$\Rightarrow \frac{CD}{BD} = \tan 60^{\circ}$$

$$\Rightarrow \frac{50}{BD} = \sqrt{3}$$

$$\Rightarrow BD = \frac{50}{\sqrt{3}}$$

In $\triangle ABD$

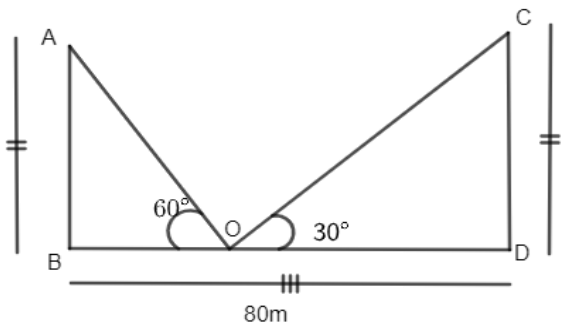
$$\Rightarrow \frac{AB}{BD} = \tan 30^{\circ}$$

$$\Rightarrow AB = \frac{50}{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\right) = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

9. Two poles of equal heights are standing opposite each other on either side of the road, which is $80m$ wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of poles and the distance of the point from the poles.

Ans: Let AB and CD be the poles and O is the point from where the elevation angles are measured.



In $\triangle ABO$,

$$\Rightarrow \frac{AB}{BO} = \tan 60^{\circ}$$

$$\Rightarrow \frac{AB}{BO} = \sqrt{3}$$

$\Rightarrow BO = \dfrac{AB}{\sqrt{3}}$

In ΔCDO,

$\Rightarrow \dfrac{CD}{DO} = \tan 30^\circ$

$\Rightarrow \dfrac{CD}{80 - BO} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow CD\sqrt{3} = 80 - BO$

$\Rightarrow CD\sqrt{3} = 80 - \dfrac{AB}{\sqrt{3}}$

$\Rightarrow CD\sqrt{3} + \dfrac{AB}{\sqrt{3}} = 80$

Since the poles are of equal heights, $CD = AB$

$\Rightarrow CD\left[\sqrt{3} + \dfrac{1}{\sqrt{3}}\right] = 80$

$\Rightarrow CD\left(\dfrac{3+1}{\sqrt{3}}\right) = 80$

$\Rightarrow CD = 20\sqrt{3}\,\text{m}$

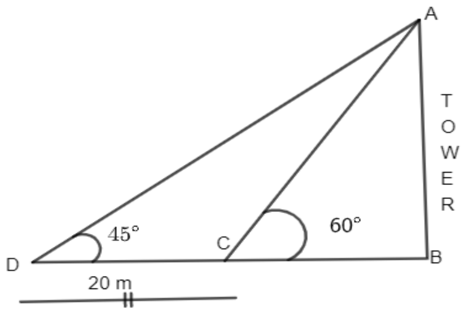
By observing the figure,

$\Rightarrow BO = \dfrac{AB}{\sqrt{3}} = \dfrac{CD}{\sqrt{3}} = \dfrac{20\sqrt{3}}{\sqrt{3}} = 20\text{m}$

$\Rightarrow DO = BD - BO = 80 - 20 = 60\text{m}$

Therefore, the height of poles is $20\sqrt{3}$ and the point is 20m and 60m far from these poles.

10.A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is $60°$. From another point 20m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is $30°$. Find the height of the tower and the width of the canal.



Ans:

In $\triangle ABC$,

$$\Rightarrow \frac{AB}{BC} = \tan 60^\circ$$

$$\Rightarrow \frac{AB}{BO} = \sqrt{3}$$

$$\Rightarrow BC = \frac{AB}{\sqrt{3}} \ldots. (1)$$

In $\triangle ABD$,

$$\Rightarrow \frac{AB}{BD} = \tan 30^\circ$$

$$\Rightarrow \frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 3AB = AB + 20\sqrt{3}$$

$$\Rightarrow 2AB = 20\sqrt{3}$$

$$\Rightarrow AB = 10\sqrt{3}m$$

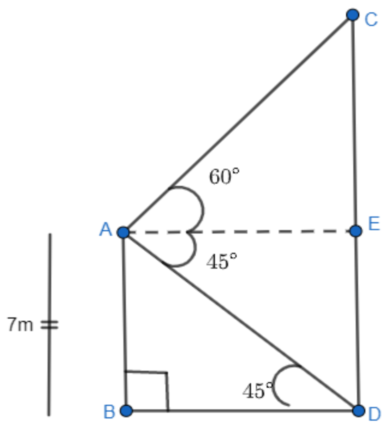
Substitute $AB = 10\sqrt{3}m$ in $BC = \frac{AB}{\sqrt{3}}$,

$$\Rightarrow BC = \frac{AB}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10m$$

Therefore, the height of the tower is $10\sqrt{3}\text{m}$ and the width of the canal is 10m.

11. From the top of a 7m high building, the angle of elevation of the top of a cable tower is $60°$ and the angle of depression of its foot is $45°$. Determine the height of the tower.

Ans: Let AB be a building and CD be a cable tower.



In ΔABD,

$$\Rightarrow \frac{AB}{BD} = \tan 45^\circ$$

$$\Rightarrow \frac{7}{BD} = 1$$

$$\Rightarrow BD = 7\text{m}$$

In ΔACE, $AE = BD = 7\text{m}$

$$\Rightarrow \frac{CE}{AE} = \tan 60^\circ$$

$$\Rightarrow \frac{CE}{7} = \sqrt{3}$$

$$\Rightarrow CE = 7\sqrt{3}\text{m}$$

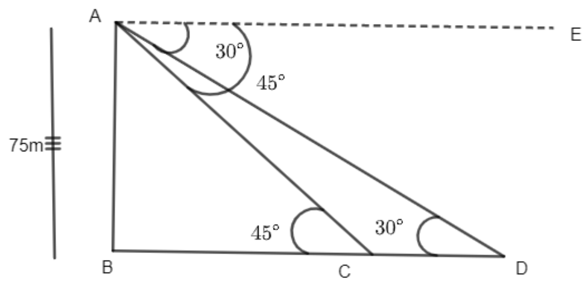
$$\Rightarrow CD = CE + ED = \left(7\sqrt{3} + 7\right) = 7\left(\sqrt{3} + 1\right)\text{m}$$

Therefore, the height of the cable tower is $7\left(\sqrt{3}+1\right)\text{m}$.

12. As observed from the top of a 75m high lighthouse from the sea-level, the

angles of depression of two ships are $30°$ and $45°$. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Ans: Let AB be the lighthouse and the two ships be at point C and D respectively.



In ΔABC,

$$\Rightarrow \frac{AB}{BC} = \tan 45^{\circ}$$

$$\Rightarrow \frac{75}{BC} = 1$$

$$\Rightarrow BC = 75m$$

In ΔABD,

$$\Rightarrow \frac{AB}{BD} = \tan 30^{\circ}$$

$$\Rightarrow \frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$

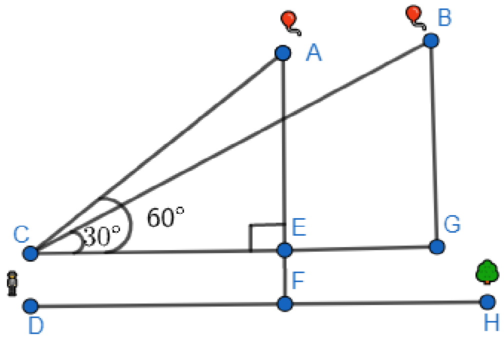
$$\Rightarrow \frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 75\sqrt{3} = 75 + CD$$

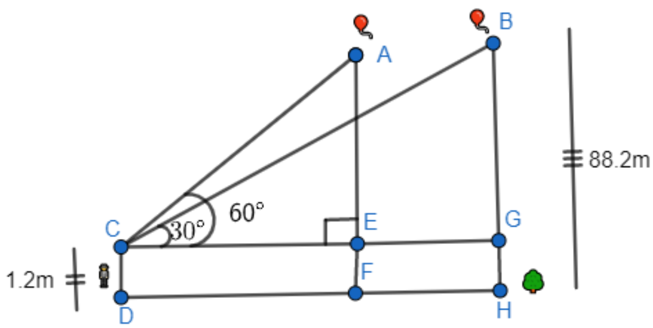
$$\Rightarrow 75\left(\sqrt{3} - 1\right) m = CD$$

Therefore, the distance between the two ships is $75\left(\sqrt{3} - 1\right)$ m.

13. A $1.2m$ tall girl spots a balloon moving with the wind in a horizontal line at a height of $88.2m$ from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is $60°$. After some time, the angle of elevation reduces to $30°$. Find the distance travelled by the balloon during the interval.



Ans: Let the initial position A of balloon change to B after some time and CD be the girl.



In ΔACE,

$$\Rightarrow \frac{AE}{CE} = \tan 60^\circ$$

$$\Rightarrow \frac{AF\text{-}EF}{CE} = \tan 60^\circ$$

$$\Rightarrow \frac{88.2\text{-}1.2}{CE} = \sqrt{3}$$

$$\Rightarrow \frac{87}{CE} = \sqrt{3}$$

$$\Rightarrow CE = \frac{87}{\sqrt{3}} = 29\sqrt{3}m$$

In ΔBCG,

$$\Rightarrow \frac{BG}{CG} = \tan 30^\circ$$

$$\Rightarrow \frac{BH\text{-}GH}{CG} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \frac{88.2\text{-}1.2}{CG} = \frac{1}{\sqrt{3}}$

$\Rightarrow 87\sqrt{3} = CG$

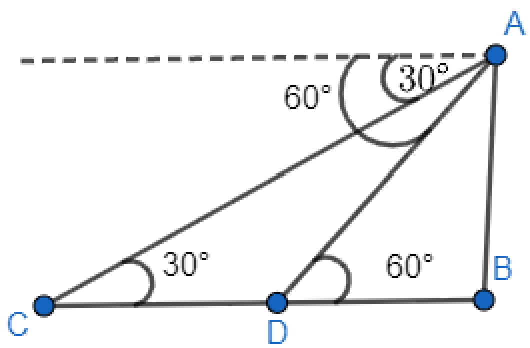
Distance travelled by balloon $= EG = CG - CE$

$= \ 87\sqrt{3} - 29\sqrt{3}$

$= 58\sqrt{3}m$

14. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of $30°$, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be $60°$. Find the time taken by the car to reach the foot of the tower from this point.

Ans: Let AB be the tower. Initial position of the car is C, which changes to D after six seconds.



In ΔADB,

$\Rightarrow \frac{AB}{CB} = \tan 60^{\circ}$

$\Rightarrow \frac{AB}{DB} = \sqrt{3}$

$\Rightarrow DB = \frac{AB}{\sqrt{3}}$

In ΔABC,

$\Rightarrow \frac{AB}{BC} = \tan 30^{\circ}$

$$\Rightarrow \frac{AB}{BD+DC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AB\sqrt{3} = BD + DC$$

$$\Rightarrow AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$\Rightarrow DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2AB}{\sqrt{3}}$$

Time taken by the car to travel distance DC $\left(\text{i.e}\frac{2AB}{\sqrt{3}}\right) = 6$ seconds.

Time taken by the car to travel distance DB $\left(\text{ie.,}\frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}}\left(\frac{AB}{\sqrt{3}}\right) = \frac{6}{2} = 3$

seconds.

15. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Ans: Let AB be the length of the statue and BC be the length of the pedestal.

In $\triangle$BCD,

$\frac{BC}{CD}$ = tan45°

∴BC = CD

In $\triangle$ACD,

tan60° = $\frac{AC}{CD}$

$\frac{AB + BC}{CD} = \sqrt{3}$

BC + 1.6 = $\sqrt{3}$ CD

BC + 1.6 = $\sqrt{3}$ BC

BC = $\frac{1.6}{\sqrt{3} - 1}$

BC = $\frac{1.6(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

= 1.6 ($\sqrt{3}$ + 1) $\frac{1.6(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2}$ $\frac{1.6(\sqrt{3} + 1)}{2}$

= 0.8($\sqrt{3}$ + 1) m