

introduction to trigonometry

8

Chapter

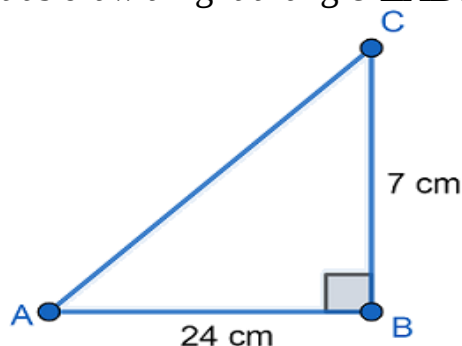
Exercise 8.1

1. In $\triangle ABC$ right angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine

(i). $\sin A, \cos A$

Ans: Given that in right angle triangle $\triangle ABC$, $AB = 24$ cm, $BC = 7$ cm.

Let us draw a right triangle $\triangle ABC$, also $AB = 24$ cm, $BC = 7$ cm. We get



We have to find $\sin A, \cos A$.

We know that for right triangle

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \text{ and}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

Here, $AB = 24$ cm, $BC = 7$ cm

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = (24)^2 + (7)^2$$

$$\Rightarrow (AC)^2 = 576 + 49$$

$$\Rightarrow (AC)^2 = 625 \text{ cm}^2$$

$$\Rightarrow AC = 25 \text{ cm}$$

Now,

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\Rightarrow \sin A = \frac{BC}{AC}$$

$$\therefore \sin A = \frac{7}{25}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

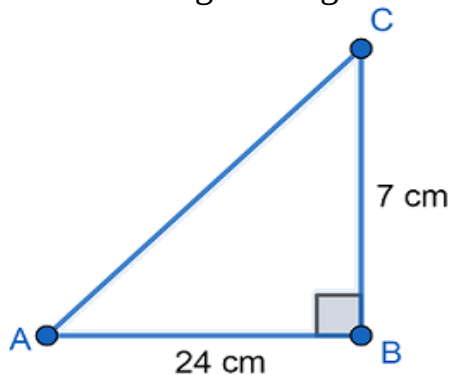
$$\Rightarrow \cos A = \frac{AB}{AC}$$

$$\therefore \cos A = \frac{24}{25}$$

(ii). $\sin C, \cos C$

Ans: Given that in right angle triangle $\triangle ABC$, $AB = 24 \text{ cm}$, $BC = 7 \text{ cm}$.

Let us draw a right triangle $\triangle ABC$, also $AB = 24 \text{ cm}$, $BC = 7 \text{ cm}$. We get



We have to find $\sin C, \cos C$.

We know that for right triangle

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \text{ and}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem ,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

Here, $AB = 24 \text{ cm}$, $BC = 7 \text{ cm}$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = (24)^2 + (7)^2$$

$$\Rightarrow (AC)^2 = 576 + 49$$

$$\Rightarrow (AC)^2 = 625 \text{ cm}^2$$

$$\Rightarrow AC = 25 \text{ cm}$$

Now,

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\Rightarrow \sin C = \frac{AB}{AC}$$

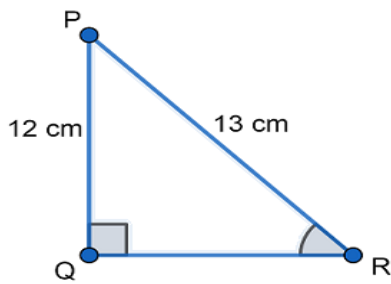
$$\therefore \sin C = \frac{24}{25}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\Rightarrow \cos C = \frac{BC}{AC}$$

$$\therefore \cos A = \frac{7}{25}$$

2. In the given figure find $\tan P - \cot R$.



Ans: Given in the figure,

$$PQ = 12 \text{ cm}$$

$$PR = 13 \text{ cm}$$

We know that for right triangle

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} \text{ and}$$

$$\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$$

Now, we need to apply the Pythagoras theorem to find the measure of adjacent side/base.

In $\triangle PQR$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (PR)^2 = (PQ)^2 + (QR)^2$$

$$\Rightarrow (13)^2 = (12)^2 + (QR)^2$$

$$\Rightarrow 169 = 144 + (QR)^2$$

$$\Rightarrow (QR)^2 = 169 - 144$$

$$\Rightarrow (QR)^2 = 25 \text{ cm}^2$$

$$\Rightarrow QR = 5 \text{ cm}$$

Now,

$$\tan P = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\Rightarrow \tan P = \frac{QR}{PQ}$$

$$\therefore \tan P = \frac{5}{12}$$

$$\cot R = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\Rightarrow \cot R = \frac{QR}{PQ}$$

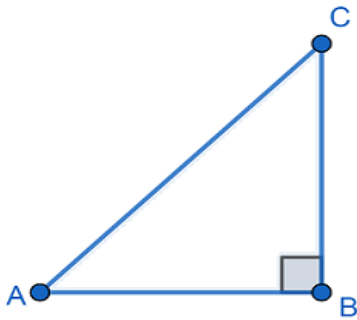
$$\therefore \cot R = \frac{5}{12}$$

$$\Rightarrow \tan P - \cot R = \frac{5}{12} - \frac{5}{12}$$

$$\therefore \tan P - \cot R = 0$$

3. If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Given that $\sin A = \frac{3}{4}$.

We know that $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$.

From the above figure, we get

$$\sin A = \frac{BC}{AC}$$

Therefore, we get

$$\Rightarrow BC = 3 \text{ and}$$

$$\Rightarrow AC = 4$$

Now, we have to find the values of $\cos A$ and $\tan A$.

We know that $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ and $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$.

Now, we need to apply the Pythagoras theorem to find the measure of adjacent side/base.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

Here, $AC = 4 \text{ cm}$, $BC = 3 \text{ cm}$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow 4^2 = AB^2 + 3^2$$

$$\Rightarrow 16 = AB^2 + 9$$

$$\Rightarrow AB^2 = 16 - 9$$

$$\Rightarrow AB^2 = 7$$

$$\Rightarrow AB = \sqrt{7} \text{ cm}$$

Now, we get

$$\cos A = \frac{AB}{AC}$$

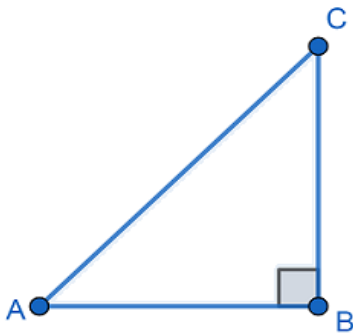
$$\therefore \cos A = \frac{\sqrt{7}}{4}$$

$$\text{And } \tan A = \frac{BC}{AB}$$

$$\therefore \tan A = \frac{3}{\sqrt{7}}$$

4. Given $15\cot A = 8$. Find $\sin A$ and $\sec A$.

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Given that $15\cot A = 8$.

$$\text{We get } \cot A = \frac{8}{15}.$$

$$\text{We know that } \cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}.$$

From the above figure, we get

$$\cot A = \frac{AB}{BC}$$

Therefore, we get

$$\Rightarrow BC = 15 \text{ and}$$

$$\Rightarrow AB = 8$$

Now, we have to find the values of $\sin A$ and $\sec A$.

We know that $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ and $\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$.

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow AC^2 = 8^2 + 15^2$$

$$\Rightarrow AC^2 = 64 + 225$$

$$\Rightarrow AC^2 = 289$$

$$\Rightarrow AC = 17 \text{ cm}$$

Now, we get

$$\sin A = \frac{BC}{AC}$$

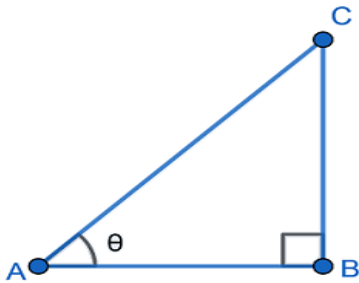
$$\therefore \sin A = \frac{15}{17}$$

$$\text{And } \sec A = \frac{AC}{AB}$$

$$\therefore \sec A = \frac{17}{8}$$

5. Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Given that $\sec \theta = \frac{13}{12}$.

We know that $\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$.

From the above figure, we get

$$\sec \theta = \frac{AC}{AB}$$

Therefore, we get

$$\Rightarrow AC = 13 \text{ and}$$

$$\Rightarrow AB = 12$$

Now, we need to apply the Pythagoras theorem to find the measure of the perpendicular/opposite side.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow 13^2 = 12^2 + BC^2$$

$$\Rightarrow 169 = 144 + BC^2$$

$$\Rightarrow BC^2 = 25$$

$$\Rightarrow BC = 5 \text{ cm}$$

Now, we know that

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{Here, } \sin \theta = \frac{BC}{AC}$$

$$\therefore \sin \theta = \frac{5}{13}$$

We know that $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$$\text{Here, } \cos \theta = \frac{AB}{AC}$$

$$\therefore \cos \theta = \frac{12}{13}$$

We know that $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$

$$\text{Here, } \tan \theta = \frac{BC}{AB}$$

$$\therefore \tan \theta = \frac{5}{12}$$

We know that $\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{opposite side}}$

$$\text{Here, } \operatorname{cosec} \theta = \frac{AC}{BC}$$

$$\therefore \operatorname{cosec} \theta = \frac{13}{5}$$

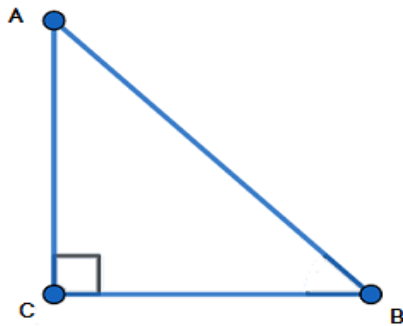
We know that $\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$

$$\text{Here, } \cot \theta = \frac{AB}{BC}$$

$$\therefore \cot \theta = \frac{12}{5} .$$

6. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Given that $\cos A = \cos B$.

In a right triangle $\triangle ABC$, we know that

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Here,

$$\cos A = \frac{AC}{AB}$$

$$\text{And } \cos B = \frac{BC}{AB}$$

As given $\cos A = \cos B$, we get

$$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$$

$$\Rightarrow AC = BC$$

Now, we know that angles opposite to the equal sides are also equal in measure.

Then, we get

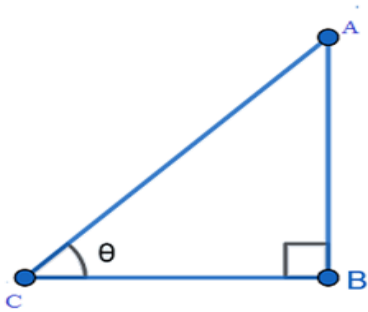
$$\angle A = \angle B$$

Hence proved.

7. Evaluate the following if $\cot \theta = \frac{7}{8}$

(i). $\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Now, in a right triangle we know that $\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$.

Here, from the figure $\cot \theta = \frac{BC}{AB}$.

We get

$$AB = 8 \text{ and}$$

$$BC = 7$$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AC)^2 = 8^2 + 7^2$$

$$\Rightarrow (AC)^2 = 64 + 49$$

$$\Rightarrow (AC)^2 = 113$$

$$\Rightarrow AC = \sqrt{113}$$

Now, we know that

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Here, we get

$$\sin \theta = \frac{AB}{AC} = \frac{8}{\sqrt{113}} \text{ and}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Here, we get

$$\cos \theta = \frac{BC}{AC} = \frac{7}{\sqrt{113}}$$

Now, we have to evaluate

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

Applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \sin^2 \theta}{1 - \cos^2 \theta}$$

Substituting the values, we get

$$\Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2}$$

$$\Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}}$$

$$\Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{\frac{113 - 64}{113}}{\frac{113 - 49}{113}}$$

$$\Rightarrow \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{49}{64}$$

$$\therefore \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{49}{64}$$

(ii). $\cot^2 \theta$

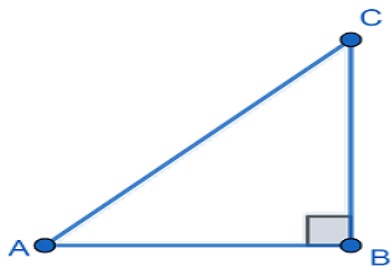
Ans: Given that $\cot \theta = \frac{7}{8}$.

$$\text{Now, } \cot^2 \theta = \left(\frac{7}{8}\right)^2$$

$$\therefore \cot^2 \theta = \frac{49}{64}$$

8. If $3\cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Given that $3\cot A = 4$.

We get $\cot A = \frac{4}{3}$.

We know that $\cot \theta = \frac{\text{adjacent side}}{\text{opposite side}}$.

From the above figure, we get

$$\cot A = \frac{AB}{BC}$$

Therefore, we get

$$\Rightarrow BC = 3 \text{ and}$$

$$\Rightarrow AB = 4$$

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow AC^2 = 4^2 + 3^2$$

$$\Rightarrow AC^2 = 16 + 9$$

$$\Rightarrow AC^2 = 25$$

$$\Rightarrow AC = 5$$

Now, let us consider LHS of the expression $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$, we get

$$\text{LHS} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

Now, we know that $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Here, we get

$$\tan A = \frac{BC}{AB} = \frac{3}{4}$$

Substitute the value, we get

$$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\frac{16 - 9}{16}}{\frac{16 + 9}{16}}$$

$$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{\frac{7}{16}}{\frac{25}{16}}$$

$$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{7}{25}$$

Now, let us consider RHS of the expression $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$, we get

$$\text{RHS} = \cos^2 A - \sin^2 A$$

We know that $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ and $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$.

Here, we get

$$\sin A = \frac{BC}{AC} = \frac{3}{5}$$

$$\text{And } \cos A = \frac{AB}{AC} = \frac{4}{5}$$

Substitute the values, we get

$$\Rightarrow \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\Rightarrow \cos^2 A - \sin^2 A = \frac{7}{25}$$

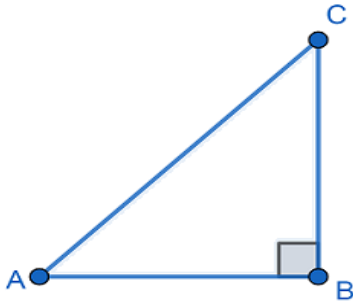
Hence, we get LHS=RHS

$$\therefore \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A.$$

9. In $\triangle ABC$, right angled at B . If $\tan A = \frac{1}{\sqrt{3}}$, find the value of

(i). $\sin A \cos C + \cos A \sin C$

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



$$\text{Given that } \tan A = \frac{1}{\sqrt{3}}.$$

In a right triangle, we know that $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Here, from the figure we get

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

We get $BC = 1$ and $AB = \sqrt{3}$.

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow AC^2 = (\sqrt{3})^2 + 1^2$$

$$\Rightarrow AC^2 = 3 + 1$$

$$\Rightarrow AC^2 = 4$$

$$\Rightarrow AC = 2$$

We know that $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ and $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$.

Here, we get

$$\sin A = \frac{BC}{AC} = \frac{1}{2} \text{ and } \sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\text{And } \cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2} \text{ and } \cos C = \frac{BC}{AC} = \frac{1}{2}$$

Now, we have to find the value of the expression $\sin A \cos C + \cos A \sin C$.

Substituting the values we get

$$\Rightarrow \sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

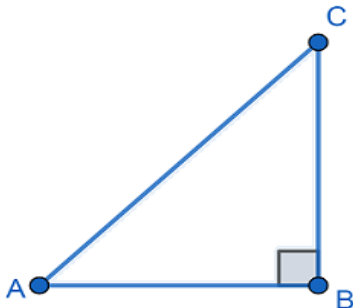
$$\Rightarrow \sin A \cos C + \cos A \sin C = \frac{1}{4} + \frac{3}{4}$$

$$\Rightarrow \sin A \cos C + \cos A \sin C = \frac{4}{4}$$

$$\therefore \sin A \cos C + \cos A \sin C = 1$$

(ii). $\cos A \cos C - \sin A \sin C$

Ans: Let us consider a right angled triangle $\triangle ABC$. We get



Given that $\tan A = \frac{1}{\sqrt{3}}$.

In a right triangle, we know that $\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Here, from the figure we get

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

We get $BC = 1$ and $AB = \sqrt{3}$.

Now, we need to apply the Pythagoras theorem to find the measure of hypotenuse.

In $\triangle ABC$, by Pythagoras theorem,

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow AC^2 = (\sqrt{3})^2 + 1^2$$

$$\Rightarrow AC^2 = 3 + 1$$

$$\Rightarrow AC^2 = 4$$

$$\Rightarrow AC = 2$$

We know that $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ and $\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$.

Here, we get

$$\sin A = \frac{BC}{AC} = \frac{1}{2} \text{ and } \sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\text{And } \cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2} \text{ and } \cos C = \frac{BC}{AC} = \frac{1}{2}$$

Now, we have to find the value of the expression $\cos A \cos C - \sin A \sin C$.

Substituting the values we get

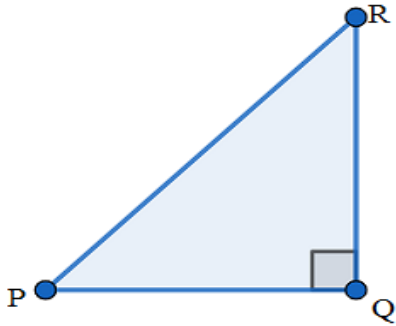
$$\Rightarrow \cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$\therefore \Rightarrow \cos A \cos C - \sin A \sin C = 0$$

10. In $\triangle PQR$, right angled at Q , $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Ans: Let us consider a right angled triangle ΔPQR , we get



Given that $PR + QR = 25$ cm and $PQ = 5$ cm.

Let $QR = 25 - PR$

Now, applying the Pythagoras theorem in ΔPQR , we get

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$$

We get

$$\Rightarrow (PR)^2 = (PQ)^2 + (QR)^2$$

$$\Rightarrow PR^2 = 5^2 + (25 - PR)^2$$

$$\Rightarrow PR^2 = 25 + 25^2 + PR^2 - 50PR$$

$$\Rightarrow PR^2 = PR^2 + 25 + 625 - 50PR$$

$$\Rightarrow 50PR = 650$$

$$\Rightarrow PR = 13 \text{ cm}$$

Therefore,

$$QR = 25 - 13$$

$$\Rightarrow QR = 12 \text{ cm}$$

Now, we know that in right triangle,

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \text{ and } \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}.$$

Here, we get

$$\sin P = \frac{QR}{PR}$$

$$\therefore \sin P = \frac{12}{13}$$

$$\cos P = \frac{PQ}{PR}$$

$$\therefore \cos P = \frac{5}{13}$$

$$\tan P = \frac{QR}{PQ}$$

$$\therefore \tan P = \frac{12}{5}$$

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

Ans: The given statement is false. The value of $\tan A$ depends on the length of sides of a right triangle and sides of a triangle may have any measure.

(ii) For some value of angle A , $\sec A = \frac{12}{5}$.

Ans: We know that in right triangle $\sec A = \frac{\text{hypotenuse}}{\text{adjacent side of } \angle A}$.

We know that in right triangle hypotenuse is the largest side.

Therefore, the value of $\sec A$ must be greater than 1.

In the given statement $\sec A = \frac{12}{5}$, which is greater than 1.

Therefore, the given statement is true.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A .

Ans: The given statement is false because $\cos A$ is the abbreviation used for the cosine of angle A . Abbreviation used for the cosecant of angle A is $\operatorname{cosec} A$.

(iv) $\cot A$ is the product of $\cot$ and A .

Ans: $\cot A$ is the abbreviation used for the cotangent of angle A . Hence the given statement is false.

(v) For some angle θ , $\sin \theta = \frac{4}{3}$.

Ans: We know that in right triangle $\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$.

We know that in right triangle hypotenuse is the largest side.

Therefore, the value of $\sin \theta$ must be less than 1.

In the given statement $\sin \theta = \frac{4}{3}$, which is greater than 1.

Therefore, the given statement is false.

Exercise 8.2

1. Evaluate the following:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	N o t d e f i n e d

We have to evaluate $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$.

Substitute the values from the above table, we get

$$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \frac{3}{4} + \frac{1}{4}$$

$$\Rightarrow \frac{4}{4}$$

$$\therefore \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ = 1.$$

$$\text{(ii) } 2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$.

Substitute the values from the above table, we get

$$\Rightarrow 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow 2 + \frac{3}{4} - \frac{3}{4}$$

$$\Rightarrow 2$$

$$\therefore 2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2.$$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$.

Substitute the values from the above table, we get

$$\begin{aligned} & \Rightarrow \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} \\ & \Rightarrow \frac{\frac{1}{\sqrt{2}}}{\frac{2 + 2\sqrt{3}}{\sqrt{3}}} \\ & \Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2 + 2\sqrt{3}} \end{aligned}$$

Multiplying and dividing by $\sqrt{3} - 1$, we get

$$\begin{aligned}
&\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\
&\Rightarrow \frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{2}(2+2\sqrt{3})(\sqrt{3}-1)} \\
&\Rightarrow \frac{\sqrt{3}(\sqrt{3}-1)}{2\sqrt{2}(\sqrt{3}+1)(\sqrt{3}-1)} \\
&\Rightarrow \frac{3-\sqrt{3}}{2\sqrt{2}((\sqrt{3})^2-1^2)} \\
&\Rightarrow \frac{3-\sqrt{3}}{2\sqrt{2}(3-1)} \\
&\Rightarrow \frac{3-\sqrt{3}}{4\sqrt{2}} \\
&\therefore \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{3-\sqrt{3}}{4\sqrt{2}}
\end{aligned}$$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ}$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ}$.

Substitute the values from the above table, we get

$$\Rightarrow \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$\Rightarrow \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}}$$

$$\Rightarrow \frac{\frac{3\sqrt{3} - 4}{2\sqrt{3}}}{\frac{2\sqrt{3}}{3\sqrt{3} + 4}}$$

$$\Rightarrow \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

Multiplying and dividing by $3\sqrt{3} - 4$, we get

$$\Rightarrow \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$

Now, applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - 4^2}$$

$$\begin{aligned}
&\Rightarrow \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2-4^2} \\
&\Rightarrow \frac{27+16-24\sqrt{3}}{27-16} \\
&\Rightarrow \frac{43-24\sqrt{3}}{11} \\
\therefore \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ - \cot 45^\circ} &= \frac{43-24\sqrt{3}}{11}
\end{aligned}$$

$$(v) \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sec^2 30^\circ + \cos^2 30^\circ}$$

Ans: With the help of trigonometric ratio table we can find the values of standard trigonometric angles. The trigonometric ratio table is as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	Not defined

We have to evaluate $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sec^2 30^\circ + \cos^2 30^\circ}$.

Substitute the values from the above table, we get

$$\begin{aligned}
&\Rightarrow \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\
&\Rightarrow \frac{5\left(\frac{1}{4}\right) + 4\left(\frac{4}{3}\right) - 1}{\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)} \\
&\quad \frac{15 + 64 - 12}{1 + 3} \\
&\Rightarrow \frac{12}{4} \\
&\quad \frac{15 + 64 - 12}{1 + 3} \\
&\Rightarrow \frac{12}{4} \\
&\quad \frac{15 + 64 - 12}{4} \\
&\Rightarrow \frac{12}{4} \\
&\quad \frac{67}{1} \\
&\Rightarrow \frac{12}{1} \\
&\therefore \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sec^2 30^\circ + \cos^2 30^\circ} = \frac{67}{12}.
\end{aligned}$$

2. Choose the correct option and justify your choice.

(i) $\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ} = \dots\dots\dots$

- (A) $\sin 60^\circ$
- (B) $\cos 60^\circ$
- (C) $\tan 60^\circ$
- (D) $\sin 30^\circ$

Ans: The given expression is $\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}$.

We know that from the trigonometric ratio table we have $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

Substitute the value in the given expression we get

$$\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 + \left(\frac{1}{\sqrt{3}} \right)^2}$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{\sqrt{3}}{2}$$

From the trigonometric table we know that

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\text{Hence, } \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \sin 60^\circ.$$

Therefore, option (A) is the correct answer.

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \dots\dots\dots$

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

Ans: The given expression is $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$.

We know that from the trigonometric ratio table we have $\tan 45^\circ = 1$.

Substitute the value in the given expression we get

$$\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - 1^2}{1 + 1^2}$$

$$\Rightarrow \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{1 - 1}{1 + 1}$$

$$\Rightarrow \Rightarrow \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = \frac{0}{2}$$

$$\therefore \frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} = 0$$

Therefore, option (D) is the correct answer.

(iii) $\sin 2A = 2\sin A$ is true when $A = \dots\dots\dots$

(A) 0°

(B) 30°

(C) 45°

(D) 60°

Ans: The given expression is $\sin 2A = 2\sin A$.

We know that from the trigonometric ratio table we have

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 90^\circ = 1$$

The given statement is true when $A = 0^\circ$.

Substitute the value in the given expression we get

$$\Rightarrow \sin 2A = 2 \sin A$$

$$\Rightarrow \sin 2 \times 0^\circ = 2 \sin 0^\circ$$

$$0 = 0$$

Therefore, option (A) is the correct answer.

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \dots\dots\dots$

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

Ans: The given expression is $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$.

We know that from the trigonometric ratio table we have $\tan 30^\circ = \frac{1}{\sqrt{3}}$.

Substitute the value in the given expression we get

$$\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \left(\frac{1}{\sqrt{3}} \right)}{1 - \left(\frac{1}{\sqrt{3}} \right)^2}$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$\Rightarrow \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \sqrt{3}$$

From the trigonometric table we know that

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\text{Hence, } \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \tan 60^\circ.$$

Therefore, option (C) is the correct answer.

3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0^\circ < A + B \leq 90^\circ$. Find A and B.

Ans: Given that $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$.

From the trigonometric ratio table we know that $\tan 60^\circ = \sqrt{3}$ and $\tan 30^\circ = \frac{1}{\sqrt{3}}$

Then we get

$$\tan(A + B) = \sqrt{3}$$

$$\Rightarrow \tan(A + B) = \tan 60^\circ$$

$$\Rightarrow A + B = 60^\circ \dots\dots\dots(1)$$

$$\text{Also, } \tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A - B) = \tan 30^\circ$$

$$\Rightarrow A - B = 30^\circ \dots\dots\dots(2)$$

Adding eq. (1) and (2), we get

$$2A = 90^\circ$$

$$\therefore A = 45^\circ$$

Substitute the obtained value in eq. (1), we get

$$45^\circ + B = 60^\circ$$

$$\Rightarrow B = 60^\circ - 45^\circ$$

$$\therefore B = 15^\circ$$

Therefore, the values of A and B is 45° and 15° respectively.

4. State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$.

Ans: Let us assume $A = 30^\circ$ and $B = 60^\circ$.

Now, let us consider LHS of the given expression, we get

$$\sin(A + B)$$

Substitute the assumed values in the LHS, we get

$$\sin(A + B) = \sin(30^\circ + 60^\circ)$$

$$\Rightarrow \sin(A + B) = \sin(90^\circ)$$

From the trigonometric ratio table we know that $\sin 90^\circ = 1$, we get

$$\Rightarrow \sin(A + B) = 1$$

Now, let us consider the RHS of the given expression and substitute the values, we get

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

From the trigonometric ratio table we know that $\sin 30^\circ = \frac{1}{2}$ and $\sin 60^\circ = \frac{\sqrt{3}}{2}$,

we get

$$\Rightarrow \sin A + \sin B = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin A + \sin B = \frac{1 + \sqrt{3}}{2}$$

Thus, $\text{LHS} \neq \text{RHS}$.

Therefore, the given statement is false.

(ii) The value of $\sin \theta$ increases as θ increases.

Ans: The value of sine from the trigonometric ratio table is as follows:

$$\sin 0^\circ = 0$$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\sin 90^\circ = 1$$

Therefore, we can conclude that the value of $\sin \theta$ increases as θ increases.
Therefore, the given statement is true.

(iii) The value of $\cos \theta$ increases as θ increases.

Ans: The value of cosine from the trigonometric ratio table is as follows:

$$\cos 0^\circ = 1$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

Therefore, we can conclude that the value of $\cos \theta$ decreases as θ increases.
Therefore, the given statement is false.

(iv) $\sin \theta = \cos \theta$ for all values of θ .

Ans: The trigonometric ratio table is given as follows:

Exact Values of Trigonometric Functions				
Angle θ		$\sin \theta$	$\cos \theta$	$\tan \theta$
Degrees	Radians			
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

90°	$\frac{\pi}{2}$	1	0	Not defi ned
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From the above table we can conclude that $\sin \theta = \cos \theta$ is true only for $\theta = 45^\circ$
 $\sin \theta = \cos \theta$ is not true for all values of θ .

Therefore, the given statement is false.

(iv) $\cot A$ is not defined for $A = 0^\circ$.

Ans: We know that $\cot A = \frac{\cos A}{\sin A}$.

If $A = 0^\circ$, then $\cot 0^\circ = \frac{\cos 0^\circ}{\sin 0^\circ}$

From trigonometric ratio table we get

$\sin 0^\circ = 0$ and $\cos 0^\circ = 1$

We get

$\cot 0^\circ = \frac{1}{0}$, which is undefined.

Therefore, the given statement is true.

Exercise 8.3

1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans: For a right triangle we have an identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

Let us consider the above identity, we get

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Now, reciprocating both sides we get

$$\Rightarrow \frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

Now, we know that $\frac{1}{\operatorname{cosec}^2 A} = \sin^2 A$, we get

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \pm \frac{1}{\sqrt{1 + \cot^2 A}}$$

Now, we know that sine value will be negative for angles greater than 180° , for a triangle sine value is always positive with respect to an angle. Then we will consider only positive value.

$$\therefore \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

We know that $\tan A = \frac{1}{\cot A}$

Also, we will use the identity $\sec^2 A = 1 + \tan^2 A$, we get

$$\sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\sqrt{\cot^2 A}}$$

$$\therefore \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

2. Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Ans: We know that $\cos A = \frac{1}{\sec A}$.

$$\therefore \cos A = \frac{1}{\sec A}$$

For a right triangle we have an identity $\sin^2 A + \cos^2 A = 1$.

Let us consider the above identity, we get

$$\sin^2 A + \cos^2 A = 1$$

Now, we know that $\cos A = \frac{1}{\sec A}$, we get

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\Rightarrow \sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$\therefore \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

Also, we will use the identity $\sec^2 A = 1 + \tan^2 A$, we get

$$\tan^2 A = \sec^2 A - 1$$

$$\therefore \tan A = \sqrt{\sec^2 A - 1}$$

Now, we know that $\cot A = \frac{\cos A}{\sin A}$, we get

$$\Rightarrow \cot A = \frac{\frac{1}{\sec A}}{\sqrt{\sec^2 A - 1}}$$

$$\therefore \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

We know that $\operatorname{cosec} A = \frac{1}{\sin A}$, we get

$$\therefore \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

3. Choose the correct option and justify your choice:

(i) $9\sec^2 A - 9\tan^2 A = \dots\dots$

(A) 1

(B) 9

(C) 8

(D) 0

Ans: The given expression is $9\sec^2 A - 9\tan^2 A$.

The given expression can be written as

$$\Rightarrow 9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$$

Now, we will use the identity $\sec^2 A = 1 + \tan^2 A$, we get

$$\sec^2 A - \tan^2 A = 1$$

$$\Rightarrow 9\sec^2 A - 9\tan^2 A = 9(1)$$

$$\therefore 9\sec^2 A - 9\tan^2 A = 9$$

Therefore, option (B) is the correct answer.

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

(A) 0

(B) 1

(C) 2

(D) -1

Ans: The given expression is $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$.

We know that the trigonometric functions have values as:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}, \operatorname{cosec} \theta = \frac{1}{\sin \theta} \text{ and } \sec \theta = \frac{1}{\cos \theta}$$

Substituting these values in the given expression, we get

$$\Rightarrow (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)$$

$$\Rightarrow (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right)$$

Now, by applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = \frac{(\sin \theta + \cos \theta)^2 - 1^2}{\sin \theta \cos \theta}$$

$$\Rightarrow (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = \frac{\sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

Now, by applying the trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = \frac{1 + 2\sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$\Rightarrow (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$\therefore (1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = 2$$

Therefore, option (C) is the correct answer.

(iii) $(\sec A + \tan A)(1 - \sin A) = \dots\dots\dots$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

Ans: Given expression is $(\sec A + \tan A)(1 - \sin A)$.

We know that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$

Substituting these values in the given expression, we get

$$(\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)(1 - \sin A)$$

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{1 + \sin A}{\cos A} \right)(1 - \sin A)$$

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{(1 + \sin A)(1 - \sin A)}{\cos A} \right)$$

Now, by applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{1^2 - \sin^2 A}{\cos A} \right)$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow (\sec A + \tan A)(1 - \sin A) = \left(\frac{\cos^2 A}{\cos A} \right)$$

$$\therefore (\sec A + \tan A)(1 - \sin A) = \cos A$$

Therefore, option (D) is the correct answer.

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

Ans: Given expression is $\frac{1 + \tan^2 A}{1 + \cot^2 A}$.

We know that the trigonometric functions have values as:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}.$$

Substituting these values in the given expression, we get

$$\begin{aligned} \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}} \\ &= \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} \end{aligned}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\begin{aligned} \Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} &= \frac{1}{\frac{\cos^2 A}{\sin^2 A}} \\ &= \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sin^2 A}{\cos^2 A} \\ &\Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A \end{aligned}$$

Therefore, option (D) is the correct answer.

4. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

Ans: Given expression is $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = (\text{cosec } \theta - \cot \theta)^2$$

Now, we know that $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$ and $\text{cosec } \theta = \frac{1}{\sin \theta}$.

By substituting the values, we get

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2$$

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \left(\frac{1 - \cos \theta}{\sin \theta} \right)^2$$

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

Now, by applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \frac{(1 - \cos \theta)^2}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \frac{(1 - \cos \theta)}{(1 + \cos \theta)}$$

$$\Rightarrow (\text{cosec } \theta - \cot \theta)^2 = \text{RHS}$$

$$\therefore (\text{cosec } \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$$

Hence proved

$$\text{(ii)} \quad \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

Ans: Given expression is $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

Now, taking LCM, we get

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{\cos^2 A + (1 + \sin A)(1 + \sin A)}{(1 + \sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{\cos^2 A + \sin^2 A + 2\sin A + 1}{(1 + \sin A)\cos A}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{1 + 2\sin A + 1}{(1 + \sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{2 + 2\sin A}{(1 + \sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{2(1 + \sin A)}{(1 + \sin A)\cos A}$$

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \frac{2}{\cos A}$$

We know that $\sec \theta = \frac{1}{\cos \theta}$, we get

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2\sec A$$

$$\Rightarrow \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = \text{RHS}$$

$$\therefore \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2\sec A$$

Hence proved

$$\text{(iii)} \quad \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

Ans: Given expression is $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$$

Now, we know that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$.

By substituting the values, we get

$$\begin{aligned} \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \left(\frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \right) \\ \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \left(\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \right) \\ \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \left(\frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \right) \\ \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{1}{(\sin \theta - \cos \theta)} \left(\frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\sin \theta} \right) \\ \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{1}{(\sin \theta - \cos \theta)} \left(\frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta} \right) \end{aligned}$$

Now, by applying the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, we get

$$\Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \right]$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\begin{aligned} \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{1}{(\sin \theta - \cos \theta)} \left[\frac{(\sin \theta - \cos \theta)(1 + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \right] \\ \Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{(1 + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \end{aligned}$$

$$\Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = \frac{1}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$\Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = \frac{1}{\sin \theta \cos \theta} + 1$$

We know that $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$, we get

$$\Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = \sec \theta \operatorname{cosec} \theta + 1$$

$$\Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\Rightarrow \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = \text{RHS}$$

$$\therefore \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

Hence proved

$$\text{(iv)} \quad \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

Ans: Given expression is $\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{1 + \sec A}{\sec A}$$

Now, we know that $\sec \theta = \frac{1}{\cos \theta}$.

By substituting the value, we get

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \frac{\frac{\cos A + 1}{\cos A}}{\frac{1}{\cos A}}$$

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \cos A + 1$$

Multiply and divide by $(1 - \cos A)$, we get

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \frac{(1 + \cos A)(1 - \cos A)}{(1 - \cos A)}$$

Now, by applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \frac{1 - \cos^2 A}{(1 - \cos A)}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{(1 - \cos A)}$$

$$\Rightarrow \frac{1 + \sec A}{\sec A} = \text{RHS}$$

$$\therefore \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

Hence proved

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Ans: Given expression is $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$.

Now, let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Dividing numerator and denominator by $\sin A$, we get

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

Now, we know that $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$, we get

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

Now, by applying the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$, substitute $1 = \cot^2 A - \operatorname{cosec}^2 A$, we get

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\cot A - (\cot^2 A - \operatorname{cosec}^2 A) + \operatorname{cosec} A}{\cot A + \cot^2 A - \operatorname{cosec}^2 A - \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\cot A - \cot^2 A + \operatorname{cosec}^2 A + \operatorname{cosec} A}{\cot A + \cot^2 A - \operatorname{cosec}^2 A - \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{\cot^2 A - 1 + \operatorname{cosec}^2 A + 2 \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{(\cot A - 1 + \operatorname{cosec} A)^2}{\cot^2 A - 1 + \operatorname{cosec}^2 A + 2 \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{2 \operatorname{cosec}^2 A + 2 \cot A \operatorname{cosec} A - 2 \cot A - 2 \operatorname{cosec} A}{\cot^2 A - 1 + \operatorname{cosec}^2 A + 2 \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{2 \operatorname{cosec} A (\cot A - \operatorname{cosec} A) - 2 (\cot A - \operatorname{cosec} A)}{\cot^2 A - 1 + \operatorname{cosec}^2 A + 2 \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{(2 \operatorname{cosec} A - 2)(\cot A - \operatorname{cosec} A)}{1 - 1 + 2 \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{(2 \operatorname{cosec} A - 2)(\cot A - \operatorname{cosec} A)}{2 \operatorname{cosec} A}$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

$$\Rightarrow \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{RHS}$$

$$\therefore \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

Hence proved

$$\text{(vi)} \quad \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

Ans: Given expression is $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

Now, multiply and divide the expression by $\sqrt{1 + \sin A}$, we get

$$\Rightarrow \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

Now, by applying the identity $(a + b)(a - b) = a^2 - b^2$, we get

$$\Rightarrow \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{1 + \sin A}{\sqrt{\cos^2 A}}$$

$$\Rightarrow \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{1 + \sin A}{\cos A}$$

$$\Rightarrow \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\Rightarrow \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$\Rightarrow \sqrt{\frac{1+\sin A}{1-\sin A}} = \text{RHS}$$

$$\therefore \sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

Hence proved

$$\text{(vii)} \quad \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \tan \theta$$

Ans: Given expression is $\frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \tan \theta$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta}$$

Taking common terms out, we get

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta (2\cos^2 \theta - 1)}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta (2(1 - 2\sin^2 \theta) - 1)}$$

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta (2 - 2\sin^2 \theta - 1)}$$

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \frac{\sin \theta (1 - 2\sin^2 \theta)}{\cos \theta (1 - 2\sin^2 \theta)}$$

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \tan \theta$$

$$\Rightarrow \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \text{RHS}$$

$$\therefore \frac{\sin \theta - 2\sin^3 \theta}{2\cos \theta - \cos \theta} = \tan \theta$$

Hence proved

$$\text{(viii)} \quad (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

Ans: Given expression is

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A.$$

Let us consider the LHS of the given expression, we get

$$\text{LHS} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

Now, by applying the identity $(a + b)^2 = a^2 + 2ab + b^2$, we get

$$\Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A +$$

$$\Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = \sin^2 A + \cos^2 A + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \operatorname{cosec} A +$$

We know that $\sin^2 \theta + \cos^2 \theta = 1$, $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$, we get

$$\Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 1 + \operatorname{cosec}^2 A + \sec^2 A + 2\sin A \frac{1}{\sin A} + 2\cos A \frac{1}{\cos A}$$

$$\Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 1 + (1 + \cot^2 A + 1 + \tan^2 A) + 2 + 2$$

$$\Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$\Rightarrow (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = \text{RHS}$$

$$\therefore (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

Hence proved

$$\text{(ix)} \quad (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

Ans: Given expression is $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

We know that $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$, we get

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right)$$

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right)$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right)$$

$$\Rightarrow (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \sin A \cos A$$

Now, consider the RHS of the given expression, we get

$$\text{RHS} = \frac{1}{\tan A + \cot A}$$

Now, we know that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$.

$$\Rightarrow \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$\Rightarrow \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$\Rightarrow \frac{1}{\tan A + \cot A} = \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$$

Now, we know that $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\Rightarrow \frac{1}{\tan A + \cot A} = \sin A \cos A$$

Here, we get LHS=RHS

$$\therefore (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

Hence proved

$$(x) \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2$$

Ans: Given expression is $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2$.

Let us consider the LHS of the given expression, we get

$$\text{LHS} = \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

By applying the identities $\sec^2 A = 1 + \tan^2 A$ and $\operatorname{cosec}^2 A = 1 + \cot^2 A$, we get

$$\Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

We know that $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\sec \theta = \frac{1}{\cos \theta}$, we get

$$\Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$\Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sin^2 A}{\cos^2 A}$$

$$\Rightarrow \frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A$$

Now, consider the RHS of the given expression, we get

$$\text{RHS} = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2$$

Now, we know that $\cot \theta = \frac{1}{\tan \theta}$, we get

$$\Rightarrow \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \left(\frac{1 - \tan A}{1 - \frac{1}{\tan A}} \right)^2$$

$$\begin{aligned} \Rightarrow \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 &= \left(\frac{\frac{1 - \tan A}{\tan A - 1}}{\tan A} \right)^2 \\ \Rightarrow \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 &= (-\tan A)^2 \\ \Rightarrow \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 &= \tan^2 A \end{aligned}$$

Here, we get LHS=RHS

$$\therefore \frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2$$

Hence proved