coordinate geometry

7 Chapter

Exercise 7.1

1. Find the distance between the following pairs of points:

(i) $(2,3),(4,1)$

Ans: Given that,

Let the points be $(2,3)$ and $(4,1)$

To find the distance between the points $(2,3),(4,1)$.

Distance between two points is given by the Distance formula

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Here, $x_1=2$

$x_2=4$

$y_1=3$

$y_2=1$

Thus, the distance between $(2,3)$ and $(4,1)$ is given by,

$$d=\sqrt{(2-4)^2+(3-1)^2}$$

$$=\sqrt{(-2)^2+(2)^2}$$

$$=\sqrt{4+4}$$

$$=\sqrt{8}$$

$$=2\sqrt{2}$$

$\therefore$The distance between $(2,3)$ and $(4,1)$ is $2\sqrt{2}$ units.

(ii) $(-5,7),(-1,3)$

Ans: Given that,

Let the points be $(-5,7)$ and $(-1,3)$

To find the distance between the points $(-5,7),(-1,3)$.

Distance between two points is given by the Distance formula

$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Here, $x_1=-5$

$x_2=-1$

$y_1=7$

$y_2=3$

Thus, the distance between $(-5,7)$ and $(-1,3)$ is given by,

$d=\sqrt{(-5-(-1))^2+(7-3)^2}$

$=\sqrt{(-4)^2+(4)^2}$

$=\sqrt{16+16}$

$=\sqrt{32}$

$=4\sqrt{2}$

$\therefore$ The distance between $(-5,7)$ and $(-1,3)$ is $4\sqrt{2}$ units.

(iii) $(a,b),(-a,-b)$

Ans: Given that,

Let the points be (a,b) and $(-a,-b)$

To find the distance between the points $(a,b),(-a,-b)$.

Distance between two points is given by the Distance formula

$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Here, $x_1=a$

$x_2=-a$

$y_1=b$

$y_2=-b$

Thus, the distance between (a,b) and $(-a,-b)$ is given by,

$$d=\sqrt{(a-(-a))^2+(b-(-b))^2}$$

$$=\sqrt{(2a)^2+(2b)^2}$$

$$=\sqrt{4a^2+4b^2}$$

$$=\sqrt{4}\sqrt{a^2+b^2}$$

$$=2\sqrt{a^2+b^2}$$

$\therefore$ The distance between (a,b) and $(-a,-b)$ is $2\sqrt{a^2+b^2}$ units.

2. Find the distance between the points $(0,0)$ and $(36,15)$. Can you now find the distance between the two towns A and B discussed in Section 7.2?

Ans: Given that,

Let the points be $(0,0)$ and $(36,15)$

To find the distance between the points $(0,0),(36,15)$.

Distance between two points is given by the Distance formula

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Here, $x_1=0$

$x_2=36$

$y_1=0$

$y_2=15$

Thus, the distance between $(0,0)$ and $(36,15)$ is given by,

$$d=\sqrt{(0-36)^2+(0-15)^2}$$

$$=\sqrt{(-36)^2+(-15)^2}$$

$$=\sqrt{1296+225}$$

$$=\sqrt{1521}$$

$$=39$$

Yes, it is possible to find the distance between the given towns A and B. The positions of this town are $A(0,0)$ and $B(36,15)$. And it can be calculated as above.

$\therefore$ The distance between $A(0,0)$ and $B(36,15)$ is 39 km.

3. Determine if the points $(1,5),(2,3)$ and $(-2,-11)$ are collinear.

Ans: Given that,

Let the three points be $(1,5),(2,3)$ and $(-2,-11)$

To determine if the given points are collinear

Let $A(1,5),B(2,3),C(-2,-11)$ be the vertices of the given triangle.

The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

To find the distance between the points $A(1,5)$ and $B(2,3)$

$$x_1=1$$
$$x_2=2$$
$$y_1=5$$
$$y_2=3$$

$$AB=\sqrt{(1-2)^2+(5-3)^2}$$
$$=\sqrt{(-1)^2+(2)^2}$$
$$=\sqrt{1+4}$$
$$=\sqrt{5}$$

To find the distance between the points $B(2,3)$ and $C(-2,-11)$

$$x_1=2$$
$$x_2=-2$$
$$y_1=3$$
$$y_2=-11$$

$$BC=\sqrt{\left(2-(-2)\right)^2+\left(3-(-11)\right)^2}$$
$$=\sqrt{(4)^2+(14)^2}$$
$$=\sqrt{16+196}$$
$$=\sqrt{212}$$

To find the distance between the points $A(1,5)$ and $C(-2,-11)$

$x_1=1$

$x_2=-2$

$y_1=5$

$y_2=-11$

$$CA=\sqrt{\left(1-(-2)\right)^2+\left(5-(-11)\right)^2}$$
$$=\sqrt{(3)^2+(16)^2}$$
$$=\sqrt{9+256}$$
$$=\sqrt{265}$$

Since $AB+AC\neq BC$ and $AB\neq BC+AC$

and $AC\neq BC$

$\therefore$ The points $A(1,5),B(2,3),C(-2,-11)$ are not collinear.

4. Check whether $(5,-2),(6,4)$ and $(7,-2)$ are the vertices of an isosceles triangle.

Ans: Given that,

Let the three points be $(5,-2),(6,4)$ and $(7,-2)$ are the vertices of the triangle.

To determine if the given points are the vertices of an isosceles triangle.

Let $A(5,-2),B(6,4),C(7,-2)$ be the vertices of the given triangle.

The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

To find the distance between the points $A(5,-2)$ and $B(6,4)$

$x_1 = 5$

$x_2 = 6$

$y_1 = -2$

$y_2 = 4$

$AB = \sqrt{(5-6)^2 + (-2-4)^2}$

$= \sqrt{(-1)^2 + (-6)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

To find the distance between the points $B(6,4)$ and $C(7,-2)$

$x_1 = 6$

$x_2 = 7$

$y_1 = 4$

$y_2 = -2$

$BC = \sqrt{(6-7)^2 + (4-(-2))^2}$

$= \sqrt{(-1)^2 + (6)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

To find the distance between the points $A(5,-2)$ and $C(7,-2)$

$x_1 = 5$

$x_2 = 7$

$y_1 = -2$

$y_2 = -2$

$CA = \sqrt{(5-7)^2 + (-2-(-2))^2}$

$= \sqrt{(-2)^2 + (0)^2}$

$= \sqrt{4+0}$

$= 2$

We can conclude that $AB = BC$.

Since two sides of the triangle are equal in length, ABC is an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A, B, C and D are shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think $ABCD$ is a square?" Chameli disagrees.

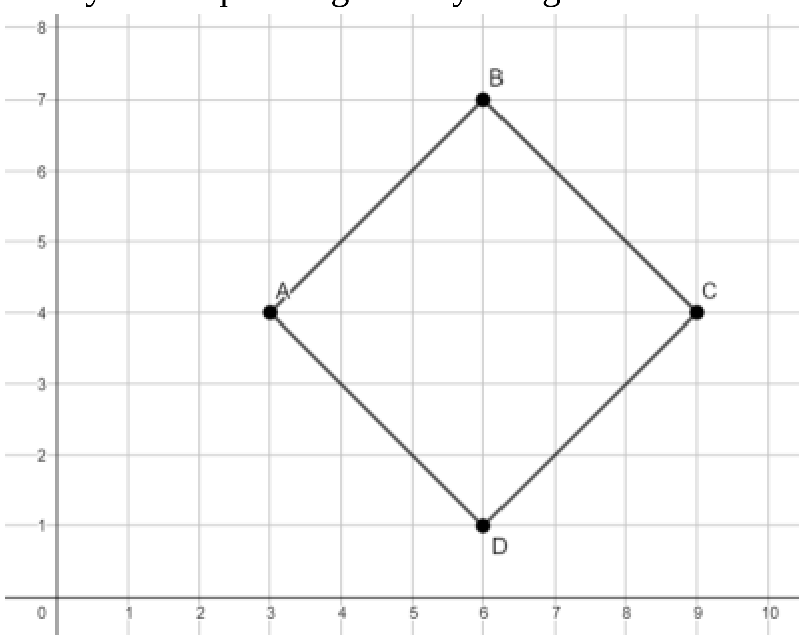
Using the distance formula, find which of them is correct.

Ans: Given that,

4 friends are seated at the points A, B, C, D

To find,

If they form square together by using distance formula



From the figure, we observe the points $A(3,4), B(6,7), C(9,4)$ and $D(6,1)$ are the positions of the four students.

The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

To find the distance between the points $A(3,4)$ and $B(6,7)$

$x_1 = 3$

$x_2 = 6$

$y_1 = 4$

$y_2 = 7$

$AB = \sqrt{(3-6)^2 + (4-7)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

To find the distance between the points $B(6,7)$ and $C(9,4)$

$x_1 = 6$

$x_2 = 9$

$y_1 = 7$

$y_2 = 4$

$BC = \sqrt{(6-9)^2 + (7-4)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

To find the distance between the points $C(9,4)$ and $(6,1)$

$x_1 = 9$

$x_2 = 6$

$y_1 = 4$

$y_2 = 1$

$CD = \sqrt{(9-6)^2 + (4-1)^2}$

$=\sqrt{(-3)^2+(3)^2}$

$=\sqrt{9+9}$

$=\sqrt{18}$

$=3\sqrt{2}$ To find the distance between the points $A(3,4)$ and $D(6,1)$

$x_1=3$

$x_2=6$

$y_1=4$

$y_2=1$

$AB=\sqrt{(3-6)^2+(4-1)^2}$

$=\sqrt{(-3)^2+(3)^2}$

$=\sqrt{9+9}$

$=\sqrt{18}$

$=3\sqrt{2}$

Since all sides of the squares are equal, now find the distance between the diagonals AC and BD.

To find the distance between the points $A(3,4)$ and $C(9,4)$

$x_1=3$

$x_2=9$

$y_1=4$

$y_2=4$

Diagonal $AC=\sqrt{(3-9)^2+(4-4)^2}$

$=\sqrt{(-6)^2+(0)^2}$

$=\sqrt{36+0}$

$=6$

Diagonal To find the distance between the points $B(6,7)$ and $D(6,1)$

$x_1 = 6$
$x_2 = 6$
$y_1 = 7$
$y_2 = 1$

Diagonal $BD = \sqrt{(6-6)^2 + (7-1)^2}$
$= \sqrt{(0)^2 + (6)^2}$
$= \sqrt{0+36}$
$= 6$

Thus, the four sides AB, BC, CD and DA are equal and its diagonals AC and BD are also equal.

$\therefore ABCD$ form a square and hence Champa was correct.

6. Name the type of quadrilateral forms, if any, by the following points, and give reasons for your answer.

(i) $(-1,-2),(1,0),(-1,2),(-3,0)$

Ans: Given that,

Let the given points denote the vertices $A(-1,-2), B(1,0), C(-1,2), D(-3,0)$ denote the vertices of the quadrilateral.

The distance between any two points is given by the Distance formula,

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

To find the distance between the points $A(-1,-2)$ and $B(1,0)$

$x_1 = -1$
$x_2 = 1$
$y_1 = -2$
$y_2 = 0$

$AB = \sqrt{(-1-1)^2 + (-2-0)^2}$
$= \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$

To find the distance between the points $B(1,0)$ and $C(-1,2)$

$x_1 = 1$
$x_2 = -1$
$y_1 = 0$
$y_2 = 2$

$BC = \sqrt{(1-(-1))^2 + (0-2)^2}$
$= \sqrt{(2)^2 + (-2)^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$

To find the distance between the points $C(-1,2)$ and $D(-3,0)$

$x_1 = -1$
$x_2 = -3$
$y_1 = 2$
$y_2 = 0$

$CD = \sqrt{(-1-(-3))^2 + (2-0)^2}$
$= \sqrt{(2)^2 + (2)^2}$
$= \sqrt{4+4}$
$= \sqrt{8}$
$= 2\sqrt{2}$

To find the distance between the points $D(-3,0)$ and $A(-1,-2)$

$x_1 = -3$
$x_2 = -1$

$y_1 = 0$

$y_2 = -2$

$AD = \sqrt{\left(-3-(-1)\right)^2 + \left(0-(-2)\right)^2}$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

To find the diagonals of the given quadrilateral

To find the distance between the points $A(-1,-2)$ and $C(-1,2)$

$x_1 = -1$

$x_2 = -1$

$y_1 = -2$

$y_2 = 2$

Diagonal $AC = \sqrt{\left(-1-(-1)\right)^2 + (-2-2)^2}$

$= \sqrt{(0)^2 + (-4)^2}$

$= \sqrt{0+16}$

$= 4$

To find the distance between the points $B(1,0)$ and $D(-3,0)$

$x_1 = 1$

$x_2 = -3$

$y_1 = 0$

$y_2 = 0$

Diagonal $BD = \sqrt{\left(1-(-3)\right)^2 + (0-0)^2}$

$= \sqrt{(4)^2 + (0)^2}$

$= \sqrt{16+0}$

$= 4$

Since all sides of the given quadrilateral are of the same measure and the diagonals are also the same length.

$\therefore$ The given points of the quadrilateral form a square.

(ii) $(-3,5),(3,1),(0,3),(-1,-4)$

Ans: Given that,

Let the given points denote the vertices $A(-3,5),B(3,1),C(0,3),D(-1,-4)$ denote the vertices of the quadrilateral.

The distance between any two points is given by the Distance formula,

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

To find the distance between the points $A(-3,5)$ and $B(3,1)$

$x_1=-3$

$x_2=3$

$y_1=5$

$y_2=1$

$AB=\sqrt{(-3-3)^2+(5-1)^2}$

$=\sqrt{(-6)^2+(4)^2}$

$=\sqrt{36+16}$

$=\sqrt{52}$

$=2\sqrt{13}$

To find the distance between the points $B(3,1)$ and $C(0,3)$

$x_1=3$

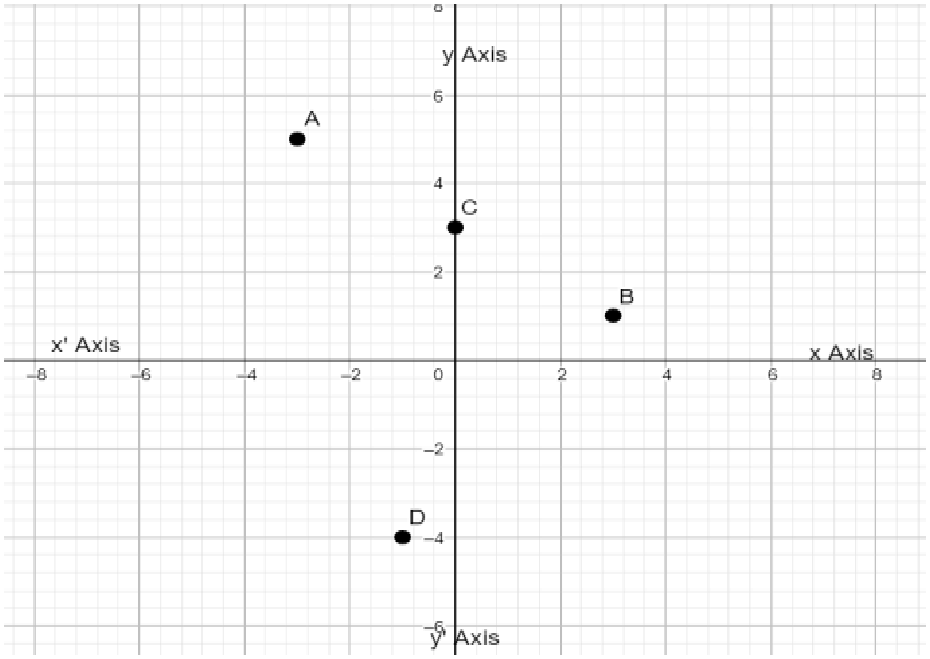
$x_2=0$

$y_1=1$

$y_2=3$

$BC=\sqrt{(3-0)^2+(1-3)^2}$

$=\sqrt{(3)^2+(-2)^2}$
$=\sqrt{9+4}$
$=\sqrt{13}$
To find the distance between the points $C(0,3)$ and $D(-1,-4)$
$x_1=0$
$x_2=-1$
$y_1=3$
$y_2=-4$
$CD=\sqrt{(0-(-1))^2+(3-(-4))^2}$
$=\sqrt{(1)^2+(7)^2}$
$=\sqrt{1+49}$
$=\sqrt{50}$
$=5\sqrt{2}$
To find the distance between the points $A(-3,5)$ and $D(-1,-4)$
$x_1=-3$
$x_2=-1$
$y_1=5$
$y_2=-4$
$AD=\sqrt{(-3-(-1))^2+(5-(-4))^2}$
$=\sqrt{(-2)^2+(9)^2}$
$=\sqrt{4+81}$
$=\sqrt{85}$
From the distance we found that,
$AB\neq BC\neq CD\neq AD$
By plotting the graph, we get,



From the graph above, we can note that the points ABC are collinear.

$\therefore$ The quadrilateral cannot be formed by using the above points.

(iii) $(4,5),(7,6),(4,3),(1,2)$

Ans: Given that,

Let the given points denote the vertices $A(4,5), B(7,6), C(4,3), D(1,2)$ denote the vertices of the quadrilateral.

The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

To find the distance between the points $A(4,5)$ and $B(7,6)$

$x_1 = 4$

$x_2 = 7$

$y_1 = 5$

$y_2 = 6$

$$AB = \sqrt{(4-7)^2 + (5-6)^2}$$

$$= \sqrt{(-3)^2 + (-1)^2}$$

$=\sqrt{9+1}$
$=\sqrt{10}$

To find the distance between the points $B(7,6)$ and $C(4,3)$

$x_1=7$
$x_2=4$
$y_1=6$
$y_2=3$

$BC=\sqrt{(7-4)^2+(6-3)^2}$
$=\sqrt{(3)^2+(3)^2}$
$=\sqrt{9+9}$
$=\sqrt{18}$
$=3\sqrt{2}$

To find the distance between the points $C(4,3)$ and $D(1,2)$

$x_1=4$
$x_2=1$
$y_1=3$
$y_2=2$

$CD=\sqrt{(4-1)^2+(3-2)^2}$
$=\sqrt{(3)^2+(1)^2}$
$=\sqrt{9+1}$
$=\sqrt{10}$

To find the distance between the points $A(4,5)$ and $D(1,2)$

$x_1=4$
$x_2=1$
$y_1=5$
$y_2=2$

$AD=\sqrt{(4-1)^2+(5-2)^2}$

$=\sqrt{(3)^2+(3)^2}$

$=\sqrt{9+9}$

$=\sqrt{18}$

$=3\sqrt{2}$

To find the diagonals of the given quadrilateral

To find the distance between the points $A(4,5)$ and $C(4,3)$

$x_1=4$

$x_2=4$

$y_1=5$

$y_2=3$

Diagonal $AC=\sqrt{(4-4)^2+(5-3)^2}$

$=\sqrt{(0)^2+(2)^2}$

$=\sqrt{0+4}$

$=2$

To find the distance between the points $B(7,6)$ and $D(1,2)$

$x_1=7$

$x_2=1$

$y_1=6$

$y_2=2$

Diagonal $BD=\sqrt{(7-1)^2+(6-2)^2}$

$=\sqrt{(6)^2+(4)^2}$

$=\sqrt{36+16}$

$=\sqrt{52}$

$=2\sqrt{13}$

Form the above calculation, the opposite sides of the quadrilateral are of same length and the diagonals are not of same length

$\therefore$ The given points of the quadrilateral form a parallelogram.

7. Find the point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$.

Ans: Given that,

$(2,-5)$

$(-2,9)$

To find,

The point that is equidistant from the points $(2,-5)$ and $(-2,9)$

Let us consider the points as $A(2,-5)$ and $B(-2,9)$ and to find the equidistant point P.

Since the point is on x-axis, the coordinates of the required point is of the form $P(x,0)$.

The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

To find the distance between the points $P(x,0)$ and $A(2,-5)$

$x_1=x$

$x_2=2$

$y_1=0$

$y_2=-5$

$$PA=\sqrt{(x-2)^2+(0-(-5))^2}$$

$$=\sqrt{(x-2)^2+25}$$

To find the distance between the points $A(x,0)$ and $B(-2,9)$

$x_1=x$

$x_2=-2$

$y_1=0$

$y_2 = 9$

$$PB = \sqrt{\left(x-(-2)\right)^2 + (0-9)^2}$$

$$= \sqrt{(x+2)^2 + (-9)^2}$$

$$= \sqrt{(x+2)^2 + 81}$$

Since the distance are equal in measure,

$PA = PB$

$$\sqrt{(x-2)^2 + 25} = \sqrt{(x+2)^2 + 81}$$

Taking square on both sides,

$$(x-2)^2 + 25 = (x+2)^2 + 81$$

By solving, we get,

$$x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$$

$8x = -56$

$x = -7$

The coordinate is $(-7,0)$.

$\therefore$ The point that is equidistant from $(2,-5)$ and $(-2,9)$ is $(-7,0)$.

8. Find the values of y for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10 units.

Ans: Given that,

$P(2,-3)$

$Q(10,y)$

The distance between these points are 10 units.

The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

To find the distance between the points $P(2,-3)$ and $Q(10,y)$

$x_1 = 2$

$x_2 = 10$

$y_1 = -3$

$y_2 = y$

$PQ = \sqrt{(10-2)^2 + (y-(-3))^2}$

$= \sqrt{(-8)^2 + (y+3)^2}$

$= \sqrt{64 + (y+3)^2}$

Since the distance between them is 10 units,

$\sqrt{64+(y+3)^2} = 10$

Squaring on both sides, we get,

$64 + (y+3)^2 = 100$

$(y+3)^2 = 36$

$y + 3 = \pm 6$

So, $y + 3 = 6$

$y = 3$

And, $y + 3 = -6$

$y = -9$

$\therefore$ The possible values of y are $y = 3$ or $y = -9$.

9. If $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$, find the values of x. Also find the distance of QR and PR.

Ans: Given that,

$Q(0,1)$

$P(5,-3)$

$R(x,6)$

To find,

- The values of x
- The distance of QR and PR

Q is equidistant between P and R.

SO $PQ = QR$

The distance between any two points is given by the Distance formula,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2 + 25}$$

Squaring on both sides, we get,

$$41 = x^2 + 25$$

$$x^2 = 16$$

$$x = \pm 4$$

Thus the point R is $R(4,6)$ or $(-4,6)$.

To find the distance PR and QR

Case (1):

When the point is $R(4,6)$

Distance between the point $P(5,-3)$ and $R(4,6)$ is,

$$PR = \sqrt{(5-4)^2 + (-3-6)^2}$$

$$= \sqrt{(1)^2 + (-9)^2}$$

$$= \sqrt{1+81}$$

$$= \sqrt{82}$$

Distance between the point $Q(0,1)$ and $R(4,6)$,

$$QR = \sqrt{(0-4)^2 + (1-6)^2}$$

$$= \sqrt{(-4)^2 + (-5)^2}$$

$$= \sqrt{16+25}$$

$$= \sqrt{41}$$

Case (2):

When the point is $R(-4,6)$

Distance between the point $P(5,-3)$ and $R(4,6)$,

$$PR=\sqrt{(5-(-4))^2+(-3-6)^2}$$
$$=\sqrt{(9)^2+(-9)^2}$$
$$=\sqrt{81+81}$$
$$=\sqrt{162}$$
$$=9\sqrt{2}$$

Distance between the point $Q(0,1)$ and $R(4,6)$,

$$QR=\sqrt{(0-(-4))^2+(1-6)^2}$$
$$=\sqrt{(4)^3+(-5)^2}$$
$$=\sqrt{16+25}$$
$$=\sqrt{41}$$

10. Find a relation between x and y such that the point (x,y) is equidistant from the point $(3,6)$ and $(-3,4)$.

Ans: Given that,

(x,y) is equidistant from $(3,6)$ and $(-3,4)$

To find,

The value of (x,y)

Let $P(x,y)$ is equidistant from $A(3,6)$ and $B(-3,4)$

Since they are equidistant,

$$PA=PB$$
$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{\left(x-(-3)^2\right)+(y-4)^2}$$
$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$$

Squaring on both sides, we get,

$$\left(x-3\right)^2+\left(y-6\right)^2=\left(x+3\right)^2+\left(y-4\right)^2$$
$$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$$
$$36-16=12x+4y$$
$$3x+y=5$$
$$3x+y-5=0$$
$\therefore$ The relation between x and y is given by $3x+y-5=0$

Exercise 7.2

1. Find the coordinates of the point which divides the join of $\left(-1,7\right)$ and $\left(4,-3\right)$ in the ratio $2:3$

Ans: Given that,

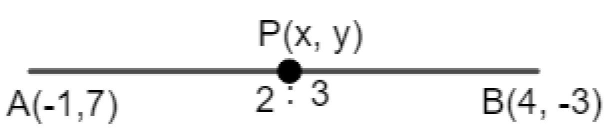
The points $A\left(-1,7\right)$ and $B\left(4,-3\right)$

Ratio $m:n=\ 2:3$

To find,

The coordinates

Let $P\left(x,y\right)$ be the required coordinate



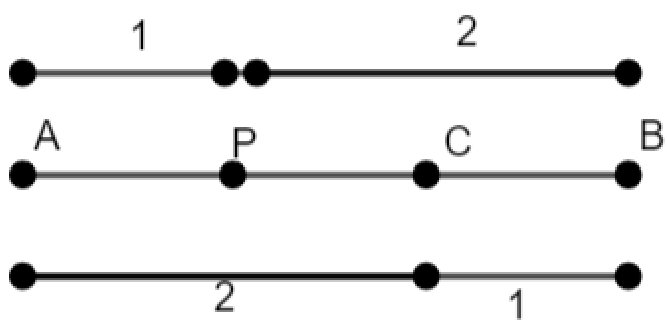
By section formula,

$$P\left(x,y\right)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$
$$P\left(x,y\right)=\left[\frac{2\left(4\right)+3\left(-1\right)}{2+3},\frac{2\left(-3\right)+3\left(7\right)}{2+3}\right]$$
$$=\left[\frac{8-3}{5},\frac{-6+21}{5}\right]$$

$=\left[\frac{5}{5},\frac{15}{5}\right]$
$=(1,3)$
$\therefore$ The coordinates of P is $P(1,3)$.

2. Find the coordinates of the point of trisection of the line segment joining $(4,-1)$ and $(-2,-3)$.

Ans: Given that,
The line segment joining $(4,-1)$ and $(-2,-3)$
To find,
The coordinates of the point
Let the line segment joining the points be $A(4,-1)$ and $B(-2,-3)$
Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ are the points of trisection of the line segment joining the points
$AP=PQ=QB$



From the diagram, the point P divides AB internally in the ratio of $1:2$
Hence $m:n=1:2$
By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P(x_1,y_1)=\left[\frac{1(-2)+1(4)}{1+2},\frac{1(-3)+2(-1)}{1+2}\right]$$

$=\left[\frac{-2+8}{3},\frac{-3-2}{3}\right]$

$=\left[\frac{6}{3},\frac{-5}{3}\right]$

$=\left(2,-\frac{5}{3}\right)$

$\therefore$ The coordinates of P is $P\left(2,-\frac{5}{3}\right)$.

From the diagram, the point Q divides AB internally in the ratio of $2:1$

Hence $m:n=2:1$

By section formula,

$Q(x_2,y_2)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$

$Q(x_2,y_2)=\left[\frac{2(-2)+1(4)}{2+1},\frac{2(-3)+3(-1)}{2+1}\right]$

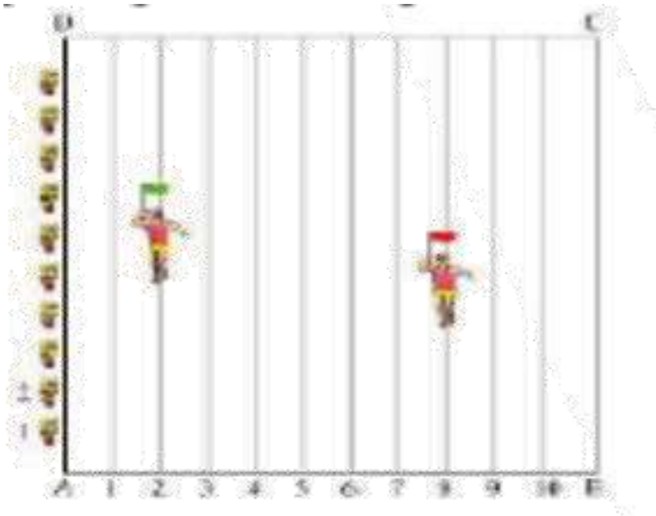
$=\left[\frac{-4+4}{3},\frac{-6-1}{3}\right]$

$=\left[0,-\frac{7}{3}\right]$

$\therefore$ The coordinates of Q is $Q\left(0,-\frac{7}{3}\right)$.

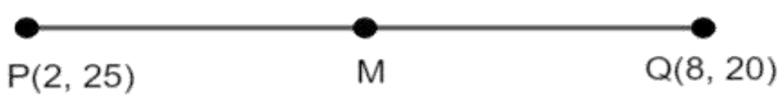
3. To conduct Sports day activities, in your rectangular shaped school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}$ th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If

Rashmi has to post a blue flag exactly halfway between the line segments joining the two flags, where should she post her flag?



Ans: Given that,

- Niharika posted her green flag at a distance of P which is $\frac{1}{4}\times 100 = 25\text{ m}$ from the starting point of the second line. The coordinate of P is $P(2,25)$
- Preet posted red flag at $\frac{1}{5}$ of a distance Q which is $\frac{1}{5}\times 100 = 20\text{ m}$ from the starting point of the eighth line. The coordinate of Q is $(8,20)$



The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

The distance between the flags $P(2,25), Q(8,20)$ is given by,

$$PQ=\sqrt{(8-2)^2+(25-20)^2}$$

$$=\sqrt{(6)^2+(5)^2}$$

$$=\sqrt{36+25}$$

$=\sqrt{61}\text{ m}$

Rashmi should post her blue flag in the mid-point of the line joining points P and Q. Let this point be $M(x,y)$.

By section formula,

$$M(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$M(x,y)=\left[\frac{1(2)+1(8)}{1+1},\frac{1(25)+1(20)}{1+1}\right]$$

$$=\left[\frac{2+8}{2},\frac{25+20}{2}\right]$$

$$=\left[\frac{10}{2},\frac{45}{2}\right]$$

$$=(5,22.5)$$

$\therefore$Rashmi should place her blue flag at 22.5 m in the fifth line.

4. Find the ratio in which the line segment joining the points $(-3,10)$ and $(6,-8)$ is divided by $(-1,6)$

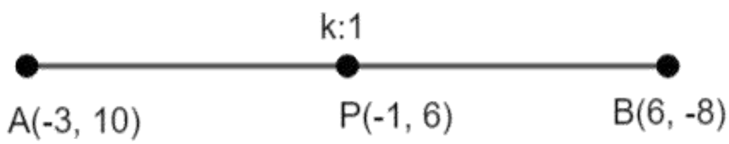
Ans: Given that,

The line segment $(-3,10)$ and $(6,-8)$

The point $(-1,6)$ divides the line segment

To find,

The ratio of dividing line segment



Let the line segment be $A(-3,10)$ and $B(6,-8)$ divided by point $P(-1,6)$ in the ratio of $k:1$

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

Equating the x term,

$$-1=\frac{6k-3}{k+1}$$

$$-k-1=6k-3$$

$$7k=2$$

$$k=\frac{2}{7}$$

$\therefore$ The point P divides the line segment AB in the ratio of $2:7$

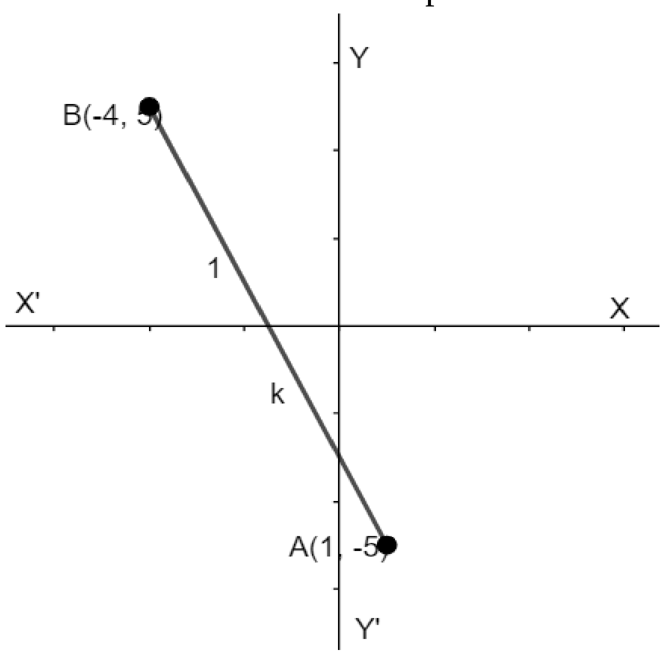
5. Find the ratio in which the line segment joining $A(1,-5)$ and $B(-4,5)$ is divided by the x axis. Also find the coordinates of the point of division.

Ans: Given that,

The line segment joining the points $A(1,-5)$ and $B(-4,5)$

To find,

- The ratio
- The coordinates of the point of division



Let the ratio be $k:1$

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P(x,y)=\left[\frac{k(-4)+1(1)}{k+1},\frac{k(5)+1(-5)}{k+1}\right]$$

$$=\left[\frac{-4k+1}{k+1},\frac{5k-5}{k+1}\right]$$

We know that y coordinate on x axis is zero.

$$\frac{5k-5}{k+1}=0$$

$$5k-5=0$$

$$5k=5$$

$$k=1$$

Therefore x axis divides it in the ratio of $1:1$

Division point, $P=\left(\frac{-4(1)+1}{1+1},\frac{5(1)-5}{1+1}\right)$

$$=\left(\frac{-4+1}{2},\frac{5-5}{2}\right)$$

$$=\left(-\frac{3}{2},0\right)$$

$\therefore$ The ratio at which the line segment is divided is $1:1$ and the point of division is $\left(-\frac{3}{2},0\right)$.

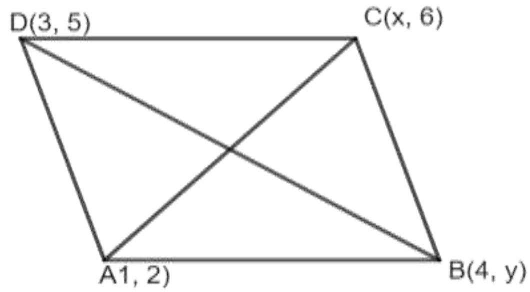
6. If $(1,2),(4,y),(x,6)$ and $(3,5)$ are the vertices of the parallelogram taken in order, find x and y.

Ans: Given that,

The vertices of the parallelogram are $A(1,2),B(4,y),C(x,6),D(3,5)$

To find,

The value of x and y



- The diagonals of the parallelogram bisect each other at O.
- Intersection point O of diagonal AC and BD divides these diagonals. So O is the midpoint of AC and BD.

If O is the midpoint of AC,

$$O=\left(\frac{1+x}{2},\frac{2+6}{2}\right)$$

$$=\left(\frac{1+x}{2},\frac{8}{2}\right)$$

$$=\left(\frac{1+x}{2},4\right)$$

If O is the midpoint of BD,

$$O=\left(\frac{4+3}{2},\frac{5+y}{2}\right)$$

$$=\left(\frac{7}{2},\frac{5+y}{2}\right)$$

Equating the points of O,

$\frac{x+1}{2}=\frac{7}{2}$ and

$$4=\frac{5+y}{2}$$

Finding x term,

$$\frac{x+1}{2}=\frac{7}{2}$$

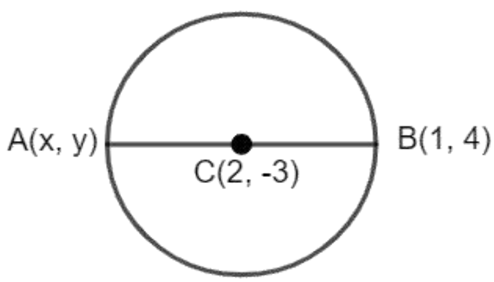
$x+1=7$
$x=6$
Finding y term,
$$4=\frac{5+y}{2}$$
$5+y=8$
$y=3$
The value of x and y are $x=6$ and $y=3$

7. Find the coordinates of point A, where AB is the diameter of circle whose center is $(2,-3)$ and B is $(1,4)$

Ans: Given that,

- Center is $C(2,-3)$
- The coordinate of B is $B(1,4)$

To find,
The coordinate of A
Let the coordinate of A be $A(x,y)$



Midpoint of AB is $C(2,-3)$ and so,
$$(2,-3)=\left(\frac{x+1}{2},\frac{y+4}{2}\right)$$
Equating x term,
$$\frac{x+1}{2}=2$$

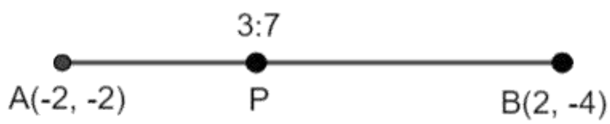
$$x+1=4$$
$$x=3$$
Equating y term,
$$\frac{y+4}{2}=-3$$
$$y+4=-6$$
$$y=-10$$
$\therefore$ The coordinate of A is $A(3,-10)$

8. If A and B are $(-2,-2)$ and $(2,-4)$ respectively, find the coordinates of P such that $AP=\frac{3}{7}AB$ and P lies on the line segment AB.

Ans: Given that,

- The coordinates are $A(-2,-2)$ and $B(2,-4)$
- $AP=\frac{3}{7}AB$

To find,
The coordinate of P



$$AP=\frac{3}{7}AB$$
$$\frac{AB}{AP}=\frac{7}{3}$$
From the figure, $AB=AP+PB$
$$\frac{AP+PB}{AP}=\frac{3+4}{3}$$
$$1+\frac{PB}{AP}=1+\frac{4}{3}$$

$\frac{PB}{AP}=\frac{4}{3}$

$\therefore AP:PB=3:4$

Thus, the point $P(x,y)$ divides the line segment AB in the ratio of $3:4$

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P(x,y)=\left[\frac{3(2)+4(-2)}{3+4},\frac{3(-4)+4(-2)}{3+4}\right]$$

$$=\left[\frac{6-8}{7},\frac{-12-8}{7}\right]$$

$$=\left[-\frac{2}{7},-\frac{20}{7}\right]$$

$\therefore$ The coordinates of P is $P\left(-\frac{2}{7},-\frac{20}{7}\right)$.

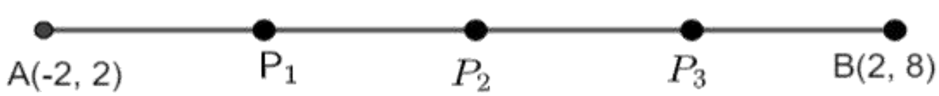
7. Find the coordinates of the points which divide the line segment joining $A(-2,2)$ and $B(2,8)$ into four equal parts.

Ans: Given that,

The line segment $A(-2,2)$ and $B(2,8)$

To find,

The coordinate that divides the line segment into four equal parts



Form the figure, P_1,P_2,P_3 be the points that divide the line segment AB into four equal parts.

Point P_1 divides the line segment AB in the ratio of $1:3$, so,

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P_1(x,y)=\left[\frac{1(2)+3(-2)}{1+3},\frac{1(8)+3(2)}{1+3}\right]$$

$$=\left[\frac{-4}{4},\frac{14}{4}\right]$$

$$=\left(-1,\frac{7}{2}\right)$$

Point P_2 divides the line segment AB in the ratio of $1:1$, so,

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P_2(x,y)=\left[\frac{1(2)+1(-2)}{1+1},\frac{1(8)+1(2)}{1+1}\right]$$

$$=\left[\frac{2+(-2)}{2},\frac{2+8}{2}\right]$$

$$=(0,5)$$

Point P_3 divides the line segment AB in the ratio of $3:1$, so,

By section formula,

$$P(x,y)=\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$$

$$P_3(x,y)=\left[\frac{3(2)+1(-2)}{3+1},\frac{3(8)+1(2)}{3+1}\right]$$

$$=\left[\frac{6-2}{4},\frac{24+2}{4}\right]$$

$$=\left(1,\frac{13}{2}\right)$$

The coordinates that divide the line segment into four equal parts are

$P_1\left(-1,\frac{7}{2}\right), P_2(0,5)$ and $P_3\left(1,\frac{13}{2}\right)$

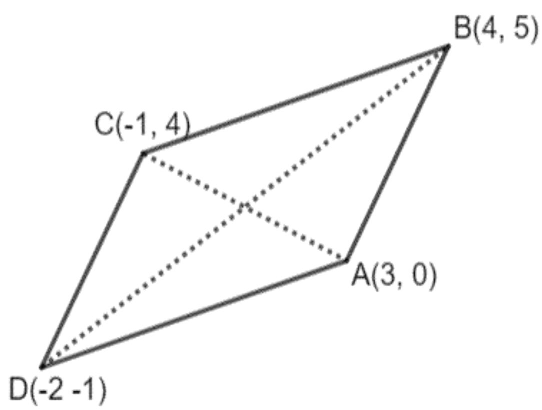
10. Find the area of the rhombus if its vertices are $(3,0),(4,5),(-1,4)$ and $(-2,-1)$ taken in order. [Hint: Area of a rhombus = $\frac{1}{2}$(Product of its diagonals)]

Ans: Given that,

The vertices of the rhombus are $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

To find,

The area of the rhombus



The distance between any two points is given by the Distance formula,

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Distance between the diagonal AC is given by,

$$AC=\sqrt{(3-(-1))^2+(0-4)^2}$$

$$=\sqrt{(4)^2+(-4)^2}$$

$$=\sqrt{16+16}$$

$$=\sqrt{32}$$

$= 4\sqrt{2}$

Distance between the diagonal BD si given by,

$BD = \sqrt{\left(4-(-2)\right)^2 + \left(5-(-1)\right)^2}$

$= \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= 6\sqrt{2}$

Area of the rhombus = $\frac{1}{2} \times$(Products of lengths of diagonals

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$square units