triangles

Chapter 6

EXERCISE 6.1

Question 1:

Fill in the blanks using correct word given in the brackets:–

(i) All circles are ___________. (congruent, similar)

(ii) All squares are ___________. (similar, congruent)

(iii) All ___________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___________ and (b) their corresponding sides are ___________. (equal, proportional)

Solution 1:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

Question 2:

Give two different examples of pair of

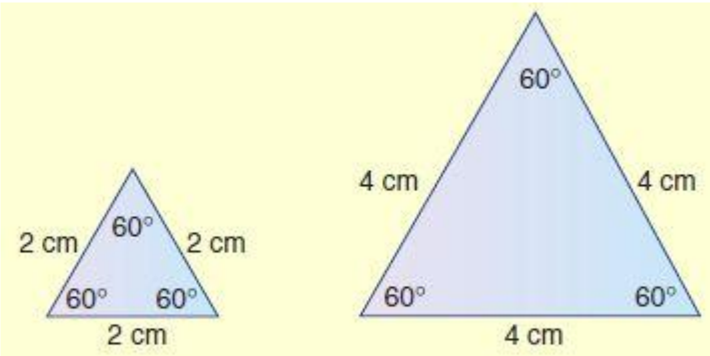
(i) Similar figures

(ii) Non-similar figures

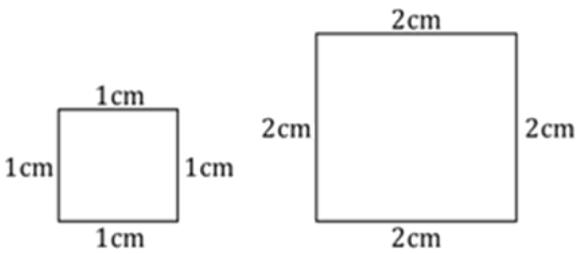
Solution 2:

(i) Similar figures

- Two equilateral triangles with sides 2 cm and 4 cm

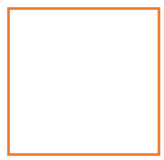


- Two squares with sides 1 cm and 2 cm

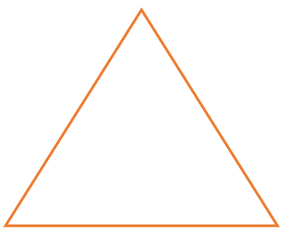


(ii) Non-similar figures

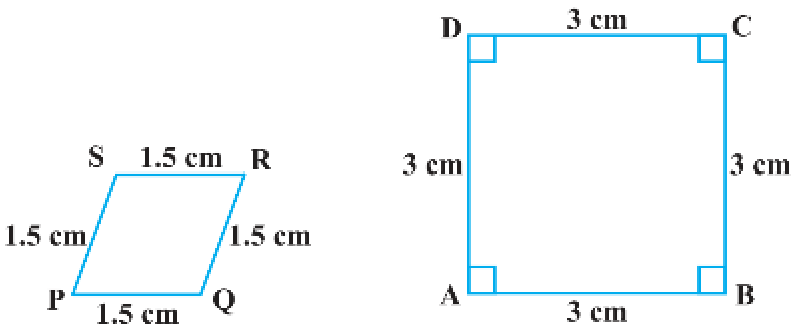
- Trapezium and square



- Triangle and parallelogram



Question 3:
State whether the following quadrilaterals are similar or not:

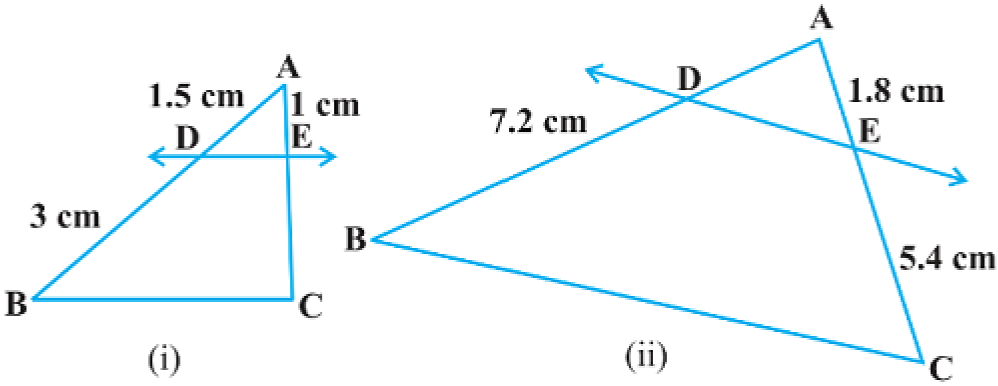


Solution 3:
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

EXERCISE 6.2

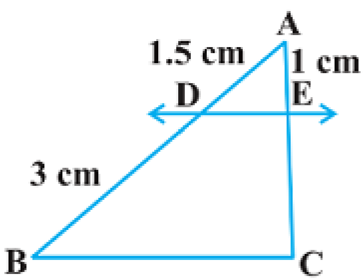
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Solution 1:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

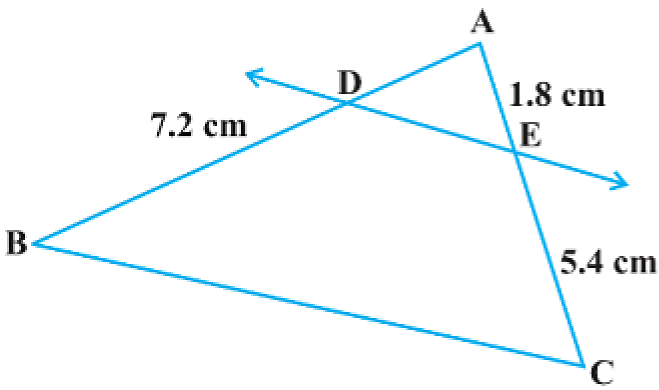
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore \ EC = 2 \text{ cm}$$

(ii)



Let AD = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore \ AD = 2.4 \text{cm}$$

Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

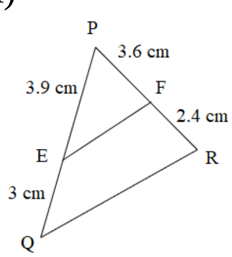
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Solution 2:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

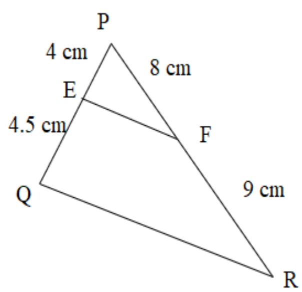
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

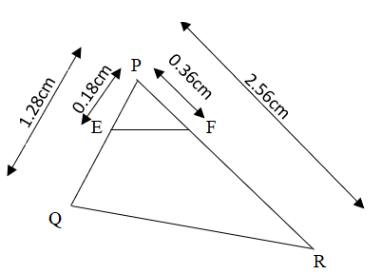
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ}=\frac{0.18}{1.28}=\frac{18}{128}=\frac{9}{64}$$

$$\frac{PF}{PR}=\frac{0.36}{2.56}=\frac{9}{64}$$

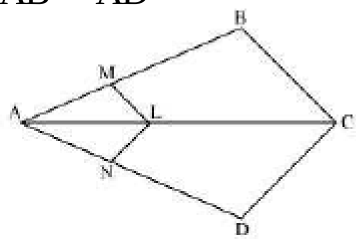
Hence, $\frac{PE}{PQ}=\frac{PF}{PR}$

Therefore, EF is parallel to QR.

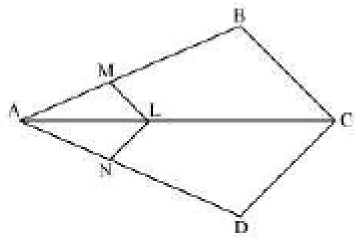
Question 3:

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB}=\frac{AN}{AD}$$



Solution 3:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB}=\frac{AL}{AC} \quad \text{(i)}$$

Similarly, $LN \| CD$

$$\therefore \frac{AN}{AD}=\frac{AL}{AC} \qquad \text{(ii)}$$

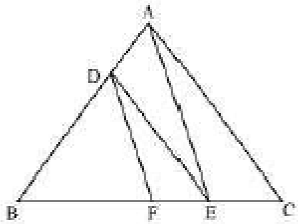
From (i) and (ii), we obtain

$$\frac{AM}{AB}=\frac{AN}{AD}$$

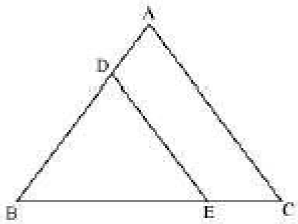
Question 4:

In the following figure, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$

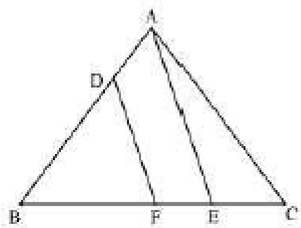


Solution 4:



In ΔABC, DE || AC

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ (Basic proportionality Theorem) (i)



In ΔBAE, DF || AE

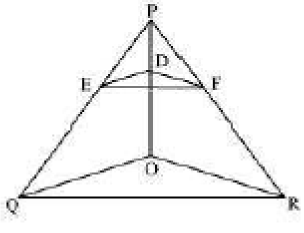
$\therefore \frac{BD}{DA} = \frac{BE}{FE}$ (Basic proportionality Theorem) (ii)

From (i) and (ii), we obtain

$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

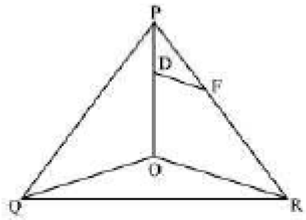
In the following figure, DE || OQ and DF || OR, show that EF || QR.



Solution 5:

In ΔPOQ, DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ (basic proportionality theorem) (i)



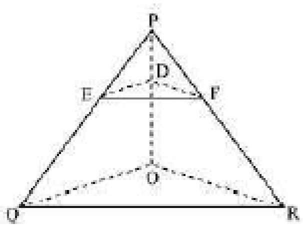
In ΔPOR, DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (basic proportionality theorem) (ii)

From (i) and (ii), we obtain

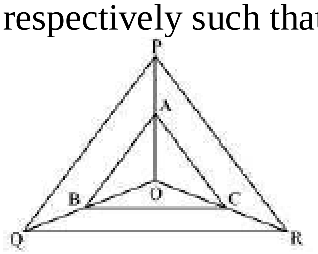
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \| QR$ (Converse of basic proportionality theorem)

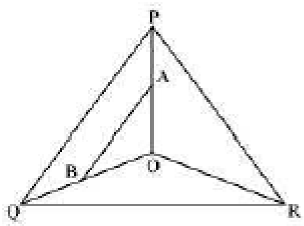


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

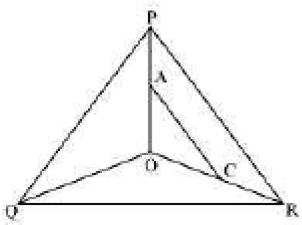


Solution 6:



In ΔPOQ, $AB \parallel PQ$

$\therefore \dfrac{OA}{AP} = \dfrac{OB}{BQ}$ (Basic proportionality theorem) (i)



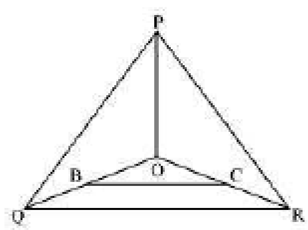
In ΔPOR, $AC \parallel PR$

$\therefore \dfrac{OA}{AP} = \dfrac{OC}{CR}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

$\dfrac{OB}{BQ} = \dfrac{OC}{CR}$

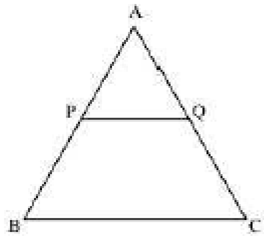
$\therefore BC \parallel QR$ (By the converse of basic proportionality theorem)



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Solution 7:



Consider the given figure in which PQ is a line segment drawn through the mid-pointP of line AB, such that $PQ \| BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad \text{(P is the mid point of AB} \therefore \text{AP = PB)}$$

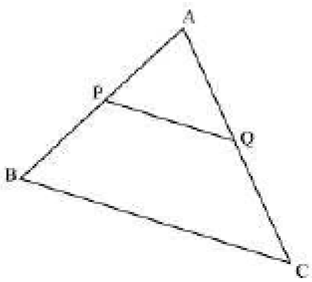
$\Longrightarrow$ AQ = QC

Or Q is the mid point of AC

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Solution 8:



Consider the given figure in which PQ is a line segment joining the mid-points P and
Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

And $\frac{AQ}{QC} = \frac{1}{1}$

$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$

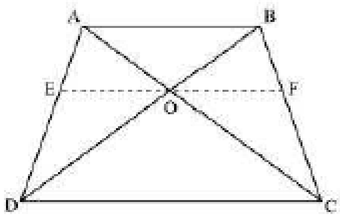
Hence, by using basic proportionality theorem, we obtain

PQ ∥ BC

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$

Solution 9:



Draw a line EF through point O, such that

In ΔADC, EO ∥ CD

By using basic proportionality theorem, we obtain

$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$

In ΔABD, OE ∥ AB

So, by using basic proportionality theorem, we obtain

$\frac{ED}{AE} = \frac{OD}{BO}$

$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$

From equation (1) and (2), we obtain

$\frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$

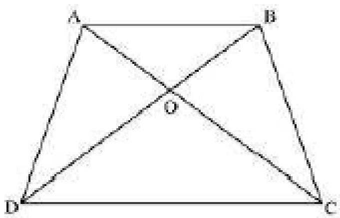
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$

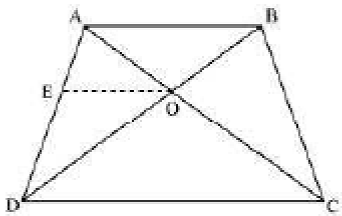
Show that ABCD is a trapezium.

Solution 10:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ;ABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

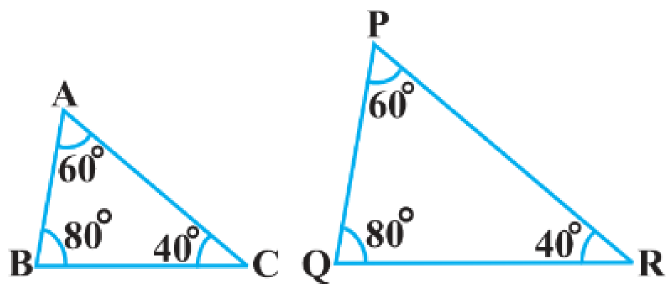
⇒ AB || CD

∴ ABCD is a trapezium.

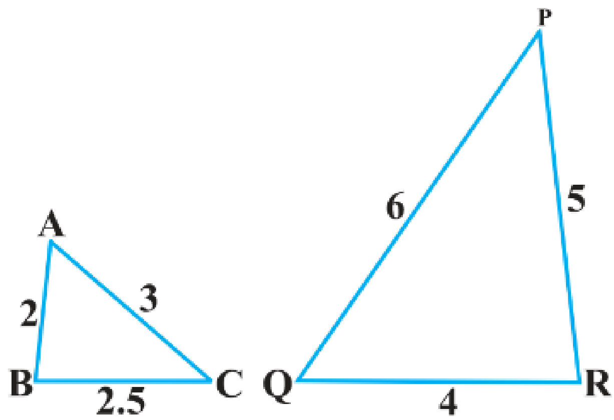
EXERCISE 6.3

Question 1:

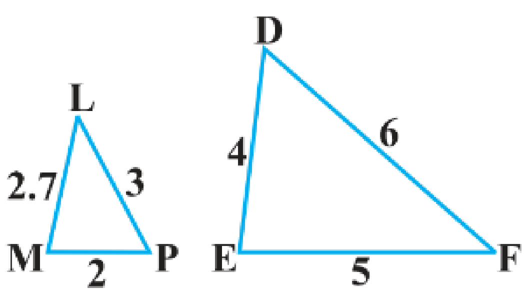
State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



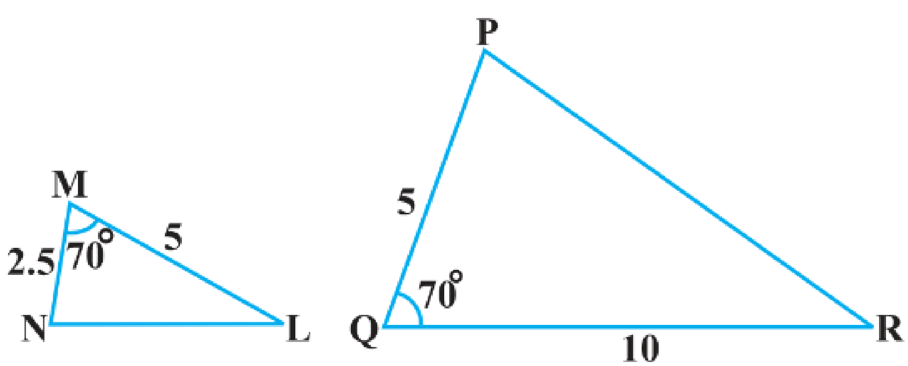
i.



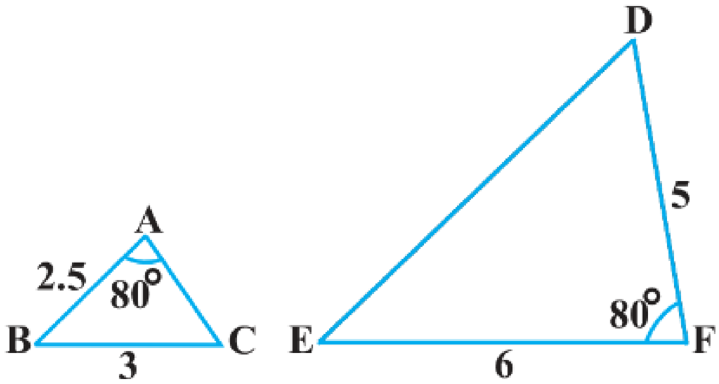
ii.

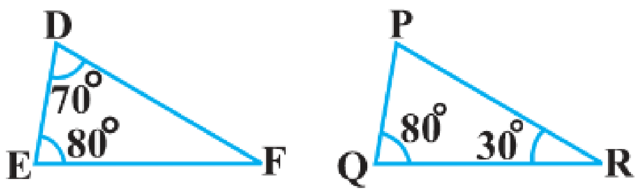


iii.



iv.

v. 

vi. 

Solution 1:

i. $\angle A = \angle P = 60°$
$\angle B = \angle Q = 80°$
$\angle C = \angle R = 40°$
Therefore, $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]
$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

ii. As corresponding sides are proportional
$\therefore$ $\Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

iii. The given triangles are not similar as the corresponding sides are not proportional.

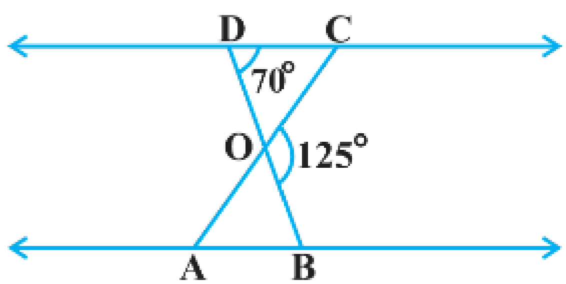
iv. The given triangles are similar By SAS similarity criteria
$\therefore$ $\Delta MNL \sim \Delta QPR$

v. The given triangles are not similar as the corresponding angle is not contained by the two corresponding sides.

vi. In ΔDEF,
$\angle D + \angle E + \angle F = 180°$
(Sum of the measures of the angles of a triangle is 180°.)

$70^o + 80^o + \angle F = 180^o$
$\angle F = 30^o$
Similarly, in ΔPQR,
$\angle P + \angle Q + \angle R = 180^o$
(Sum of the measures of the angles of a triangle is 180^o.)
$\angle P + 80^o + 30^o = 180^o$
$\angle P = 70^o$
In ΔDEF and ΔPQR,
$\angle D = \angle P$ (Each $70°$)
$\angle E = \angle Q$ (Each $80°$)
$\angle F = \angle R$ (Each $30°$)
$\therefore \Delta DEF \sim \Delta PQR$ [By AAA similarity criterion]

Question 2:
In the following figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$.
Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Solution 2:
DOB is a straight line.
$\therefore \angle DOC + \angle COB = 180°$
$\Rightarrow \angle DOC = 180° - 125°$
$= 55°$
In ΔDOC,
$\angle DCO + \angle CDO + \angle DOC = 180°$
(Sum of the measures of the angles of a triangle is 180^o.)
$\Rightarrow \angle DCO + 70^o + 55^o = 180°$
$\Rightarrow \angle DCO = 55°$
It is given that $\Delta ODC \sim \Delta OBA$.
$\therefore \angle OAB = \angle OCD$ [Corresponding angles are equal in similar triangles.]
$\Rightarrow \angle OAB = 55°$

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that

$$\frac{AO}{OC} = \frac{OB}{OD}$$

Solution 3:

In ΔDOC and ΔBOA,

∠CDO = ∠ABO [Alternate interior angles as AB || CD]

∠DCO = ∠BAO [Alternate interior angles as AB || CD]

∠DOC = ∠BOA [Vertically opposite angles]

∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

$\therefore \frac{DO}{BO} = \frac{OC}{OA}$ [Corresponding sides are proportional]

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$

Question 4:

In the figure 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that ΔPQS ~ ΔTQR

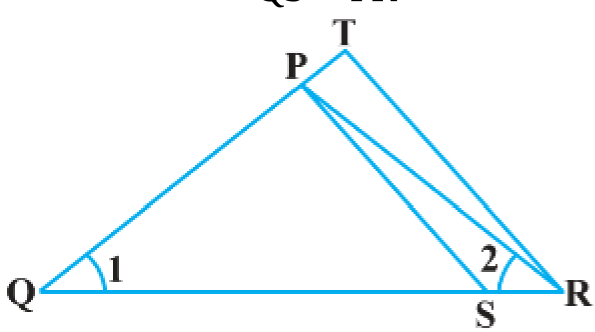
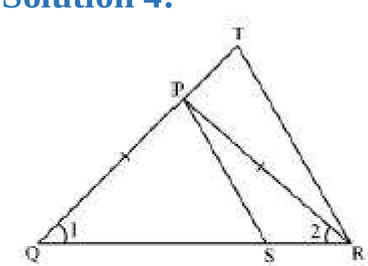


Fig. 6.36

Solution 4:



In ΔPQR, ∠PQR = ∠PRQ

$\therefore$ PQ = PR (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP}$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad \text{[Using (ii)]}$$

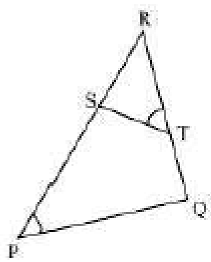
$$\angle Q = \angle Q$$

$\therefore$ ΔPQS ~ ΔTQR   [SAS similarity criterion]

Question 5:

S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Solution 5:



In ΔRPQ and ΔRST,

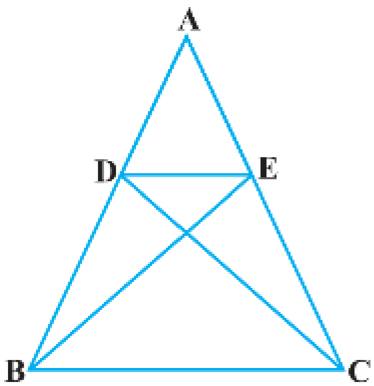
∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

$\therefore$ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Solution 6:

It is given that $\Delta ABE \cong \Delta ACD$.

$\therefore$ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

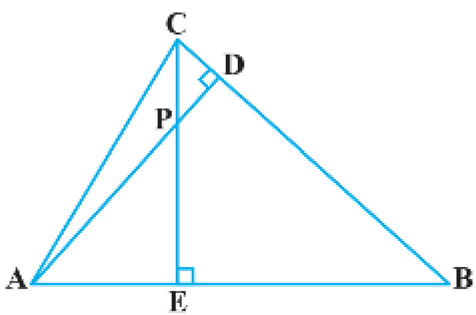
[Dividing equation (2) by (1)]

$\angle A = \angle A$ [Common angle]

$\therefore \Delta ADE \sim \Delta ABC$ [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) $\Delta AEP \sim \Delta CDP$

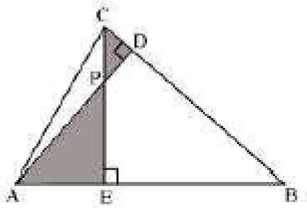
(ii) $\Delta ABD \sim \Delta CBE$

(iii) $\Delta AEP \sim \Delta ADB$

(v) $\Delta PDC \sim \Delta BEC$

Solution 7:

(i)



In ΔAEP and ΔCDP,

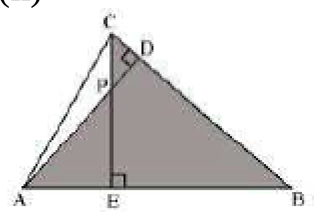
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

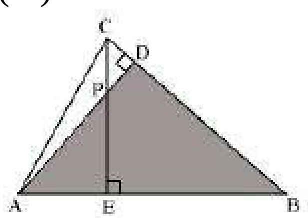
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

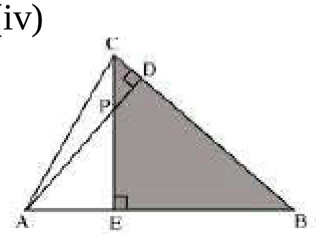
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

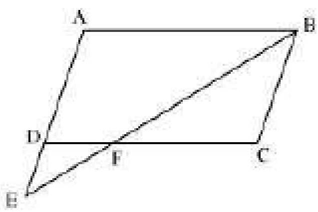
(iv)



In ΔPDC and ΔBEC,
$\angle PDC = \angle BEC$ (Each 90°)
$\angle PCD = \angle BCE$ (Common angle)
Hence, by using AA similarity criterion,
$\Delta PDC \sim \Delta BEC$

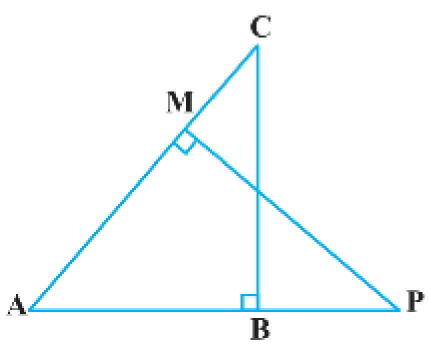
Question 8:
E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$

Solution 8:



In ΔABE and ΔCFB,
$\angle A = \angle C$ (Opposite angles of a parallelogram)
$\angle AEB = \angle CBF$ (Alternate interior angles as AE || BC)
$\therefore \Delta ABE \sim \Delta CFB$ (By AA similarity criterion)

Question 9:
In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



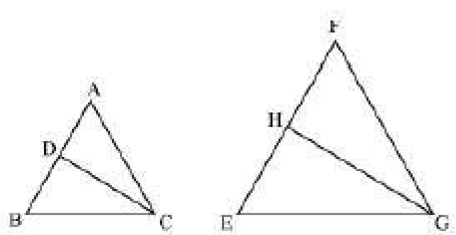
(i) $\Delta ABC \sim \Delta AMP$
(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

Solution 9:

In ΔABC and ΔAMP,
∠ABC = ∠AMP (Each 90°)
∠A = ∠A (Common)
∴ ΔABC ~ ΔAMP (By AA similarity criterion)
$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

Question 10:
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:
(i) $\frac{CD}{GH} = \frac{AC}{FG}$
(ii) ΔDCB ~ ΔHGE
(iii) ΔDCA ~ ΔHGF

Solution 10:

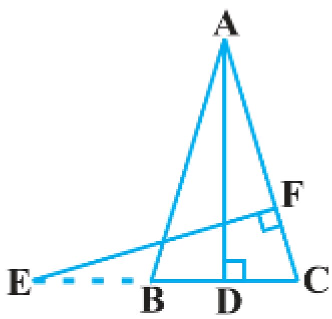


i. It is given that ΔABC ~ ΔFEG.
∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE
Since, ∠ACB = ∠FGE
∴ ∠ACD = ∠FGH (Angle bisector)
And, ∠DCB = ∠HGE (Angle bisector)
In ΔACD and ΔFGH,
∠A = ∠F (Proved above)
∠ACD = ∠FGH (Proved above)
∴ ΔACD ~ ΔFGH (By AA similarity criterion)
$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

ii. In ΔDCB and ΔHGE,
∠DCB = ∠HGE (Proved above)
∠B = ∠E (Proved above)
∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

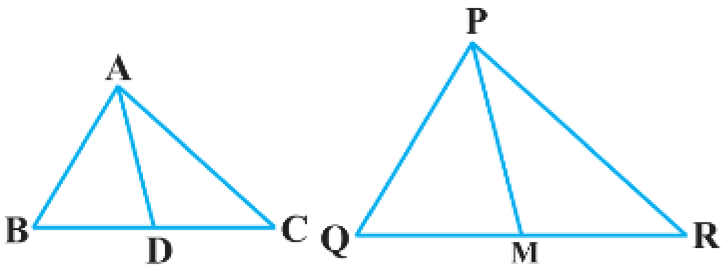
iii. In ΔDCA and ΔHGF,
∠ACD = ∠FGH (Proved above)
∠A = ∠F (Proved above)
∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

Question 11:
In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF



Solution 11:
It is given that ABC is an isosceles triangle.
∴ AB = AC
⇒ ∠ABD = ∠ECF
In ΔABD and ΔECF,
∠ADB = ∠EFC (Each 90°)
∠BAD = ∠CEF (Proved above)
∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:
Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.



Solution 12:

Median equally divides the opposite side.

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

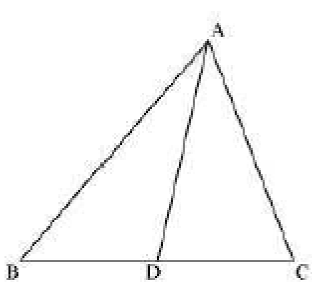
∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC.

Show that $CA^2 = CB.CD$

Solution 13:



In ΔADC and ΔBAC,

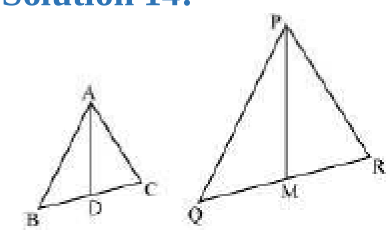
$\angle ADC = \angle BAC$ (Given)
$\angle ACD = \angle BCA$ (Common angle)
$\therefore \Delta ADC \sim \Delta BAC$ (By AA similarity criterion)
We know that corresponding sides of similar triangles are in proportion.

$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB.CD$$

Question 14:
Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

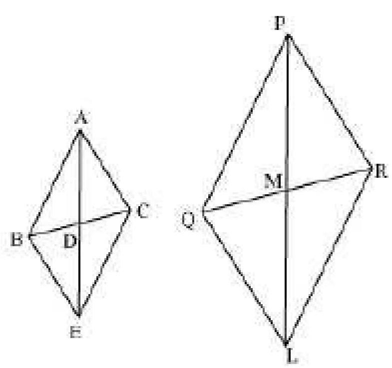
Solution 14:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM
= ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.
Therefore, BD = DC and QM = MR
Also, AD = DE (By construction)
And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.
Therefore, quadrilateral ABEC is a parallelogram.
∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)
Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR
It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

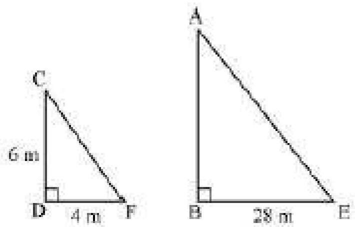
∴ $\Delta ABE \sim \Delta PQL$ (By SSS similarity criterion)
We know that corresponding angles of similar triangles are equal.
∴ $\angle BAE = \angle QPL$ … (1)
Similarly, it can be proved that $\Delta AEC \sim \Delta PLR$ and
$\angle CAE = \angle RPL$ … (2)
Adding equation (1) and (2), we obtain
$\angle BAE + \angle CAE = \angle QPL + \angle RPL$
$\Rightarrow \angle CAB = \angle RPQ$ … (3)
In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{AC}{PR} \text{(Given)}$$

$\angle CAB = \angle RPQ$ [Using equation (3)]
∴ $\Delta ABC \sim \Delta PQR$ (By SAS similarity criterion)

Question 15:
A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Solution 15:



Let AB and CD be a tower and a pole respectively.
Let the shadow of BE and DF be the shadow of AB and CD respectively.
At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.
Therefore, $\angle DCF = \angle BAE$
And, $\angle DFC = \angle BEA$
$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)
$\therefore \Delta ABE \sim \Delta CDF$ (AAA similarity criterion)

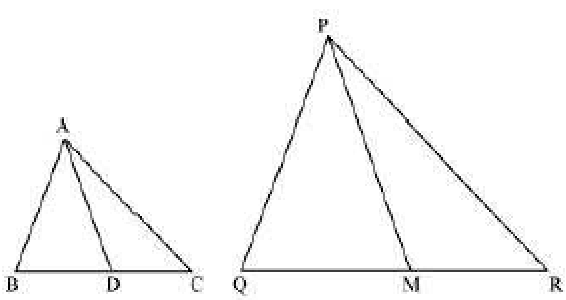
$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ cm}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

Therefore, the height of the tower will be 42 metres.

Question 16:
If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$



Solution 16:

It is given that $\Delta ABC \sim \Delta PQR$

We know that the corresponding sides of similar triangles are in proportion.

$\therefore \dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{BC}{QR}$ … (1)

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ … (2)

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD = \dfrac{BC}{2}$ and $QM = \dfrac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\dfrac{AB}{PQ} = \dfrac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

$\angle B = \angle Q$ [Using equation (2)]

$\dfrac{AB}{PQ} = \dfrac{BD}{QM}$ [Using equation (4)]

$\therefore \Delta ABD \sim \Delta PQM$ (By SAS similarity criterion)

$\Rightarrow \dfrac{AB}{PQ} = \dfrac{BD}{QM} = \dfrac{AD}{PM}$