quadratic equations

4
Chapter

Exercise 4.1

1. Check whether the following are quadratic equations:

i. $\mathbf{(x+1)^2=2(x-3)}$

Ans: $(x+1)^2=2(x-3)$

$\Rightarrow x^2+2x+1=2x-6$

$\Rightarrow x^2+7=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

ii. $\mathbf{x^2-2x=(-2)(3-x)}$

Ans: $x^2-2x=(-2)(3-x)$

$\Rightarrow x^2-2x=-6+2x$

$\Rightarrow x^2-4x+6=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

iii. $\mathbf{(x-2)(x+1)=(x-1)(x+3)}$

Ans: $(x-2)(x+1)=(x-1)(x+3)$

$\Rightarrow x^2-x-2=x^2+2x-3$

$\Rightarrow 3x-1=0$

Since, it is not in the form of $ax^2+bx+c=0$.

Therefore, the given equation is not a quadratic equation.

iv. $\mathbf{(x-3)(2x+1)=x(x+5)}$

Ans: $(x-3)(2x+1)=x(x+5)$

$\Rightarrow 2x^2-5x-3=x^2+5x$

$\Rightarrow x^2-10x-3=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

v. $\mathbf{(2x-1)(x-3)=(x+5)(x-1)}$

Ans: $(2x-1)(x-3)=(x+5)(x-1)$

$\Rightarrow 2x^2-7x+3=x^2+4x-5$

$\Rightarrow x^2-11x+8=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

vi. $\mathbf{x^2+3x+1=(x-2)^2}$

Ans: $x^2+3x+1=(x-2)^2$

$\Rightarrow x^2+3x+1=x^2+4-2x$

$\Rightarrow 7x-3=0$

Since, it is not in the form of $ax^2+bx+c=0$.

Therefore, the given equation is not a quadratic equation.

vii. $\mathbf{(x+2)^3=2x(x^2-1)}$

Ans: $(x+2)^3=2x(x^2-1)$

$\Rightarrow x^3+8+6x^2+12x=2x^3-2x$

$\Rightarrow x^3-14x-6x^2-8=0$

Since, it is not in the form of $ax^2+bx+c=0$.

Therefore, the given equation is not a quadratic equation.

viii. $\mathbf{x^3-4x^2-x+1=(x-2)^3}$

Ans: $x^3-4x^2-x+1=(x-2)^3$

$\Rightarrow x^3-4x^2-x+1=x^3-8-6x^2+12x$

$\Rightarrow 2x^2-13x+9=0$

Since, it is in the form of $ax^2+bx+c=0$.

Therefore, the given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations.

i. The area of a rectangular plot is $\mathbf{528\ m^2}$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Ans: Let the breath of the plot be $x\ m$.

Thus, length would be-

$\text{Length}=(2x+1)\text{m}$

Hence, Area of rectangle $=\text{Length}\times\text{breadth}$

So, $528=x(2x+1)$

$\Rightarrow 2x^2-x-528=0$

ii. The product of two consecutive positive integers is 306. We need to find the integers.

Ans: Let the consecutive integers be x and $x+1$.

Thus, according to question-

$x(x+1)=306$

$\Rightarrow x^2+x-306=0$

iii. Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Ans: Let Rohan's age be x.

Hence, his mother's age is $x+26$.

Now, after 3 years.

Rohan's age will be $x+3$.

His mother's age will be $x+29$.

So, according to question-

$(x+3)(x+29)=360$

$\Rightarrow x^2+3x+29x+87=360$

$\Rightarrow x^2+32x-273=0$

iv. A train travels a distance of 480 km at a uniform speed. If the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Ans: Let the speed of train be $x\text{ km/h}$.

Thus, time taken to travel 482 km is $\frac{480}{x}\text{hrs}$.

Now, let the speed of train $=(x-8)\text{km/h}$.

Therefore, time taken to travel 482 km is $\left(\frac{480}{x}+3\right)\text{hrs}$.

Hence, speed×time=distance

i.e $(x-8)\left(\frac{480}{x}+3\right)=480$

$\Rightarrow 480+3x-\frac{3840}{x}-24=480$

$\Rightarrow 3x-\frac{3840}{x}=24$

$\Rightarrow 3x^2-24x-3840=0$

$\Rightarrow x^2-8x-1280=0$

Exercise 4.2

1. Find the roots of the following quadratic equations by factorisation:

i. x^2-3x-10=0

Ans: $x^2-3x-10=0$

$\Rightarrow x^2-5x+2x-10$

$\Rightarrow x(x-5)+2(x-5)$

$\Rightarrow (x-5)(x+2)$

Therefore, roots of this equation are –

$x-5=0$ or $x+2=0$

i.e $x=5$ or $x=-2$

ii. $2x^2$+x-6=0

Ans: $2x^2+x-6=0$

$\Rightarrow 2x^2+4x-3x-6$

$\Rightarrow 2x(x+2)-3(x+2)$

$\Rightarrow (x+2)(2x-3)$

Therefore, roots of this equation are –

$x+2=0$ or $2x-3=0$

i.e $x=-2$ or $x=\frac{3}{2}$

iii. $\sqrt{2}x^2$+7x+5$\sqrt{2}$=0

Ans: $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2+5x+2x+5\sqrt{2}$

$\Rightarrow x\left(\sqrt{2}x+5\right)+\sqrt{2}\left(\sqrt{2}x+5\right)$

$\Rightarrow \left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$

Therefore, roots of this equation are –

$\sqrt{2}x+5=0$ or $x+\sqrt{2}=0$

i.e $x=\frac{-5}{\sqrt{2}}$ or $x=-\sqrt{2}$

iv. $\mathbf{2x^2-x+\frac{1}{8}=0}$

Ans: $2x^2-x+\frac{1}{8}=0$

$\Rightarrow \frac{1}{8}\left(16x^2-8x+1\right)$

$\Rightarrow \frac{1}{8}\left(4x\left(4x-1\right)-1\left(4x-1\right)\right)$

$\Rightarrow \left(4x-1\right)^2$

Therefore, roots of this equation are –

$4x-1=0$ or $4x-1=0$

i.e $x=\frac{1}{4}$ or $x=\frac{1}{4}$

v. $\mathbf{100x^2-20x+1=0}$

Ans: $100x^2-20x+1=0$

$\Rightarrow 100x^2-10x-10x+1$

$\Rightarrow 10x\left(10x-1\right)-1\left(10x-1\right)$

$\Rightarrow \left(10x-1\right)\left(10x-1\right)$

Therefore, roots of this equation are –

$\left(10x-1\right)=0$ or $\left(10x-1\right)=0$

i.e $x=\frac{1}{10}$ or $x=\frac{1}{10}$

2. Solve the problems given in Example 1

Ans: Let the number of john's marbles be x.
Thus, number of Jivanti's marble be $45-x$.
According to question i.e,
After losing 5 marbles.
Number of john's marbles be $x-5$
And number of Jivanti's marble be $40-x$.
Therefore, $(x-5)(40-x)=124$

$\Rightarrow x^2-45x+324=0$
$\Rightarrow x^2-36x-9x+324=0$
$\Rightarrow x(x-36)-9(x-36)=0$
$\Rightarrow (x-36)(x-9)=0$

So now,
Case 1- If $x-36=0$ i.e $x=36$
So, the number of john's marbles be 36.
Thus, number of Jivanti's marble be 9.
Case 2- If $x-9=0$ i.e $x=9$
So, the number of john's marbles be 9.
Thus, number of Jivanti's marble be 36.

ii. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.
Ans: Let the number of toys produced be x.
Therefore, Cost of production of each toy be $Rs(55-x)$.
Thus, $(55-x)x=750$

$\Rightarrow x^2-55x+750=0$
$\Rightarrow x^2-25x-30x+750=0$
$\Rightarrow x(x-25)-30(x-25)=0$
$\Rightarrow (x-25)(x-30)=0$

Case 1- If $x-25=0$ i.e $x=25$
So, the number of toys be 25.

Case 2- If $x-30=0$ i.e $x=30$
So, the number of toys be 30.

3. Find two numbers whose sum is 27 and product is 182.
Ans: Let the first number be x,
Thus, the second number be $27-x$.
Therefore,
$x\left(27-x\right)=182$
$\Rightarrow x^2-27x+182=0$
$\Rightarrow x^2-13x-14x+182=0$
$\Rightarrow x\left(x-13\right)-14\left(x-13\right)=0$
$\Rightarrow \left(x-13\right)\left(x-14\right)=0$
Case 1- If $x-13=0$ i.e $x=13$
So, the first number be 13,
Thus, the second number be 14.
Case 2- If $x-14=0$ i.e $x=14$
So, the first number be 14.
Thus, the second number be 13.

4. Find two consecutive positive integers, sum of whose squares is 365.
Ans: Let the consecutive positive integers be x and $x+1$.
Thus, $x^2+\left(x+1\right)^2=365$
$\Rightarrow x^2+x^2+1+2x=365$
$\Rightarrow 2x^2+2x-364=0$
$\Rightarrow x^2+x-182=0$
$\Rightarrow x^2+14x-13x-182=0$
$\Rightarrow x\left(x+14\right)-13\left(x+14\right)=0$
$\Rightarrow \left(x+14\right)\left(x-13\right)=0$
Case 1- If $x+14=0$ i.e $x=-14$.
This case is rejected because number is positive.
Case 2- If $x-13=0$ i.e $x=13$
So, the first number be 13.
Thus, the second number be 14.
Hence, the two consecutive positive integers are 13 and 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Ans: Let the base of the right-angled triangle be $x\text{ cm}$.

Its altitude be $(x-7)\text{cm}$.

Thus, by pythagores theorem-

$\text{base}^2+\text{altitude}^2=\text{hypotenuse}^2$

$\therefore x^2+(x-7)^2=13^2$

$\Rightarrow x^2+x^2+49-14x=169$

$\Rightarrow 2x^2-14x-120=0$

$\Rightarrow x^2-7x-60=0$

$\Rightarrow x^2+12x+5x-60=0$

$\Rightarrow x(x-12)+5(x-12)=0$

$\Rightarrow (x-12)(x+5)=0$

Case 1- If $x-12=0$ i.e $x=12$.

So, the base of the right-angled triangle be 12 cm and Its altitude be 5cm

Case 2- If $x+5=0$ i.e $x=-5$

This case is rejected because side is always positive.

Hence, the base of the right-angled triangle be 12 cm and Its altitude be 5cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Ans: Let the number of articles produced be x.

Therefore, cost of production of each article be $\text{Rs}(2x+3)$.

Thus, $x(2x+3)=90$

$\Rightarrow 2x^2+3x-90=0$

$\Rightarrow 2x^2+15x-12x-90=0$

$\Rightarrow x(2x+15)-6(2x+15)=0$

$\Rightarrow (2x+15)(x-6)=0$

Case 1- If $2x-15=0$ i.e $x=\frac{-15}{2}$.

This case is rejected because number of articles is always positive.

Case 2- If $x-6=0$ i.e $x=6$

Hence, the number of articles produced be 6.

Therefore, cost of production of each article be $\text{Rs}15$.

Exercise 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them-

i. $\mathbf{2x^2-3x+5=0}$

Ans: For a quadratic equation $ax^2+bx+c=0$.

Where Discriminant $=b^2-4ac$

Then –

Case 1- If $b^2-4ac>0$ then there will be two distinct real roots.

Case 2- If $b^2-4ac=0$ then there will be two equal real roots.

Case 3- If $b^2-4ac<0$ then there will be no real roots.

Thus, for $2x^2-3x+5=0$ **.**

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2,\ b=-3,\ c=5$.

Discriminant $=(-3)^2-4(2)(5)$

$=9-40$

$=-31$

Since, Discriminant: $b^2-4ac<0$.

Therefore, there is no real root for the given equation.

ii. $\mathbf{3x^2-4\sqrt{3}x+4=0}$

Ans: For a quadratic equation $ax^2+bx+c=0$.

Where Discriminant $=b^2-4ac$

Then –

Case 1- If $b^2-4ac>0$ then there will be two distinct real roots.

Case 2- If $b^2-4ac=0$ then there will be two equal real roots.

Case 3- If $b^2-4ac<0$ then there will be no real roots.

Thus, for $3x^2-4\sqrt{3}x+4=0$ **.**

On comparing this equation with $ax^2+bx+c=0$.

So, $a=3,\ b=-4\sqrt{3},\ c=4$.

Discriminant $=\left(-4\sqrt{3}\right)^2-4(3)(4)$

$=48-48$

$=0$

Since, Discriminant: $b^2-4ac=0$.

Therefore, there is equal real root for the given equation and the roots are-

$\frac{-b}{2a}$ and $\frac{-b}{2a}$.

Hence, roots are-

$$\frac{-b}{2a}=\frac{-\left(-4\sqrt{3}\right)}{6}$$

$$=\frac{4\sqrt{3}}{6}$$

$$=\frac{2\sqrt{3}}{3}$$

Therefore, roots are $\frac{2\sqrt{3}}{3}$ and $\frac{2\sqrt{3}}{3}$.

iii. $2x^2-6x+3=0$

Ans: For a quadratic equation $ax^2+bx+c=0$.

Where Discriminant $=b^2-4ac$

Then –

Case 1- If $b^2-4ac>0$ then there will be two distinct real roots.

Case 2- If $b^2-4ac=0$ then there will be two equal real roots.

Case 3- If $b^2-4ac<0$ then there will be no real roots.

Thus, for $2x^2-6x+3=0$ **.**

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2$, $b=-6$, $c=3$.

Discriminant $=\left(-6\right)^2-4\left(2\right)\left(3\right)$

$=36-24$

$=12$

Since, Discriminant: $b^2-4ac>0$.

Therefore, distinct real roots exists for the given equation and the roots are-

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Hence, roots are-

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{4}$$

$$=\frac{6\pm\sqrt{36-24}}{4}$$

$$=\frac{6\pm\sqrt{12}}{4}$$

$$=\frac{6\pm2\sqrt{3}}{4}$$

$$=\frac{3\pm\sqrt{3}}{2}$$

Therefore, roots are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

i. $2x^2+kx+3=0$

Ans: If a quadratic equation $ax^2+bx+c=0$ has two equal roots, then its discriminant will be 0 i.e., $b^2-4ac=0$

So, for $2x^2+kx+3=0$.

On comparing this equation with $ax^2+bx+c=0$.

So, $a=2$, $b=k$, $c=3$.

Discriminant $=(k)^2-4(2)(3)$

$=k^2-24$

For equal roots-

$b^2-4ac=0$

$\therefore k^2-24=0$

$\Rightarrow k^2=24$

$\Rightarrow k=\sqrt{24}$

$\Rightarrow k=\pm2\sqrt{6}$

ii. $kx(x-2)+6=0$

Ans: If a quadratic equation $ax^2+bx+c=0$ has two equal roots, then its discriminant will be 0 i.e., $b^2-4ac=0$

So, for $kx(x-2)+6=0$

$\Rightarrow kx^2-2kx+6=0$
On comparing this equation with $ax^2+bx+c=0$.
So, $a=k$, $b=-2k$, $c=6$.
Discriminant $=(-2k)^2-4(k)(6)$
$=4k^2-24k$
For equal roots-
$b^2-4ac=0$
$\therefore 4k^2-24k=0$
$\Rightarrow 4k(k-6)=0$
$\Rightarrow k=0$ or $k=6$
But k cannot be zero. Thus, this equation has two equal roots when k should be 6.

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800m^2$? If so, find its length and breadth.
Ans: Let the breadth of mango grove be x.
So, length of mango grove will be $2x$.
Hence, Area of mango grove is $=(2x)x$
$=2x^2$.
So, $2x^2=800$
$\Rightarrow x^2=400$
$\Rightarrow x^2-400=0$
On comparing this equation with $ax^2+bx+c=0$.
So, $a=1$, $b=0$, $c=400$.
Discriminant $=(0)^2-4(1)(-400)$
$=1600$
Since, Discriminant: $b^2-4ac>0$.
Therefore, distinct real roots exist for the given equation and the roots are-
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
Hence, roots are-
$$x=\frac{-(0)\pm\sqrt{(0)^2-4(1)(-400)}}{2}$$
$$=\frac{\pm\sqrt{1600}}{2}$$

$=\frac{\pm 40}{2}$
$=\pm 20$
Since, length cannot be negative.
Therefore, breadth of the mango grove is 20m.
And length of the mango grove be $2(20)\text{m}$ i.e., 40m.

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.
Ans: Let the age of one friend be x years.
So, age of the other friend will be $(20-x)$ years.
Thus, four years ago, the age of one friend be $(x-4)$ years.
And age of the other friend will be $(16-x)$ years.
Hence, according to question-
$(x-4)(16-x)=48$
$\Rightarrow 16x-64-x^2+4x=48$
$\Rightarrow 20x-112-x^2=0$
$\Rightarrow x^2-20x+112-=0$
On comparing this equation with $ax^2+bx+c=0$.
So, $a=1$, $b=-20$, $c=112$.
Discriminant $=(-20)^2-4(1)(112)$
$=400-448$
$=-48$
Since, Discriminant: $b^2-4ac<0$.
Therefore, there is no real root for the given equation and hence, this situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 m and area 400m^2? If so find its length and breadth.
Ans: Let the length of the park be $x\text{ m}$ and breadth of the park be $x\text{ m}$.
Thus, $\text{Perimeter}=2(x+y)$.
Hence, according to question-
$2(x+y)=80$
$\Rightarrow x+y=40$
$\Rightarrow y=40-x$.
Now, $\text{Area}=x\times y$.

Substituting value of y.

$\text{Area}=x(40-x)$

So, according to question-

$x(40-x)=400$

$\Rightarrow x^2-40x+400=0$

On comparing this equation with $ax^2+bx+c=0$.

So, $a=1$, $b=-40$, $c=400$.

Discriminant $=(-40)^2-4(1)(400)$

$=1600-1600$

$=0$

Since, Discriminant: $b^2-4ac=0$.

Therefore, there is equal real roots for the given equation and hence, this situation is possible.

Hence, roots are-

$$\frac{-b}{2a}=\frac{-(-40)}{2}$$

$$=\frac{40}{2}$$

$$=20$$

Therefore, length of park is $x=20m$.

And breadth of park be $y=(40-20)m$ i.e., $y=20m$.