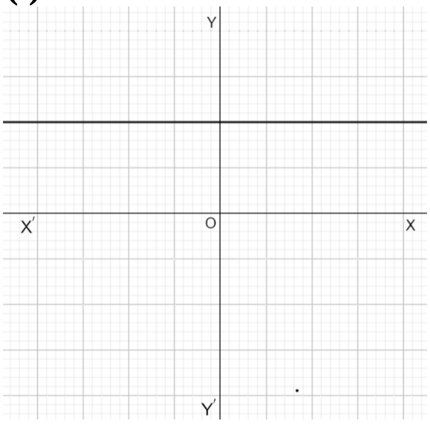
polynomials

2 Chapter

Exercise 2.1

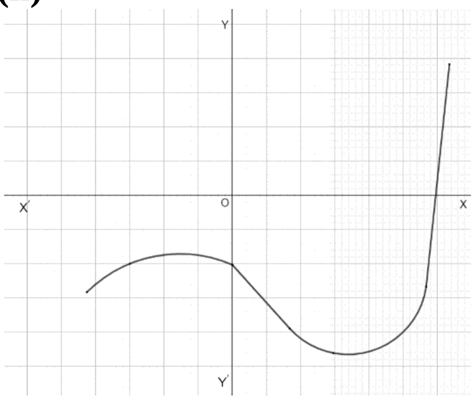
1. The graphs of $y = p(x)$ are given in following figure, for some Polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

(i)



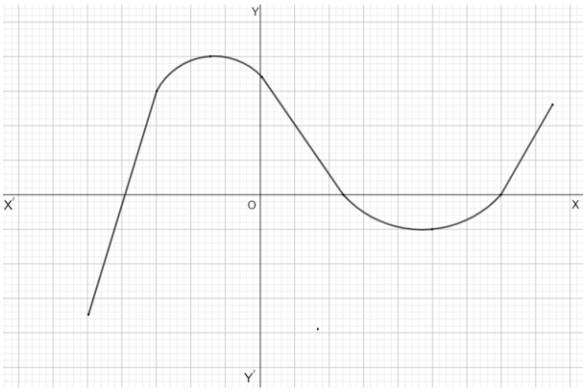
Ans: The graph does not intersect the x-axis at any point. Therefore, it does not have any zeroes.

(ii)



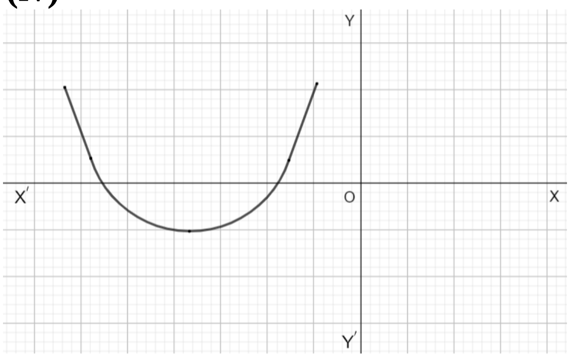
Ans: The graph intersects at the x-axis at only 1point. Therefore, the number of zeroes is 1.

(iii)



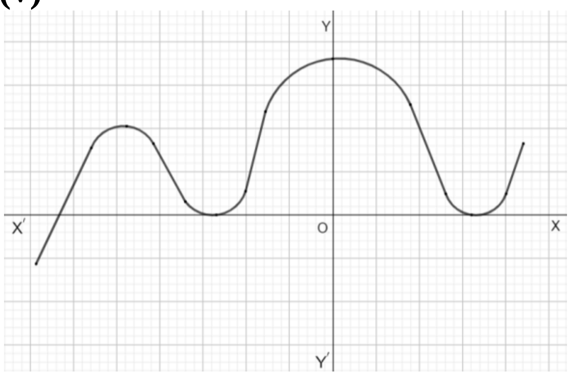
Ans: The graph intersects at the x-axis at 3 points. Therefore, the number of zeroes is 3.

(iv)



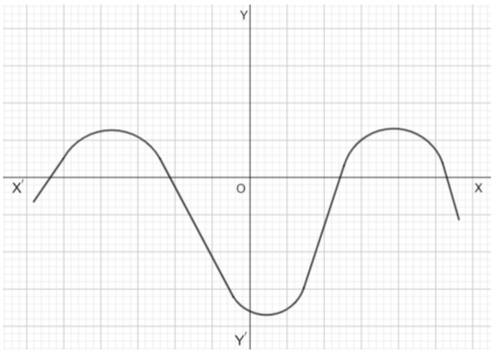
Ans: The graph intersects at the x-axis at 2 points. Therefore, the number of zeroes is 2.

(v)



Ans: The graph intersects at the x-axis at 4 points. Therefore, the number of zeroes is 4.

(vi)



Ans: The graph intersects at the x-axis at 3 points. Therefore, the number of zeroes is 3.

Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients. $x^2 - 2x - 8$

Given: $x^2 - 2x - 8$.

Now factorize the given polynomial to get the roots.

$\Rightarrow (x-4)(x+2)$

Ans: The value of $x^2 - 2x - 8$ is zero.

when $x - 4 = 0$ or $x + 2 = 0$. i.e., $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

Now, Sum of zeroes $= 4 - 2 = 2 = -\frac{2}{1} = -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$

$$\therefore \text{Sum of zeroes} = -\frac{\text{Coefficient of x}}{\text{Coefficient of } x^2}$$

Product of zeroes $= 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\therefore \text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

(i) $4s^2 - 4s + 1$

Ans: Given: $4s^2 - 4s + 1$

Now factorize the given polynomial to get the roots.

$\Rightarrow (2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero.

when $2s - 1 = 0$, $2s - 1 = 0$. i.e., $s = \frac{1}{2}$ and $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2-4s+1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Now, Sum of zeroes $=\frac{1}{2}+\frac{1}{2}=1=\frac{(-4)}{4}=\frac{-(\text{Coefficient of s})}{\text{Coefficient of s}^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of s})}{\text{Coefficient of s}^2}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of s}^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of s}^2}$.

(ii) $6x^2-3-7x$

Ans: Given: $6x^2-3-7x$

$\Rightarrow 6x^2-7x-3$

Now factorize the given polynomial to get the roots.

$\Rightarrow (3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero.

when $3x+1=0$ or $2x-3=0$. i.e., $x=\frac{-1}{3}$ or $x=\frac{3}{2}$.

Therefore, the zeroes of $6x^2-3-7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Now, Sum of zeroes $=\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of x})}{\text{Coefficient of x}^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of x})}{\text{Coefficient of x}^2}$

Product of zeroes $=\frac{-1}{3}\times\frac{3}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of x}^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of x}^2}$

(iii) $4u^2+8u$

Ans: Given: $4u^2+8u$

$\Rightarrow 4u^2+8u+0$

$\Rightarrow 4u(u+2)$

The value of $4u^2+8u$ is zero.

when $4u=0$ or $u+2=0$. i.e., $u=0$ or $u=-2$

Therefore, the zeroes of $4u^2+8u$ are 0 and -2.

Now, Sum of zeroes $=0+(-2)=-2=\frac{-8}{4}=\frac{-(\text{Coefficient of u})}{\text{Coefficient of } u^2}$

$\therefore$ Sum of zeroes$=\frac{-(\text{Coefficient of u})}{\text{Coefficient of } u^2}$

Product of zeroes $=0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

$\therefore$ Product of zeroes$=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(iv) t^2-15

Ans: Given: t^2-15

$\Rightarrow t^2-0t-15$

Now factorize the given polynomial to get the roots.

$\Rightarrow (t-\sqrt{15})(t+\sqrt{15})$

The value of t^2-15 is zero.

when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., $t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of t^2-15 are $\sqrt{15}$ and $-\sqrt{15}$.

Now, Sum of zeroes $=\sqrt{15}+(-\sqrt{15})=0=\frac{-0}{1}=\frac{-(\text{Coefficient of t})}{\text{Coefficient of } t^2}$

$\therefore$ Sum of zeroes $=\frac{-(\text{Coefficient of t})}{\text{Coefficient of } t^2}$

Product of zeroes $=\left(\sqrt{15}\right)\times\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } t^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of } t^2}$.

(v) $3x^2-x-4$

Ans: Given: $3x^2-x-4$

Now factorize the given polynomial to get the roots.

$\Rightarrow \left(3x-4\right)(x+1)$

The value of $3x^2-x-4$ is zero.

when $3x-4=0$ or $x+1=0$, i.e., $x=\frac{4}{3}$ or $x=-1$

Therefore, the zeroes of $3x^2-x-4$ are $\frac{4}{3}$ and -1.

Now, Sum of zeroes $=\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of x})}{\text{Coefficient of }x^2}$

$\therefore$ Sum of zeroes$=\frac{-(\text{Coefficient of x})}{\text{Coefficient of }x^2}$

Product of zeroes $=\frac{4}{3}\times(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of }x^2}$

$\therefore$ Product of zeroes $=\frac{\text{Constant term}}{\text{Coefficient of }x^2}$.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$

Ans:Given: $\frac{1}{4}$,-1

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=\frac{1}{4}$

$\alpha\beta=-1$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-\frac{1}{4}x-1$

$\Rightarrow 4x^2-x-4$

Therefore, the quadratic polynomial is $4x^2-x-4$.

(ii) $\sqrt{2},\frac{1}{3}$

Ans: Given: $\sqrt{2},\frac{1}{3}$

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=\sqrt{2}$

$\alpha\beta=\frac{1}{3}$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-\sqrt{2}x+\frac{1}{3}$

$\Rightarrow 3x^2-3\sqrt{2}x+1$

Therefore, the quadratic polynomial is $3x^2-3\sqrt{2}x+1$.

(iii) $\mathbf{0,\sqrt{5}}$

[here, root is missing]

Ans: Given: $0,\sqrt{5}$

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=0$

$\alpha\beta=\sqrt{5}$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-0x+\sqrt{5}$

$\Rightarrow x^2+\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1,1

Ans: Given: 1,1

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=1$

$\alpha\beta=1$

Hence, the required polynomial is $x^2-(\alpha+\beta)x+\alpha\beta$.

$\Rightarrow x^2-1x+1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $\mathbf{-\frac{1}{4},\frac{1}{4}}$

Ans: Given: $-\frac{1}{4},\frac{1}{4}$

Let the zeroes of polynomial be α and β.

Then,

$\alpha+\beta=-\frac{1}{4}$

$\alpha\beta = \frac{1}{4}$

Hence, the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\Rightarrow x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$\Rightarrow 4x^2 + x + 1$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4,1

Ans: Given: 4,1

Let the zeroes of polynomial be α and β.

Then,

$\alpha + \beta = 4$

$\alpha\beta = 1$

Hence, the required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$.

$\Rightarrow x^2 - 4x + 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.