real numbers

1 Chapter

Exercise 1.1

1. Express each number as product of its prime factors:

(i) 140

Ans: We know that the procedure of writing a number as the product of prime numbers is known as the prime factorization. Prime numbers that can be multiplied to obtain the original number are known as prime factors.

$\Rightarrow 140 = 2\times 2\times 5\times 7$

$\therefore 140 = 2^2 \times 5\times 7$

Therefore, the prime factors of 140 are $2,5,7$.

(ii) 156

Ans: We know that the procedure of writing a number as the product of prime numbers is known as the prime factorization. Prime numbers that can be multiplied to obtain the original number are known as prime factors.

$\Rightarrow 156 = 2 \times 2 \times 3 \times 13$

$\therefore 156 = 2^2 \times 3 \times 13$

Therefore, the prime factors of 156 are $2, 3, 13$.

(iii) 3825

Ans: We know that the procedure of writing a number as the product of prime numbers is known as the prime factorization. Prime numbers that can be multiplied to obtain the original number are known as prime factors.

$\Rightarrow 3825 = 3 \times 3 \times 5 \times 5 \times 17$

$\therefore 3825 = 3^2 \times 5^2 \times 17$

Therefore, the prime factors of 3825 are $3, 5, 17$.

(iv) 5005

Ans: We know that the procedure of writing a number as the product of prime numbers is known as the prime factorization. Prime numbers that can be multiplied to obtain the original number are known as prime factors.

$\Rightarrow 5005 = 5 \times 7 \times 11 \times 13$

$\therefore 5005 = 5 \times 7 \times 11 \times 13$

Therefore, the prime factors of 5005 are $5, 7, 11, 13$.

(v) 7429

Ans: We know that the procedure of writing a number as the product of prime numbers is known as the prime factorization. Prime numbers that can be multiplied to obtain the original number are known as prime factors.

$\Rightarrow 7429 = 17 \times 19 \times 23$

$\therefore 7429 = 17 \times 19 \times 23$

Therefore, the prime factors of 7429 are $17, 19, 23$.

2. Find the LCM and HCF of the following pairs of integers and verify that $\mathbf{LCM \times HCF = Product\ of\ two\ numbers}$.

(i) 26 and 91

Ans: First we write the prime factors of 26 and 91. We get

$\Rightarrow 26 = 2 \times 13$ and

$\Rightarrow 91 = 7 \times 13$

Now, we know that HCF is the highest factor, among the common factors of two numbers.

Therefore, the HCF of 26 and 91 is 13.
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of 26 and 91 will be
$\Rightarrow 2\times7\times13=182$
Therefore, the LCM of 26 and 91 is 182.
Now, the product of two numbers is
$\Rightarrow 26\times91=2366$
Product of LCM and HCF is
$\Rightarrow 13\times182=2366$
We get $\text{LCM}\times\text{HCF}=\text{Product of two numbers}$.
The desired result has been verified.

(ii) 510 and 92
Ans: First we write the prime factors of 510 and 92. We get
$\Rightarrow 510=2\times3\times5\times17$ and
$\Rightarrow 92=2\times2\times23$
Now, we know that HCF is the highest factor, among the common factors of two numbers.
Therefore, the HCF of 510 and 92 is 2.
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of 510 and 92 will be
$\Rightarrow 2\times2\times3\times5\times17\times23=23460$
Therefore, the LCM of 510 and 92 is 23460.
Now, the product of two numbers is
$\Rightarrow 510\times92=46920$
Product of LCM and HCF is
$\Rightarrow 2\times23460=46920$
We get $\text{LCM}\times\text{HCF}=\text{Product of two numbers}$.
The desired result has been verified.

(iii) 336 and 54
Ans: First we write the prime factors of 336 and 54. We get
$\Rightarrow 336=2\times2\times2\times2\times3\times7$ and
$\Rightarrow 54=2\times3\times3\times3$
Now, we know that HCF is the highest factor, among the common factors of two

numbers.
Therefore, the HCF of 336 and 54 is $2\times 3=6$.
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of 336 and 54 will be
$\Rightarrow 2\times 2\times 2\times 2\times 3\times 3\times 3\times 7=3024$
Therefore, the LCM of 336 and 54 is 3024.
Now, the product of two numbers is
$\Rightarrow 336\times 54=18144$
Product of LCM and HCF is
$\Rightarrow 6\times 3024=18144$
We get $\text{LCM}\times \text{HCF}=\text{Product of two numbers}$.
The desired result has been verified.

3. Find the LCM and HCF of the following integers by applying the prime factorization method.
(i) $12,15$ and 21
Ans: The procedure of writing a number as the product of prime numbers is known as the prime factorization.
The prime factors of $12,15$ and 21 are as follows:
$\Rightarrow 12=2\times 2\times 3$
$\Rightarrow 15=3\times 5$ and
$\Rightarrow 21=3\times 7$
Now, we know that HCF is the highest factor, among the common factors of two numbers.
Therefore, the HCF of $12,15$ and 21 is 3.
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of $12,15$ and 21 will be
$\Rightarrow 2\times 2\times 3\times 5\times 7=420$
Therefore, the LCM of $12,15$ and 21 is 420.

(ii) $17,23$ and 29
Ans: The procedure of writing a number as the product of prime numbers is known as the prime factorization.
The prime factors of $17,23$ and 29 are as follows:

$\Rightarrow 17 = 17 \times 1$
$\Rightarrow 23 = 23 \times 1$ and
$\Rightarrow 29 = 29 \times 1$
Now, we know that HCF is the highest factor, among the common factors of two numbers.
Therefore, the HCF of $17, 23$ and 29 is 1.
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of $17, 23$ and 29 will be
$\Rightarrow 17 \times 23 \times 29 = 11339$
Therefore, the LCM of $17, 23$ and 29 is 11339.

(iii) $8, 9$ and 25
Ans: The procedure of writing a number as the product of prime numbers is known as the prime factorization.
The prime factors of $8, 9$ and 25 are as follows:
$\Rightarrow 8 = 2 \times 2 \times 2$
$\Rightarrow 9 = 3 \times 3$ and
$\Rightarrow 25 = 5 \times 5$
Now, we know that HCF is the highest factor, among the common factors of two numbers. as there is no common factor.
Therefore, the HCF of $8, 9$ and 25 is 1.
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of $8, 9$ and 25 will be
$\Rightarrow 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$
Therefore, the LCM of $8, 9$ and 25 is 1800.

4. Given that HCF $(306, 657) = 9$, find LCM $(306, 657)$.

Ans: We have been given the HCF of two numbers $(306, 657) = 9$.

We have to find the LCM of $(306, 657)$.

Now, we know that $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$
Substitute the values, we get
$\text{LCM} \times 9 = 306 \times 657$

$\Rightarrow \text{LCM} = \dfrac{306 \times 657}{9}$

$\therefore \text{LCM} = 22338$

Therefore, the LCM of $(306, 657) = 22338$.

5. Check whether 6^n can end with the digit 0 for any natural number n.

Ans: We have to check whether 6^n can end with the digit 0 for any natural number n.

By divisibility rule we know that if any number ends with the digit 0, it is divisible by 2 and 5.

Thus, the prime factors of 6^n is

$\Rightarrow 6^n = (2 \times 3)^n$

Now, we will observe that for any value of n, 6^n is not divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

6. Explain why $7\times11\times13+13$ and $7\times6\times5\times4\times3\times2\times1+5$ are composite numbers.

Ans: The given numbers are $7\times11\times13+13$ and $7\times6\times5\times4\times3\times2\times1+5$.

We can rewrite the given numbers as

$\Rightarrow 7\times11\times13+13 = 13\times(7\times11+1)$

$\Rightarrow 7\times11\times13+13 = 13\times(77+1)$

$\Rightarrow 7\times11\times13+13 = 13\times78$

$\Rightarrow 7\times11\times13+13 = 13\times13\times6$

And,

$\Rightarrow 7\times6\times5\times4\times3\times2\times1+5 = 5\times(7\times6\times4\times3\times2\times1+1)$

$\Rightarrow 7\times6\times5\times4\times3\times2\times1+5 = 5\times(1008+1)$

$\Rightarrow 7\times6\times5\times4\times3\times2\times1+5 = 5\times1009$

Here, we can observe that the given expressions has its factors other than 1 and the number itself.

A composite number have factors other than 1 and the number itself.

Therefore, $7\times11\times13+13$ and $7\times6\times5\times4\times3\times2\times1+5$ are composite numbers.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose

they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Ans: It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. Both are going in the same direction, they will meet again when Ravi will have completed 1 round of that circular path with respect to Sonia.
The total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for ending 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.
The prime factors of 12 and 18 are as follows:
$\Rightarrow 12 = 2 \times 2 \times 3$ and
$\Rightarrow 18 = 2 \times 3 \times 3$
Now, we know that LCM is least common multiple. To find the LCM multiplies each factor to the number of times it occurs in any number.
Then the LCM of 12 and 18 will be
$\Rightarrow 2 \times 2 \times 3 \times 3 = 36$
Therefore, Ravi and Sonia meet again at the starting point after 36 minutes.

Exercise 1.2

1. Prove that $\sqrt{5}$ is irrational.

Ans: We have to prove that $\sqrt{5}$ is irrational.
We will use contradiction method to prove it.
Let $\sqrt{5}$ is a rational number of the form $\frac{a}{b}$, where $b \neq 0$ and a and b are co-prime i.e. a and b have only 1 as a common factor.
Let $\sqrt{5} = \frac{a}{b}$
Now, squaring both sides, we get
$$\left(\sqrt{5}\right)^2 = \left(\frac{a}{b}\right)^2$$
$$\Rightarrow 5 = \frac{a^2}{b^2}$$
$$\Rightarrow a^2 = 5b^2 \quad \ldots\ldots.(1)$$
If a^2 is divisible by 5 than a is also divisible by 5.
Let $a = 5k$, where, k is any integer.

Again squaring both sides, we get

$\Rightarrow a^2 = (5k)^2$

Substitute the value in eq. (1), we get

$\Rightarrow (5k)^2 = 5b^2$

$\Rightarrow b^2 = 5k^2$ …..(2)

If b^2 is divisible by 5 than b is also divisible by 5.

From, eq. (1) and (2), we can conclude that a and b have 5 as a common factor.

This contradicts our assumption.

Therefore, we can say that $\sqrt{5}$ is irrational.

Hence proved.

2. Prove that $3+2\sqrt{5}$ is irrational.

Ans: We have to prove that $3+2\sqrt{5}$ is irrational.

We will use contradiction method to prove it.

Let $3+2\sqrt{5}$ is a rational number of the form $\frac{a}{b}$, where $b \neq 0$ and a and b are co-prime i.e. a and b have only 1 as a common factor.

Let $3+2\sqrt{5} = \frac{a}{b}$

$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$

$\Rightarrow \sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$ ……..(1)

From eq. (1) we can say that $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ is rational so $\sqrt{5}$ must be rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. Hence the assumption is false.

Therefore, we can say that $3+2\sqrt{5}$ is irrational.

Hence proved.

3. Prove that following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

Ans: We have to prove that $\frac{1}{\sqrt{2}}$ is irrational.

We will use contradiction method to prove it.

Let $\frac{1}{\sqrt{2}}$ is a rational number of the form $\frac{a}{b}$, where $b \neq 0$ and a and b are co-prime i.e. a and b have only 1 as a common factor.

Let $\frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$ ………..(1)

From eq. (1) we can say that $\frac{b}{a}$ is rational so $\sqrt{2}$ must be rational.

But this contradicts the fact that $\sqrt{2}$ is irrational. Hence the assumption is false.

Therefore, we can say that $\frac{1}{\sqrt{2}}$ is irrational.

Hence proved.

(ii) $7\sqrt{5}$

Ans: We have to prove that $7\sqrt{5}$ is irrational.

We will use contradiction method to prove it.

Let $7\sqrt{5}$ is a rational number of the form $\frac{a}{b}$, where $b \neq 0$ and a and b are co-prime i.e. a and b have only 1 as a common factor.

Let $7\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ………..(1)

From eq. (1) we can say that $\frac{a}{7b}$ is rational so $\sqrt{5}$ must be rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. Hence the assumption is false.

Therefore, we can say that $7\sqrt{5}$ is irrational.

Hence proved.

(iii) $6+\sqrt{2}$

Ans: We have to prove that $6+\sqrt{2}$ is irrational.
We will use contradiction method to prove it.

Let $6+\sqrt{2}$ is a rational number of the form $\frac{a}{b}$, where $b \neq 0$ and a and b are co-prime i.e. a and b have only 1 as a common factor.

Let $6+\sqrt{2}=\frac{a}{b}$

$\Rightarrow \sqrt{2}=\frac{a}{b}-6$ ....(1)

From eq. (1) we can say that $\frac{a}{b}-6$ is rational so $\sqrt{2}$ must be rational.

But this contradicts the fact that $\sqrt{2}$ is irrational. Hence the assumption is false.
Therefore, we can say that $6+\sqrt{2}$ is irrational.
Hence proved.

1.3 Revisiting Irrational Numbers

In Class IX, you were introduced to irrational numbers and many of their properties.
You studied about their existence and how the rationals and the irrationals together made up the real numbers.
You even studied how to locate irrationals on the number line. However, we did not prove that they were irrationals.
In this section, we will prove that

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and, in general, $\sqrt{p}$ is irrational, where p is a prime.

One of the theorems, we use in our proof, is the Fundamental Theorem of Arithmetic.
Recall, a number 's' is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Some examples of irrational numbers, with which you are already familiar, are :