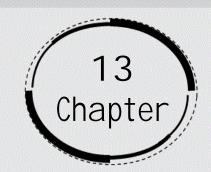
statistics



Exercise 13.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number		2-4	4-6	6-8	8-10	10-12	12-14
of plants							
Number	1	2	1	5	6	2	3
of							
houses							

Which method did you use for finding the mean, and why?

Ans: The number of houses denoted by x_i .

The mean can be found as given below:

$$\overline{X} = \frac{\sum f_i x_i}{\sum f_i}$$

Class mark (x_i) for each interval is calculated as follows:

$$xi = \frac{\text{Upper class limit+Lower class limit}}{2}$$

 x_i and $f_i x_i$ can be calculated as follows:

Number of plants	Number of houses	X _i	$f_i x_i$
	$ \mathbf{f_i} $		
0-2	1	1	1×1=1
2-4	2	3	$2\times3=6$
4-6	1	5	$1\times5=5$
6-8	5	7	$5\times7=35$
8-10	6	9	$6 \times 9 = 54$
10-12	2	11	$2 \times 11 = 22$

12-14	3	13	$3 \times 13 = 39$
Total	20		162

From the table, it can be observed that

$$\sum f_i = 20$$

$$\sum f_i x_i = 162$$

Substituting the value of $f_i x_i$ and f_i in the formula of mean we get:

Mean number of plants per house (\overline{X}) :

$$\overline{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\overline{X} = \frac{162}{20} = 8.1$$

Therefore, mean number of plants per house is 8.1.

In this case, we will use the direct method because the value of $\,x_{_i}\,$ and $\,f_{_i}\,$.

2. Consider the following distribution of daily wages of 50 worker of a factory.

Daily wages (in Rs)	100 -120	120 - 140	140 - 160	160 - 180	180 - 200
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Ans: Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Take the assured mean (a) of the given data

$$a = 150$$

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper \ class \ limit+Lower \ class \ limit}{2}$$

Class size (h) of this data is:

$$h = 120 - 100$$

$$h = 20$$

 d_i , u_i , and f_iu_i can be evaluated as follows:

Daily wages	Number	X _i	$d_i = x_i - 150$	$u_i = \frac{d_i}{d_i}$	$f_i u_i$
(In Rs)	of			20	
	Workers				
	(f_i)				
100 -120	12	110	-40	-2	-24
120 - 140	14	130	-20	-1	-14
140 - 160	8	150	0	0	0
160 - 180	6	170	20	1	6
180 - 200	10	190	40	2	20
Total	50				-12

From the table, it can be observed that

$$\sum f_i = 50$$

and

$$\sum f_i u_i = -12$$

Substituting the value of $u_{\scriptscriptstyle i}$, and $f_{\scriptscriptstyle i}u_{\scriptscriptstyle i}$ in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

$$\overline{X} = 150 + \left(\frac{-12}{50}\right) 20$$

$$\overline{X} = 150 - \frac{24}{5}$$

$$\overline{X} = 150 - 4.8$$

$$\overline{X} = 145.2$$

Hence, the mean daily wage of the workers of the factory is Rs 145.20.

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency ${\bf f}$.

Daily pocket allowance (in Rs)		13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 - 25
Number of workers	7	6	9	13	f	5	4

Ans: Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 18

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper class limit+Lower class limit}{2}$$

It is given that, mean pocket allowance, $\overline{X} = Rs 18$

Class size (h) of this data is:

$$h = 13 - 11$$

$$h = 2$$

 $\boldsymbol{d}_{_{i}},\boldsymbol{u}_{_{i}},$ and $\boldsymbol{f}_{_{i}}\boldsymbol{u}_{_{i}}$ can be evaluated as follows:

Daily	Number	Class mark	$d_i = x_i - 18$	$f_i d_i$
pocket	of	X_{i}		
allowance	workers (f _i)			
(in Rs)	-			
11 –13	7	12	-6	-42
13 - 15	6	14	-4	-24
15 - 17	9	16	-2	-18
17 - 19	13	18	0	0
19 - 21	f	20	2	2f
21 - 23	5	22	4	20
23 - 25	4	24	6	24
Total	$\sum f_i = 44 + f$			2f -40

From the table, it can be observed that

$$\sum f_{_i} = 44 + f$$

$$\sum f_i u_i = 2f - 40$$

Substituting the value of d_i , u_i , and f_iu_i in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i d_i}{\sum f_i}\right) h$$

$$18 = 18 + \left(\frac{2f - 40}{44 + f}\right) 2$$

$$0 = \left(\frac{2f - 40}{44 + f}\right)$$

$$2f - 40 = 0$$

$$f = 20$$

Hence, the value of frequency f_i is 20.

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute		68 - 71	71 - 74	74 - 77	77 - 80	80 - 83	83 - 86
Number of women	2	4	3	8	7	4	2

Ans: Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 75.5

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper \ class \ limit+Lower \ class \ limit}{2}$$

Class size (h) of this data is:

$$h = 68 - 65$$

$$h = 3$$

 d_i , u_i , and $f_i u_i$ can be evaluated as follows:

Number of heart beats per minute	Number of women f_i	X _i	$d_i = x_i - 75.5$	$u_i = \frac{d_i}{3}$	$f_i u_i$
65 - 68	2	66.5	- 9	-3	-6
68 - 71	4	69.5	-6	-2	-8
71 - 74	3	72.5	-3	-1	-3
74 - 77	8	75.5	0	0	0
77 -80	7	78.5	3	1	7
80 - 83	4	81.5	6	2	8
83 - 86	2	84.5	3	3	6
Total	30				4

From the table, it can be observed that

$$\sum f_i = 30$$

$$\sum f_{i}u_{i}=4$$

Substituting the value of $u_{_{i}}$, and $f_{_{i}}u_{_{i}}$ in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

$$\overline{X} = 75.5 + \left(\frac{4}{30}\right)3$$

$$\overline{X} = 75.5 + 0.4$$

$$\overline{X} = 75.9$$

Therefore, mean heart beats per minute for these women are 75.9 beats per minute.

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes	50-52	53-55	56-58	59-61	62-64
Number of Boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Ans:

Number of mangoes	Number of Boxes f _i
50-52	15
53-55	110
56-58	135
59-61	115
62-64	25

It can be noticed that class intervals are not continuous in the given data. There is a gap of 1 between two class intervals. Therefore, we have to subtract $\frac{1}{2}$ to lower class and have to add $\frac{1}{2}$ to upper to make the class intervals continuous.

Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 57.

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper \ class \ limit+Lower \ class \ limit}{2}$$

Class size (h) of this data is:

$$h = 52.5 - 49.5$$

$$h = 3$$

 d_i , u_i , and f_iu_i can be calculated as follows:

Class interval	f_{i}	Xi	$d_i = x_i - 57$	$u_i = \frac{d_i}{3}$	f _i u _i
49.5 – 52.5	15	51	-6	-2	-30
52.5 – 55.5	110	54	-3	-1	-110
55.5-58.5	135	57	0	0	0
58.5-61.5	115	60	3	1	115
61.5 – 64.5	25	63	6	2	50
Total	400				25

It can be observed that from the above table

$$\sum f_i = 400$$

$$\sum f_i u_i = 25$$

Substituting the value of u_i , and f_iu_i in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

$$\overline{X} = 57 + \left(\frac{25}{400}\right)3$$

$$\overline{X} = 57 + \frac{3}{16}$$

$$\overline{X} = 57.1875$$

Hence, mean number of mangoes kept in a packing box is 57.1875.

In the above case, we used step deviation method as the values of f_i , d_i are large and the class interval is not continuous.

6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (In Rs)	100-150	150-200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Ans: Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 225.

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper \ class \ limit+Lower \ class \ limit}{2}$$

Class size (h) of this data is:

$$h = 150 - 100$$

$$h = 50$$

 $\boldsymbol{d}_{i}, \boldsymbol{u}_{i}, \text{and } \boldsymbol{f}_{i}\boldsymbol{u}_{i}$ can be calculated as follows:

Daily expenditure (in Rs)	f_{i}	X _i	$d_i = x_i - 225$	$u_i = \frac{d_i}{50}$	$f_i u_i$
100-150	4	125	-100	-2	-8

150-200	5	175	-6	-1	-5
200 – 250	12	225	0	0	0
250-300	2	275	50	1	2
300-350	2	325	100	2	4
Total	25				-7

It can be observed that from the above table

$$\sum f_i = 25$$

$$\sum f_i u_i = -7$$

Substituting the value of u_i , and f_iu_i in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 225 + \left(\frac{-7}{25}\right) \times 50$$

$$\overline{X} = 221$$

Hence, mean daily expenditure on food is Rs 211.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO₂ (in ppm)	Frequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 - 0.12	9
0.12-0.16	2
0.16-0.20	4
0.20 – 0.24	2

Find the mean concentration of \mathbf{SO}_2 in the air.

Ans: Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 0.14.

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper \ class \ limit+Lower \ class \ limit}{2}$$

Class size (h) of this data is:

$$h = 0.04 - 0.00$$

$$h = 0.04$$

 d_{i} , u_{i} , and $f_{i}u_{i}$ can be calculated as follows:

Concentration of SO ₂ (in ppm)	Frequency f_i	Class mark x _i	$d_i = x_i - 0.14$	$u_i = \frac{d_i}{0.04}$	$f_i u_i$
0.00 - 0.04	4	0.02	-0.12	-3	-12
0.04 - 0.08	9	0.06	-0.08	-2	-5
0.08 - 0.12	9	0.10	-0.10	-1	_9
0.12-0.16	2	0.14	0	0	0
0.16-0.20	4	0.18	0.04	1	4
0.20-0.24	2	0.22	0.08	2	4
Total					-31

It can be observed that from the above table

$$\sum f_i = 30$$

$$\sum f_i u_i = -31$$

Substituting the value of u_i , and f_iu_i in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 0.14 + \left(\frac{-31}{30}\right) \times (0.04)$$

$$\overline{X} = 0.14 - 0.04133$$

$$\overline{X} = 0.09867$$

$$\overline{X} = 0.099 \text{ ppm}$$

Hence, mean concentration of SO_2 in the air is 0.099 ppm.

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	0-6	6-10	10-14	14-20	20-28	28-38	38-40
Number of students	11	10	7	4	4	3	1

Ans:

Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

Suppose the assured mean (a) of the data is 17.

Class mark (x_i) for each interval is calculated as follows:

$$x_{i} = \frac{Upper \ class \ limit + Lower \ class \ limit}{2}$$

 $\boldsymbol{d}_{i},\,\boldsymbol{u}_{i}, \text{and } \boldsymbol{f}_{i}\boldsymbol{u}_{i}$ can be calculated as follows:

Number of Days	Number of Students f_i	X _i	$d_i = x_i - 17$	$f_i d_i$
0-6	11	3	-14	-154
6-10	10	8	-9	-90
10-14	7	12	-5	-35
14-20	4	17	0	0
20-28	4	24	7	28
28-38	3	33	16	48
38-40	1	39	22	22
Total	40			-181

It can be observed that from the above table

$$\sum f_i = 40$$

$$\sum f_i u_i = -181$$

Substituting the value of $\,u_{_{i}}$, and $\,f_{_{i}}u_{_{i}}$ in the formula of mean we get:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

$$\overline{X} = 17 + \left(\frac{-181}{40}\right)$$

$$\overline{X} = 17 - 4.525$$

$$\overline{X} = 12.475$$

Hence, the mean number of days is 12.48 days for which a student was absent.

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in%)	45 – 55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

Ans: Let the class size of the data be h.

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

Suppose the assured mean (a) of the data is 70.

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{\text{Upper class limit+Lower class limit}}{2}$$

Class size (h) of this data is:

$$h = 55 - 45$$

$$h = 10$$

 $\boldsymbol{d}_{i},\,\boldsymbol{u}_{i}, \text{and } \boldsymbol{f}_{i}\boldsymbol{u}_{i}$ can be calculated as follows:

Literacy rate (in%)	Number of cities f_i	Xi	$d_i = x_i - 70$	$u_i = \frac{d_i}{10}$	$f_i u_i$
45-55	3	50	-20	-2	-6
55-65	10	60	-10	-1	-10

65 – 75	11	70	0	0	0
75 – 85	8	80	10	1	8
85-95	3	90	20	2	6
Total	35				-2

It can be observed that from the above table

$$\sum f_i = 35$$

$$\sum f_i u_i = -2$$

Substituting the value of u_i , and f_iu_i in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 70 + \left(\frac{-2}{35}\right) \times 10$$

$$\overline{X} = 70 - \frac{20}{35}$$

$$\overline{X} = 69.43$$

Therefore, mean literacy rate of cities is 69.43%.

Exercise 13.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5-15	15-25	25-35	35-45	45 – 55	55 – 65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Ans: For mean

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

Suppose the assured mean (a) of the data is 30.

Class mark (x_i) for each interval is calculated as follows:

$$x_i = \frac{Upper \ class \ limit+Lower \ class \ limit}{2}$$

 d_i , u_i , and f_iu_i can be evaluated as follows:

Age (in years)	Number of patients f_i	Class mark x _i	$d_i = x_i - 30$	$f_i d_i$
5-15	6	10	-20	-120
15-25	11	20	-10	-110
25-35	21	30	0	0
35-45	23	40	10	230
45-55	14	50	20	280
55-65	5	60	30	150
Total	80			430

It can be observed that from the above table

$$\sum f_i = 80$$

$$\sum f_i d_i = 430$$

Substituting the value of u_i , and f_iu_i in the formula of mean we get:

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

$$\overline{X} = 30 + \left(\frac{430}{80}\right)$$

$$\overline{X} = 30 + 5.375$$

$$\overline{X} = 35.38$$

Hence, the mean of this data is 35.38. It demonstrates that the average age of a patient admitted to hospital was 35.38 years.

For mode

Mode can be calculated as

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

It can be noticed that the maximum class frequency is 23 belonging to class interval 35 - 45.

Modal class = 35 - 45

The values of unknowns is given as below as per given data:

$$1 = 35$$

$$f_1 = 23$$

$$h = 15 - 5 = 10$$

$$f_0 = 21$$

$$f_2 = 14$$

Substituting these values in the formula of mode we get:

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$M = 35 + \left(\frac{23 - 21}{2(23) - 21 - 14}\right) \times 10$$

$$M = 35 + \left[\frac{2}{46 - 35}\right] \times 10$$

$$M = 35 + \frac{20}{11}$$

$$M=35+1.81$$

$$M = 36.8$$

Hence, the Mode of the data is 36.8. It demonstrates that the age of maximum number of patients admitted in hospital was 36.8 years.

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes	0-20	20-40	40-60	60-80	80-100	100-120
(in hours)						
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Ans: Mode can be calculated as

$$\mathbf{M} = 1 + \left(\frac{\mathbf{f}_1 - \mathbf{f}_0}{2\mathbf{f}_1 - \mathbf{f}_0 - \mathbf{f}_2}\right) \times \mathbf{h}$$

Where

1 = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

From the data given above, it can be noticed that the maximum class

frequency is 61 belongs to class interval 60 - 80.

Therefore, Modal class = 60 - 80

The values of unknowns are given as below as per given data:

$$1 = 60$$

$$f_1 = 61$$

$$h = 20$$

$$f_0 = 52$$

$$f_2 = 38$$

Substituting these values in the formula of mode we get:

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$M = 60 + \left(\frac{61 - 52}{2(61) - 52 - 38}\right) \times 20$$

$$M = 60 + \left[\frac{9}{122 - 90} \right] \times 20$$

$$M = 360 + \left(\frac{9 \times 20}{32}\right)$$

$$M = 60 + \frac{90}{16} = 60 + 5.625$$
$$M = 65.625$$

Hence, modal lifetime of electrical components is 65.625 hours.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in Rs)	Number of families
1000-1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

Ans: For mode

Mode can be calculated as

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

It can be observed from the given data that the maximum class frequency is $40\,$,

Belongs to 1500 - 2000 intervals.

Therefore, modal class = 1500 - 2000

The values of unknowns are given as below as per given data:

$$1 = 1500$$

$$f_1 = 40$$

$$f_0 = 24$$

$$f_2 = 33$$

$$h = 500$$

Substituting these values in the formula of mode we get:

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$M = 1500 + \left(\frac{40 - 24}{2(40) - 24 - 33}\right) \times 500$$

$$M = 1500 + \left[\frac{16}{80 - 57}\right] \times 500$$

$$M = 1500 + \frac{8000}{23}$$

$$M = 1500 + 347.826$$

$$M = 1847.826$$

$$M \approx 1847.83$$

Therefore, modal monthly expenditure was Rs 1847.83.

For mean

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 2750.

Class mark (x_i) for each interval is calculated as follows:

Class mark =
$$\frac{\text{Upper class limit+Lower class limit}}{2}$$

Class size (h) of this data is:

$$h = 1500 - 1000$$

$$h = 500$$

 $\boldsymbol{d}_{i}\text{, }\boldsymbol{u}_{i}\text{,and }\boldsymbol{f}_{i}\boldsymbol{u}_{i}\text{ can be evaluated as follows:}$

Expenditure (in Rs)	Number of families f_i	X _i	$d_i = x_i - 2750$	$u_i = \frac{d_i}{500}$	$f_i u_i$
1000-1500	24	1250		-3	-72
1500 – 2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750	0	0	0
3000-3500	30	3250	500	1	30
3500 – 4000	22	3750	1000	2	44
4000 – 4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
Total	200				-35

It can be observed from the above table

$$\sum f_{\rm i} = 200$$

$$\sum f_i u_i = -35$$

Substituting \boldsymbol{u}_{i} , and $\boldsymbol{f}_{i}\boldsymbol{u}_{i}$ in the formula of mean

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 2750 + \left(\frac{-35}{200}\right) \times (500)$$

$$\overline{X} = 2750 - 87.5$$

$$\overline{X} = 2662.5$$

Hence, mean monthly expenditure is Rs 2662.5.

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two Measures.

Number of students per teacher	Number of states/U.T
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

Ans: For mode

Mode can be calculated as

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

It can be observed from the given data that the maximum class frequency is 10 which belongs to class interval 30 - 35.

Therefore, modal class = 30 - 35

$$h = 5$$

$$1 = 30$$

$$f_1 = 10$$

$$f_0 = 9$$

$$f_2 = 3$$

Substituting these values in the formula of mode we get:

$$\mathbf{M} = \mathbf{l} + \left(\frac{\mathbf{f}_1 - \mathbf{f}_0}{2\mathbf{f}_1 - \mathbf{f}_0 - \mathbf{f}_2}\right) \times \mathbf{h}$$

$$M = 30 + \left(\frac{10 - 9}{2(10) - 9 - 3}\right) \times 5$$

$$M = 30 + \left(\frac{1}{20 - 12}\right) \times 5$$

$$M = 30.6$$

Hence, the mode of the given data is 30.6. It demonstrates that most of the states/U.T have a teacher-student ratio of 30.6.

For mean

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 32.5.

Class mark (x_i) for each interval is calculated as follows:

Class mark =
$$\frac{\text{Upper class limit+Lower class limit}}{2}$$

Class size (h) of this data is:

$$h = 20 - 15$$

$$h = 5$$

 d_i , u_i , and f_iu_i can be calculated as follows:

Number of students per teacher	Number of states /U.T	X _i	$D_i = x_i - 32.5$	$u_i = \frac{d_i}{5}$	$f_i u_i$
15-20	3	17.5	-15	-3	-9
20-25	8	22.5	-10	-2	-16
25-30	9	27.5	-5	-1	-9
30-35	10	32.5	0	0	0
35-40	3	37.5	5	1	3
40-45	0	42.5	10	2	0
45-50	0	47.5	15	3	0
50-55	2	52.5	20	4	8
Total	35				-23

It can be observed from the above table

$$\sum f_i = 35$$

$$\sum f_i u_i = -23$$

Substituting \boldsymbol{u}_{i} , and $\boldsymbol{f}_{i}\boldsymbol{u}_{i}$ in the formula of mean

The required mean

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 32.5 + \left(\frac{-23}{35}\right) \times 5$$

$$\overline{X} = 32.5 - \frac{23}{7}$$

$$\overline{X} = 29.22$$

Hence, mean of the data is 29.2.

It demonstrates that average teacher-student ratio was 29.2.

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen
3000-4000	4
4000 – 5000	18
5000 – 6000	9
6000 – 7000	7
7000-8000	6
8000 – 9000	3
9000-10000	1
10000-11000	1

Find the mode of the data.

Ans:

Mode can be calculated as

$$\mathbf{M} = 1 + \left(\frac{\mathbf{f}_1 - \mathbf{f}_0}{2\mathbf{f}_1 - \mathbf{f}_0 - \mathbf{f}_2}\right) \times \mathbf{h}$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

From the given data, it can be observed that the maximum class frequency is 18 Belongs to class interval 4000-5000.

Therefore, modal class = 4000 - 5000

$$1 = 4000$$

$$f_1 = 18$$

$$f_0 = 4$$

$$f_2 = 9$$

$$h = 1000$$

Substituting these values in the formula of mode we get:

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$M = 4000 + \left(\frac{18 - 4}{2(18) - 4 - 9}\right) \times 1000$$

$$M = 4000 + \left(\frac{14000}{23}\right)$$

$$M = 4608.695$$

Hence, mode of the given data is 4608.695 runs.

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	0 – 10	10 – 20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

Ans:

Mode can be calculated as

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

From the given data, it can be observed that the maximum class frequency is 20,

Belonging to 40-50 class intervals.

Therefore, modal class = 40-50

1 = 40

 $f_1 = 20$

 $f_0 = 12$

 $f_2 = 11$

h = 10

Substituting these values in the formula of mode we get:

$$\mathbf{M} = 1 + \left(\frac{\mathbf{f}_1 - \mathbf{f}_0}{2\mathbf{f}_1 - \mathbf{f}_0 - \mathbf{f}_2}\right) \times \mathbf{h}$$

$$M = 40 + \left(\frac{20 - 12}{2(20) - 12 - 11}\right) \times 10$$

$$M = 40 + \left(\frac{80}{40 - 23}\right)$$

$$M = 44.7$$

Hence, mode of this data is 44.7 cars.

Exercise 13.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units)	Number of consumers
65 – 85	4
85-105	5
105-125	13
125-145	20
145 – 165	14
165-185	8
185 – 205	4

Ans: The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 32.5.

Class mark (x_i) for each interval is calculated as follows:

To find the class mark for each interval, the following relation is used.

Class mark
$$x_i = \frac{Upper class limit+Lower class limit}{2}$$

Class size (h) of this data is:

$$h = 85 - 65$$

$$h = 20$$

 \boldsymbol{d}_{i} , \boldsymbol{u}_{i} , and $\boldsymbol{f}_{i}\boldsymbol{u}_{i}$ can be calculated according to step deviation method as follows:

Monthly consumption (in units)	Number of Consumers f_i	Class mark x _i	$d_i = x_i - 135$	$u_i = \frac{d_i}{20}$	$f_i u_i$
65 – 85	4	75	-60	-3	-12
85-105	5	95	-40	-2	-10
105 – 125	13	115	-20	-1	-13
125 – 145	20	135	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	.2.	16
185 – 205	4	195	60	3	12
Total	68				7

It can be observed from the above table

$$\sum f_i = 68$$

$$\sum f_i u_i = 7$$

Class size
$$(h) = 20$$

Substituting \boldsymbol{u}_{i} , and $\boldsymbol{f}_{i}\boldsymbol{u}_{i}$ in the formula of mean

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 135 + \left(\frac{7}{68}\right) \times (20)$$

$$\overline{X} = 135 + \frac{140}{68}$$

$$\overline{X} = 137.058$$

Hence, the mean of given data is 137.058.

For mode

Mode can be calculated as

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

From the table, it can be noticed that the maximum class frequency is 20,

Belongs to class interval 125 - 145.

Modal class = 125 - 145

1 = 125

Class size h = 20

$$f_1 = 20$$

$$f_0 = 13$$

$$f_2 = 14$$

Substituting these values in the formula of mode we get:

$$M = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$M = 125 + \left(\frac{20 - 13}{2(20) - 13 - 14}\right) \times 20$$

$$M = 125 + \frac{7}{13} \times 20$$

$$M = 135.76$$

Hence, the value of mode is 135.76

For median

We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

l = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

To find the median of the given data, cumulative frequency is calculated as follows.

Monthly consumption (in units)	Number of Consumers	Cumulative frequency
65-85	4	4
85-105	5	4+5=9
105 – 125	13	9+13=22
125 – 145	20	22 + 20 = 42
145 – 165	14	42+14=56

165 – 185	8	56+8=64
185 – 205	4	64 + 4 = 68

It can be observed from the given table

$$n = 68$$

$$\frac{n}{2} = 34$$

Cumulative frequency just greater than $\frac{n}{2}$ is 42, belonging to

Interval 125 - 145.

Therefore, median class = 125 - 145.

$$1 = 125$$

$$h = 20$$

$$f = 20$$

$$cf = 22$$

Substituting these values in the formula of median we get:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$m = 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$$m = 125 + 12$$

$$m = 137$$

Hence, median, mode, mean of the given data is 137, 135.76 and 137.05 respectively.

Mean, mode and median are almost equal in this case.

2. If the median of the distribution is given below is 28.5, find the values of \boldsymbol{x} and \boldsymbol{y} .

Class interval	Frequency
0-10	5
10-20	X
20-30	20
30-40	15
40-50	Y
50-60	5
Total	60

Ans: We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

l = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

The cumulative frequency for the given data is calculated as follows.

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	X	5 + x
20-30	20	25+x
30-40	15	40+x
40-50	Y	40 + x + y
50-60	5	45 + x + y
Total(n)	60	

It is given that the value of n is 60

From the table, it can be noticed that the cumulative frequency of last entry is 45 + x + y

Equating 45 + x + y and n, we get:

$$45 + x + y = 60$$

$$x + y = 15 \dots (1)$$

It is given that.

Median of the data is given 28.5 which lies in interval 20 - 30.

Therefore, median class = 20 - 30.

$$1 = 20$$

$$cf = 5 + x$$

$$f = 20$$

$$h = 10$$

Substituting these values in the formula of median we get:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$28.5 = 20 + \left(\frac{\frac{60}{2} - (5 + x)}{20}\right) \times 10$$

$$8.5 = \left(\frac{25 - x}{2}\right)$$

$$17 = 25 - x$$

$$x = 8$$

Substituting x = 8 in equation (1), we get:

$$8 + y = 15$$

$$y = 7$$

Hence, the values of x and y are 8 and 7 respectively.

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Ans: We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

l = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

In this case, class width is not the constant. We are not required to adjust the frequencies according to class intervals. The given frequency table is of less than type represented with upper class limits. The policies were given only to persons with age 18 years onwards but less than 60 years. Therefore, class intervals with their respective cumulative frequency can be defined as below.

Age (in years)	Number of policy holders	Cumulative frequency
	(f_i)	(cf)
18-20	2	2
20-25	6-2=4	6
25 - 30	24-6=18	24
30-35	45 - 24 = 21	45
35-40	78 - 45 = 33	78
40-45	89 - 78 = 11	89
45-50	92 - 89 = 3	92
50-55	98-92=6	98
55-60	100-98=2	100
Total(n)		

From the table, it can be observed that n = 100.

Thus,

$$\frac{n}{2} = 50$$

Cumulative frequency (cf) just greater than $\frac{n}{2}$ is 78,

belongs interval 35 - 40.

Therefore, median class = 35 - 40.

$$1 = 35$$

$$h = 5$$

$$f = 33$$

$$cf = 45$$

Substituting these values in the formula of median we get:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$m = 35 + \left(\frac{50 - 45}{33}\right) \times 5$$

$$m = 35 + \left(\frac{25}{33}\right)$$

$$m = 35.76$$

Hence, median age of people who get the policies is 35.76 years.

4. The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm)	Number of leaves f _i
118-126	3
127 – 135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

Find the median length of the leaves.

(Hint: The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5...171.5 - 180.5)

Ans: We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

1 = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

The given data does not have continuous class intervals. It can be noticed that the difference between two class intervals is 1. Therefore, we will add 0.5 in the upper class and subtract 0.5 in the lower class.

Continuous class intervals with respective cumulative frequencies can be represented as follows.

Length (in mm)	Number of leaves f _i	Cumulative
		frequency
117.5 – 126.5	3	3
126.5 – 135.5	5	3+5=8
135.5 – 144.5	9	8+9=17
144.5 – 153.5	12	17+12=29
153.5 – 162.5	5	29 + 5 = 34
162.5 – 171.5	4	34+4=38
171.5 – 180.5	2	38 + 2 = 40

It can be observed from the given table

$$n = 40$$

$$\frac{n}{2} = 20$$

From the table, it can be noticed that the cumulative frequency just greater than

$$\frac{n}{2}$$
 is 29, Belongs to interval 144.5 – 153.5.

median class = 144.5 - 153.5.

$$1 = 144.5$$

$$h = 9$$

$$f = 12$$

$$cf = 17$$

Substituting these values in the formula of median we get:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$m = 144.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

$$m = 144.5 + \left(\frac{9}{4}\right)$$

$$m = 146.75$$

Hence, median length of leaves is 146.75 mm.

5. The following table gives the distribution of the life time of 400 neon lamps:

Life time (in	Number of
hours)	lamps
1500 – 2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000 – 4500	62
4500-5000	48

Find the median life time of a lamp.

Ans: We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

1 = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

The cumulative frequencies with their respective class intervals are as follows.

Life time (in	Number of	Cumulative frequency
hours)	lamps	
1500 - 2000	14	14
2000 - 2500	56	14 + 56 = 70
2500-3000	60	70+60=130
3000-3500	86	130 + 86 = 216
3500-4000	74	216 + 74 = 290
4000 – 4500	62	290 + 62 = 352
4500 – 5000	48	352 + 48 = 400
Total(n)	400	

It can be observed from the given table

$$n = 400$$

$$\frac{n}{2} = 200$$

It can be observed that the cumulative frequency just greater than

 $\frac{n}{2}$ is 290 ,Belongs to interval 3000 - 3500 .

Median class = 3000 - 3500

$$1 = 3000$$

$$f = 86$$

$$cf = 130$$

$$h = 500$$

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$m = 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$
$$m = 3000 + \left(\frac{70 \times 500}{86}\right)$$
$$m = 3406.976$$

Hence, median life time of lamps is 3406.976 hours.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of Surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

Ans:

For median

We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

1 = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

The cumulative frequencies with their respective class intervals are as follows.

Number	Number of	Cumulative frequency
of letters	Surnames	
1-4	6	6
4-7	30	30+6=36
7-10	40	36 + 40 = 76
10-13	16	76+16=92
13-16	4	92 + 4 = 96
16-19	4	96+4=100
Total(n)	100	

It can be observed from the given table

$$n = 100$$

$$\frac{n}{2} = 50$$

It can be noticed that the cumulative frequency just greater than

$$\frac{n}{2}$$
 is 76, Belongs to interval 7 - 10.

Median class = 7 - 10

$$1 = 7$$

$$cf = 36$$

$$f = 40$$

$$h = 3$$

Substituting these values in the formula of median we get:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$m = 7 + \left(\frac{50 - 36}{40}\right) \times 3$$
$$m = 7 + \left(\frac{14 \times 3}{40}\right)$$
$$m = 8.05$$

Hence, the median number of letters in the surnames is 8.05.

For mean

The mean can be found as given below:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) h$$

Suppose the assured mean (a) of the data is 11.5.

Class mark (x_i) for each interval is calculated as follows:

Class mark
$$(x_i) = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

Class size (h) of this data is:

$$h = 4 - 1$$

$$h = 3$$

 \boldsymbol{d}_{i} , \boldsymbol{u}_{i} , and $\boldsymbol{f}_{i}\boldsymbol{u}_{i}$ can be calculated according to step deviation method as follows:

Number of letters	Number of surnames f_i	X _i	$d_i = x_i - 11.5$	$u_i = \frac{d_i}{20}$	$f_i u_i$
1-4	6	2.5	– 9	-3	-18
4-7	30	5.5	-6	-2	-60
7-10	40	8.5	-3	-1	-40
10-13	16	11.5	0	0	0

13-16	4	14.5	3	1	4
16-19	4	17.5	6	2	8
Total	100				-106

It can be observed from the above table

$$\sum f_i u_i = -106$$

$$\sum f_i = 100$$

Substituting \boldsymbol{u}_{i} , and $\boldsymbol{f}_{i}\boldsymbol{u}_{i}$ in the formula of mean

The required mean:

$$\overline{X} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\overline{X} = 11.5 + \left(\frac{-106}{100}\right) \times 3$$

$$\overline{X} = 11.5 - 3.18$$

$$\overline{X} = 8.32$$

Hence the mean of number of letters in the surnames is 8.32.

For mode

Mode can be calculated as

$$\mathbf{M} = \mathbf{l} + \left(\frac{\mathbf{f}_1 - \mathbf{f}_0}{2\mathbf{f}_1 - \mathbf{f}_0 - \mathbf{f}_2}\right) \times \mathbf{h}$$

Where

l = Lower limit of modal class

 f_1 = Frequency of modal class

 f_0 = Frequency of class preceding the modal class

 f_2 = Frequency of class succeeding the modal class

h = Class size

The data in the given table can be written as

Number of letters	Frequency (f _i)
1-4	6
4-7	30
7-10	40
10-13	16
13-16	4
16-19	4
Total(n)	100

From the table, it can be observed that the maximum class frequency is 40 Belongs to 7-10 class intervals.

Therefore, modal class = 7-10

$$1 = 7$$

$$h = 3$$

$$f_1 = 40$$

$$f_0 = 30$$

$$f_2 = 16$$

Substituting these values in the formula of mode we get:

$$m = 1 + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$m = 7 + \left(\frac{40 - 30}{2(40) - 30 - 16}\right) \times 3$$

$$m = 7 + \left(\frac{10}{34}\right) \times 3$$

$$m = 7 + \frac{30}{34}$$

$$m = 7.88$$

Hence, modal size of surnames is 7.88.

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

Ans: We can calculate the median as given below:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Where

1 = Lower limit of median class

h = Class size

f = Frequency of median class

cf = cumulative frequency of class preceding median class

The cumulative frequencies with their respective class intervals are as follows:

Weight	Number	Cumulative frequency
(in kg)	of	
	students	
40-45	2	2
45-50	3	2+3=5

50-55	8	5+8=13
55-60	6	13+6=19
60-65	6	19 + 6 = 25
65 - 70	3	25 + 3 = 28
70-75	2	28 + 2 = 30
Total(n)	30	

It can be observed from the given table

$$n = 30$$

$$\frac{n}{2} = 15$$

Cumulative frequency just greater than $\frac{n}{2}$ is 19 , Belongs to class interval 55 - 60

Median class = 55 - 60

$$1 = 55$$

$$f = 6$$

$$cf = 13$$

$$h = 5$$

Substituting these values in the formula of median we get:

$$m = 1 + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$m = 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$m = 55 + \left(\frac{10}{6}\right)$$

$$m = 56.67$$

Hence, median weight is 56.67 kg.