surface areas and volumes

12 Chapter

Exercise 12.1

1. 2 cubes of each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

Ans: Given that,

2 cubes are joined end to end as given in the following diagram.

To find the surface area of the resulting cuboid.

Volume of each cube $= 64\ cm^3$

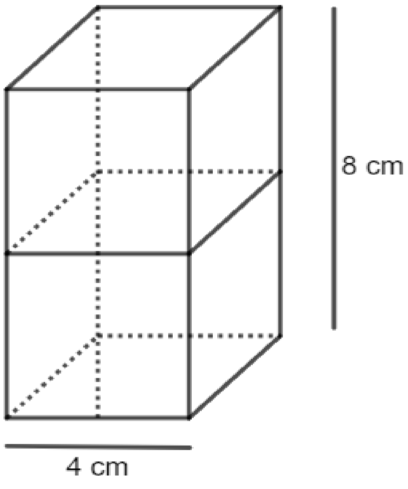
We know that,

Volume of a cube $= a^3$

$a^3 = 64$

$a = 4\ cm$

Thus the dimension of the resulting cuboid is of 4 cm, 4 cm and 8 cm when they are joined end to end. That is, $l = 4\ cm$, $b = 4\ cm$ and $h = 8\ cm$



Then,

Surface area of cuboid $= 2(lb + bh + lh)$

$= 2[(4\times4) + (4\times8) + (4\times8)]$

$= 2\ 16 + 32 + 32$

$= 2\ 80$

$= 160\ \text{cm}^2$

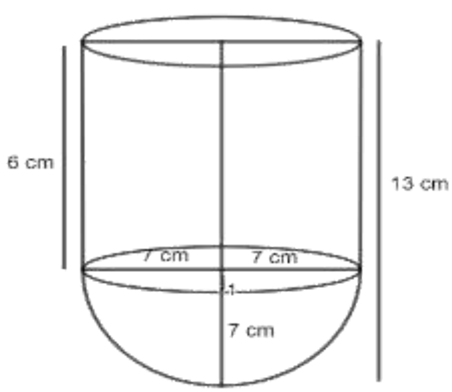
$\therefore$ The surface area of the resultant cuboid is $160\ \text{cm}^2$

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is $14\ \text{cm}$ and the total height of the vessel is $13\ \text{cm}$. Find the inner surface area of the vessel. [Use $\pi = \frac{22}{7}$]

Ans: Given that,
The diameter of the hemisphere $= 14\ \text{cm}$
The total height of the vessel $= 13\ \text{cm}$
To find,
The inner surface area of the vessel.

Thus the radius of the hollow hemisphere $= \frac{d}{2}$

$= \frac{14}{2}$
$= 7\ \text{cm}$



From the diagram, it can be observed that the radius of the cylindrical part and that of the hemispherical part is the same.
Thus, height of hemispherical part $=$ Radius $= 7\ \text{cm}$
Height of the cylindrical part $= 13 - 7$
$= 6\ \text{cm}$
Inner surface area of the vessel $=$ CSA of cylindrical part + CSA of hemispherical part

CSA of cylindrical part $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6$

$= 2 \times 22 \times 6$

$= 44 \times 6$

CSA of hemispherical part $= 2\pi r^2$

$= \left(2 \times \frac{22}{7} \times 7^2\right)$

$= 2 \times 22 \times 7$

$= 44 \times 7$

Inner surface area of the vessel $= 44\ \ 6 + 7$

$= 44\ \ 13$

$= 572\ \text{cm}^2$

$\therefore$ The inner surface area of the vessel is $572\ \text{cm}^2$

3. A toy is in the form of a cone of radius $3.5\ \text{cm}$ mounted on a hemisphere of the same radius. The total height of the toy is $15.5\ \text{cm}$. Find the total surface area of the toy. [Use $\pi = \frac{22}{7}$]

Ans: Given that,

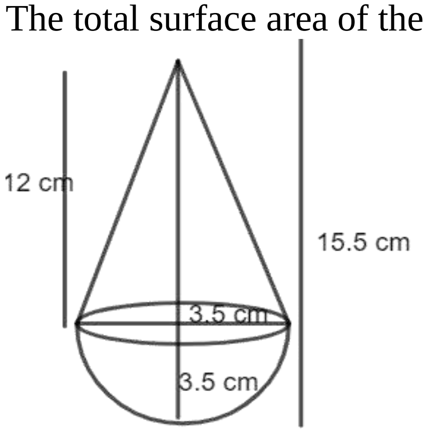
Radius of the cone $= 3.5\ \text{cm}$

Radius of the hemisphere $= 3.5\ \text{cm}$

The total height of the toy $= 15.5\ \text{cm}$

To find,

The total surface area of the toy



From the diagram, it can be observed that the radius of both conical and hemispherical parts is the same.
Height of the hemispherical part $= 3.5$ cm

$$= \frac{7}{2}$$

Height of the conical part $= 15.5 - 3.5$

$$= 12 \text{ cm}$$

Slant height of the conical part $l = \sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + 12^{\ 2}}$$

$$= \sqrt{\left(\frac{49}{4}\right) + 144}$$

$$= \sqrt{\frac{625}{4}}$$

$$= \frac{25}{2}$$

Total surface area of the toy $=$ CSA of conical part + CSA of hemispherical part
CSA of conical part $= \pi rl$

$$= \left(\frac{22}{7} \times \frac{7}{2} \times \frac{25}{2}\right)$$

$$= 137.5$$

CSA of hemispherical part $= 2\pi r^2$

$$= \left(2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2\right)$$

$$= 77$$

Total surface area of the toy $= 137.5 + 77$

$$= 214.5 \text{ cm}^2$$

$\therefore$ The total surface area of the toy is 214.5 cm^2.

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

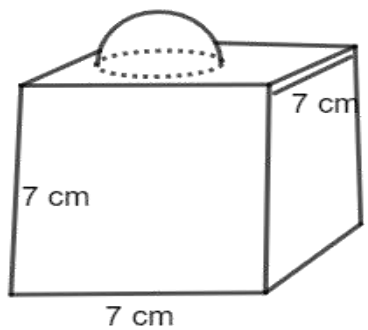
[Use $\pi = \frac{22}{7}$]

Ans: Given that,

Side of a cube $= 7$ cm

To find,

- The greatest diameter of the hemisphere.
- The surface area of the solid.



It can be observed from the diagram that the greatest possible diameter of the hemisphere is equal to the cube's edge.

Thus the greatest diameter of the hemispherical part $= 7$ cm

So the radius of the hemispherical part $= \frac{7}{2}$

$= 3.5$ cm

Total surface area of the solid = Surface area of cubical part + CSA of hemispherical part – Area of hemispherical part

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= 6\left(7\right)^2 + \left(\frac{22}{7} \times \left(\frac{7}{2}\right)^2\right)$

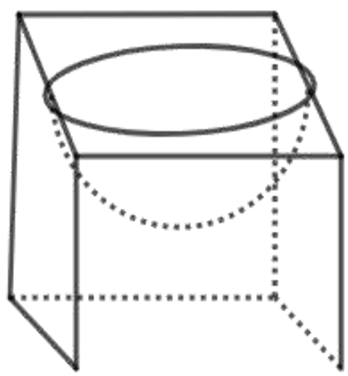
$= 294 + 38.5$

$= 332.5\ \text{cm}^2$

$\therefore$ The greatest diameter of the hemisphere is 7 cm and the surface area of the solid is $332.5\ \text{cm}^2$.

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Ans: Given that,
Diameter of the hemisphere = Edge of the cube
To determine,
The surface area of the remaining solid.



Diameter of the hemisphere = Edge of the cube $= l$

Radius of the hemisphere $= \frac{l}{2}$

Total surface area of the solid = Surface area of cubical part + CSA of hemispherical part – Area of base of hemispherical part

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= 6l^2 + \pi\left(\frac{l}{2}\right)^2$

$= 6l^2 + \frac{\pi l^2}{4}$

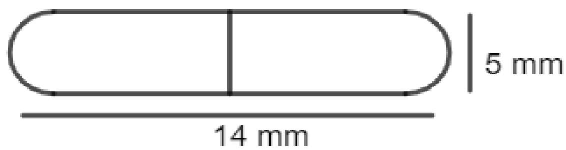
$= \frac{24l^2 + \pi l^2}{4}$

$= \frac{1}{4}\ 24 + \pi\ l^2 \text{ unit}^2$

$\therefore$ The area of the remaining solid is $\frac{1}{4}\ 24 + \pi\ l^2 \text{ unit}^2$.

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see the given figure). The length of the entire capsule is

14 mm and the diameter of the capsule is 5 mm. Find its surface area. [Use $\pi = \frac{22}{7}$]



Ans: Given that,
Length of the capsule $= 15$ mm
Diameter of the capsule $= 5$ mm
To find,
The surface area of the capsule
From the diagram,
Radius of cylindrical part = Radius of hemispherical part

$$= \frac{\text{Diameter of the capsule}}{2}$$

$$= \frac{5}{2}$$

Length of the cylindrical part (h) = Length of the entire capsule - $2r$

$$= 14 - 2\left(\frac{5}{2}\right)$$

$$= 14 - 5$$

$$= 9 \text{ mm}$$

Surface area of capsule = 2CSA of hemispherical part + CSA of cylindrical part
2CSA of hemispherical part $= 2\ 2\pi r^2$

$$= 4\pi\left(\frac{5}{2}\right)^2$$

$$= 25\pi$$

CSA of cylindrical part $= 2\pi rh$

$$= 2\pi\left(\frac{5}{2}\right) \times 9$$

$$= 45\pi$$

Surface area of the capsule $= 25\pi + 45\pi$

$$= 70\pi$$

$= 70 \times \frac{22}{7}$

$= 10 \times 22$

$= 220\ \text{mm}^2$

$\therefore$ The surface area of the capsule is $220\ \text{mm}^2$.

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are $2.1\ \text{m}$ and $4\ \text{m}$ respectively, and the slant height of the top is $2.8\ \text{m}$, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of $\text{Rs.}500$ per m^2. (Note that the base of the tent will not be covered with canvas.)

Ans: Given that,

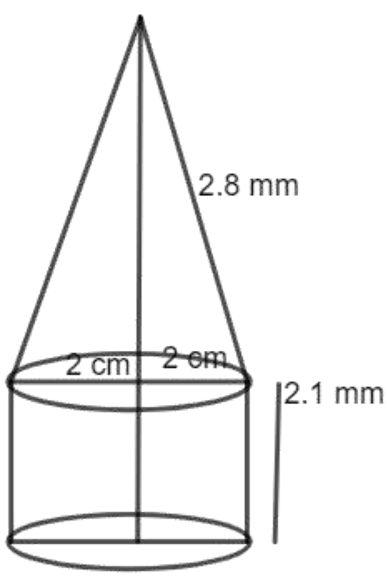
Height of the cylindrical part $= 2.1\ \text{m}$

Diameter of the cylindrical part $= 4\ \text{m}$

Radius of the cylindrical part $= 2\ \text{m}$

Slant height of conical part $= 2.8\ \text{m}$

Cost of $1\ \text{m}^2$ canvas $= \text{Rs.}500$



Area of the canvas used = CSA of conical part + CSA os cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi\ 2.8 + 4.2$

$= 2 \times \frac{22}{7} \times 7$

$= 44 \text{ m}^2$

Cost of 1 m^2 canvas $= \text{Rs.}500$

Cost of 44 m^2 canvas $= 44 \times 500$

$= \text{Rs.}22,000$

$\therefore$ The cost the canvas that is used to cover the tent is $\text{Rs.}22,000$.

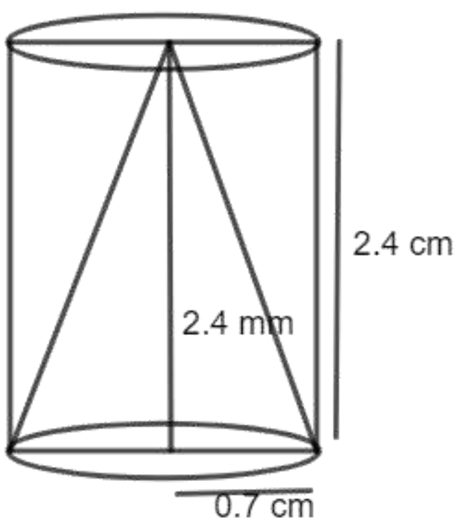
8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. [Use $\pi = \frac{22}{7}$]

Ans: Height of the cylindrical part = Height of the conical part $= 2.4$ cm

Diameter of the cylindrical part = Diameter of the conical part $= 1.4$ cm

To find,

The total surface area of the remaining solid.



Diameter of the cylindrical part $= 1.4$ cm

Radius of the cylindrical part $= \frac{1.4}{2}$

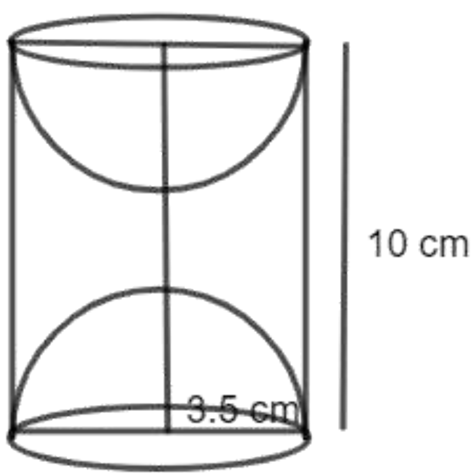
$= 0.7$ cm

Slant height of conical part $= \sqrt{r^2 + h^2}$

$=\sqrt{(0.7)^2+(2.4)^2}$
$=\sqrt{0.49+5.76}$
$=\sqrt{6.25}$
$=2.5$
The total surface area of the solid = CSA of cylindrical part + CSA of conical part + Area of cylindrical base
$=2\pi rh+\pi rl+\pi r^2$
$=\left(2\times\frac{22}{7}\times0.7\times2.4\right)+\left(\frac{22}{7}\times0.7\times2.5\right)+\left(\frac{22}{7}\times(0.7)^2\right)$
$=(4.4\times2.4)+(2.2\times2.5)+(2.2\times0.7)$
$=10.56+5.5+1.54$
$=17.6\text{ cm}^2$
$\therefore$ The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2.

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the given figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. [Use $\pi=\frac{22}{7}$]

Ans: Given,
Height of the cylindrical part $=10\text{ cm}$
Radius of the cylindrical part = Radius of the hemispherical part $=3.5$



The total surface area of the article = CSA of cylindrical part + 2CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= \left(2\pi \times 3.5 \times 10\right) + 4\pi\left(3.5\right)^2$

$= 70\pi + 49\pi$

$= 119\pi$

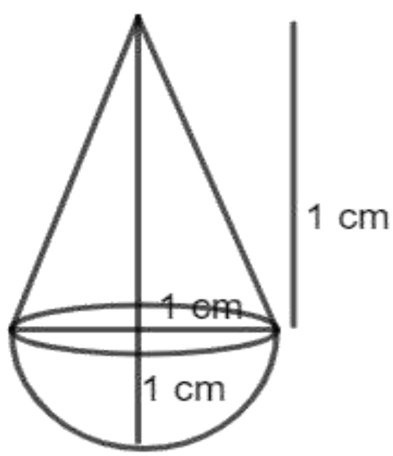
$= 119\left(\frac{22}{7}\right)$

$= 374 \text{ cm}^2$

$\therefore$ The total surface area of the article is 374 cm^2.

Exercise 12.2

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.



Ans: Given that,

Radius of cone and the hemisphere $= 1$ cm

Height of the cone = Radius of the cone $= 1$ cm

To find,

The volume of the solid in terms of π.

Volume of the given solid = Volume of the conical solid + Volume of the Hemispherical solid

$$=\frac{1}{3}\pi r^2h+\frac{2}{3}\pi r^3$$
$$=\frac{1}{3}\pi\ 1^{\ 2}\ 1\ +\frac{2}{3}\pi\ 1^{\ 3}$$
$$=\frac{1}{3}\pi+\frac{2}{3}\pi$$
$$=\frac{3}{3}\pi$$
$$=\pi$$

$\therefore$ The volume of the solid in terms of π is $\pi\ cm^3$.

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same). [Use $\pi=\frac{22}{7}$]

Ans: Given that,

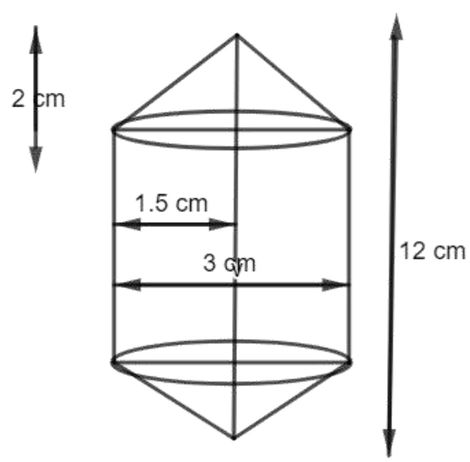
Diameter of the cylindrical part $=3\text{ cm}$

Length of the cylindrical part $=12\text{ cm}$

Height of the conical part $=2\text{ cm}$

To find,

The volume of the air contained in the model



It can be observed from the figure that the,

Height of each conical part $h_1 = 2$ cm

Height of cylindrical part $h_2 = 12 - 2 \times$ Height of the conical part

$= 12 - 2 \times 2$

$= 12 - 4$

$= 8$ cm

Diameter of the cylindrical part $= 3$ cm

Radius of the cylindrical part = Radius of the conical part

$= \frac{3}{2}$

Volume of the air in the model = Volume of cylindrical part + 2(Volume of cones)

Volume of the cylinder $= \pi r^2 h_2$

$= \pi\left(\frac{3}{2}\right)^2 \ 8$

$= \pi \times \frac{9}{4} \times 8$

$= 18\pi$

Volume of 2 cones $= 2 \times \frac{1}{3}\pi r^2 h$

$= 2 \times \frac{1}{3}\pi \times \left(\frac{3}{2}\right)^2 \times 2$

$= \frac{2}{3}\pi \times \frac{9}{4} \ 2$

$= 3\pi$

Volume of the air in cuboid $= 18\pi + 3\pi$

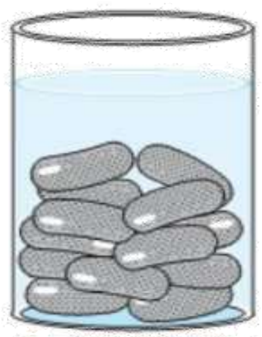
$= 21\pi$

$= 21\left(\frac{22}{7}\right)$

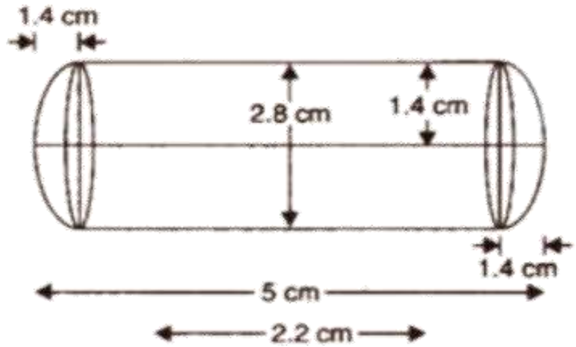
$= 66 \text{ cm}^3$

3. Ag gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each

shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see the given figure). [Use $\pi = \frac{22}{7}$]



Ans: Given that,
The length of the gulab jamun $= 5$ cm
Diameter of the gulab jamun $= 2.8$ cm
The volume of syrup in gulab jamun is 30% to its volume.
To find,
The volume of syrup in 45 gulab jamun.
The diagram of gulab jamun shaped like a cylinder with two hemispherical ends is shown in the following diagram.



From the diagram,
Radius of the cylindrical part r_1 = Radius of hemispherical part r_2

$$= \frac{2.8}{2}$$

$$= 1.4 \text{ cm}$$

The length of the hemispherical part is the same as that of the radius of the hemispherical part.

Length of each hemispherical part $= 1.4$ cm

Height of the cylindrical part $= 5 - 2 \times$ Length of hemispherical part

$= 5 - 2\ 1.4$

$= 5 - 2.8$

$= 2.2$ cm

Volume of one gulab jamun = Volume of cylindrical part $+ 2$ (Volume of hemispherical part

Volume of cylindrical part $= \pi r^2 h$

$= \pi\ 1.4\ ^2\ 2.2$

$= \frac{22}{7}\ 1.4\ ^2 \times 2.2$

$= 13.552$

2 (Volume of hemispherical part) $= 2 \times \frac{2}{3} \pi r^3$

$= \frac{4}{3} \pi r^3$

$= \frac{4}{3} \pi\ 1.4\ ^3$

$= \frac{4}{3} \times \frac{22}{7} \times\ 1.4\ ^3$

$= 11.498$

Volume of one gulab jamun $= 13.552 + 11.498$

$= 25.05\ cm^3$

Volume of 45 gulab jamuns $= 45 \times 25.05$

$= 1{,}127.25\ cm^3$

Volume of sugar syrup $= 30\%$ of volume

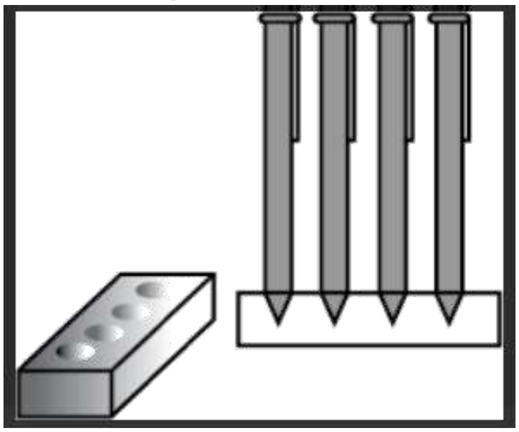
$= \frac{30}{100} \times 1127.25$

$= 338.17\ cm^3$

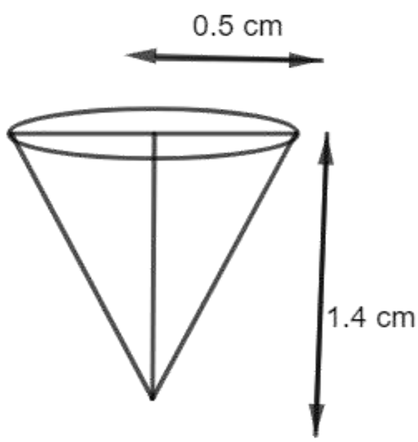
$\therefore$ The volume of sugar syrup found in 45 gulab jamuns is approximately $338\ cm^3$.

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboids are 15 cm by 10 cm

by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see the following figure0. [Use $\pi=\frac{22}{7}$]



Ans: Length of the cuboid $=15\text{ cm}$
Breadth of the cuboid $=10\text{ cm}$
Height of the cuboid $=3.5\text{ cm}$
Radius of conical depression $=0.5\text{ cm}$
Height of conical depression $=1.4\text{ cm}$
To find,
The volume of wood in entire stand



Volume of the wood = Volume of cuboid - $4\times$Volume of cones
Volume of cuboid $=$ lbh
$=15\times10\times3.5$
$=525\text{ cm}^3$

Volume of cones $= 4\times\frac{1}{3}\pi r^2 h$

$=\frac{4}{3}\times\frac{22}{7}\times\left(\frac{1}{2}\right)^2\times 1.4$

$=1.47\text{ cm}^3$

Volume of the wood $= 525-1.47$

$=523.53\text{ cm}^3$

$\therefore$ The volume of the wood in the entire stand is 523.53 cm^3.

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Ans: Given that,

Height of the conical vessel $h = 8\text{ cm}$

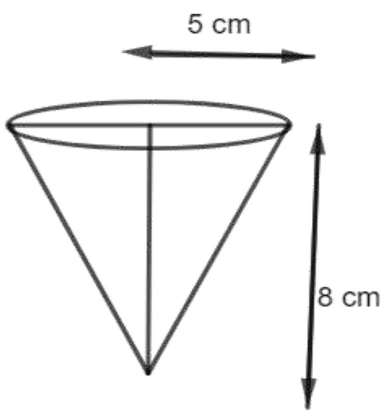
Radius of conical vessel $r_1 = 5\text{ cm}$

Radius of lead shots $r_2 = 0.5\text{ cm}$

One-fourth of water flows out from the vessel.

To find,

The number of lead shots dropped in the vessel.



Let the number of lead shots that has been dropped in the vessel be n.

Volume of water flows out = Volume of lead shots dropped in the vessel

$\frac{1}{4}\times\text{Volume of the cone} = n\times\frac{4}{3}\pi r_2^{\,3}$

$$\frac{1}{4}\times\frac{1}{3}\pi r_1^2 h = n\times\frac{4}{3}\pi r_2^3$$

$$r_1^2 h = 16 n r_2^3$$

Substituting the values we known, we obtain,

$$5^2\ 8\ = n\ 16\times\ 0.5^{\ 3}$$

$$n = \frac{200}{16\times\ 0.5^{\ 3}}$$

$$n = 100$$

$\therefore$ The number of lead shots dropped in the vessel is 100.

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. [Use $\pi = 3.14$]

Ans: Let there be two cylinders, one is of larger and the other is smaller

Given that,

Height of the larger cylinder $h_1 = 220\text{ cm}$

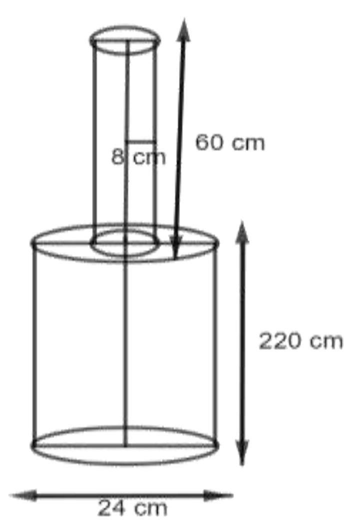
Diameter of the larger cylinder $d_1 = 24\text{ cm}$

Height of the smaller cylinder $h_2 = 60\text{ cm}$

Radius of the smaller cylinder $r_2 = 8\text{ cm}$

To find,

The mass of the iron pole.



Radius of larger cylinder $r_1 = \frac{24}{2}$

$= 12$ cm

Total volume of pole = Volume of the larger cylinder + Volume of the smaller cylinder

Volume of the larger cylinder $= \pi r_1^2 h_1$

$= \pi (12)^2 \times 220$

$= 31{,}680\pi$

Volume of the smaller cylinder $= \pi r_2^2 h_2$

$= 3840\pi$

Volume of the iron pole $= 31{,}680\pi + 3{,}840\pi$

$= 35{,}520\pi$

$= 35{,}520 \; 3.14$

$= 111{,}532.8 \text{ cm}^3$

Mass of 1 cm^3 iron $= 8$ g

Mass of $111{,}532.8 \text{ cm}^3$ iron $= 111{,}532.8 \times 8$

$= 892262.4$ g

We know that $1000 \text{ g} = 1 \text{ kg}$

$892262.4 \text{ g} = 892262.4 \times 1000 \text{ kg}$

$= 892.262$ kg

$\therefore$ The mass of the iron is 892.262 kg.

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. [Use $\pi = \frac{22}{7}$].

Ans: Given that,

A solid with a right circular cone and a hemisphere.

Height of the conical part of the cylinder $= 120$ cm

Radius of the conical part of the cylinder $= 60$ cm

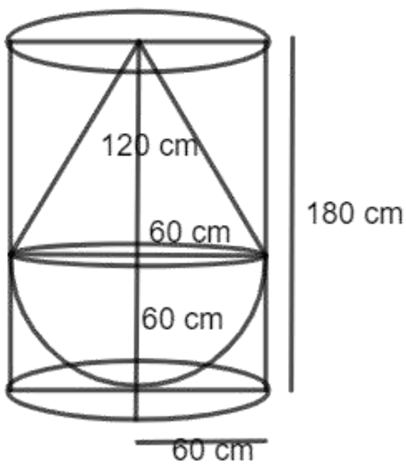
Radius of hemispherical part of the cylinder $= 60$ cm

Radius of the outer cylinder $= 60$ cm

Height of the outer cylinder $= 180$ cm

To find,

The volume of the water left in the cylinder.



Volume of the water left = Volume of cylinder – Volume of the solid

Volume of the cylinder $= \pi r^2 h$

$= \pi\ 60^{\ 2}\ 180$

$= \pi \times 3600 \times 180$

$= 648{,}000\pi\ \text{cm}^3$

Volume of the solid = Volume of cone + Volume of the hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi\ 60^{\ 2} 120 + \frac{2}{3}\pi\ 60^{\ 3}$

$= \frac{1}{3}\pi\ 432{,}000\ + \frac{2}{3}\pi\ 216{,}000$

$= 288{,}000\pi$

Volume of the water left $= 648{,}000\pi - 288{,}000\pi$

$= 360{,}000\pi$

$= 360{,}000 \times \frac{22}{7}$

$= 11311428.57\ \text{cm}^3$

$= 1.131\ \text{m}^3$

$\therefore$ The volume of the water left in the cylinder is $1.131\ \text{m}^3$

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child find its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurement and $\pi = 3.14$

Ans: Given that,

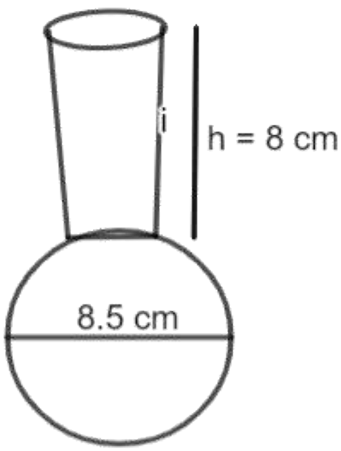
Height of the cylindrical part $= 8\text{ cm}$

Diameter of cylindrical neck $= 2\text{ cm}$

Diameter of spherical glass vessel $= 8.5\text{ cm}$

Volume of the water that the vessel holds $= 345\text{ cm}^3$

To find it the above given volume is correct.



Volume of the vessel = Volume of sphere + Volume of cylinder

Volume of sphere $= \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times 3.14 \times \left(\dfrac{8.5}{2}\right)^3$

$= 321.392\text{ cm}^3$

Volume of cylinder $= \pi r^2 h$

$= 3.14 \times (1)^2 \times 8$

$= 25.12\text{ cm}^3$

Volume of the vessel $= 321.392 + 25.12$

$= 346.51\text{ cm}^3$

$\therefore$ The volume of the vessel is 346.51 cm^3 and hence the child is wrong.