circles

10 Chapter

Exercise 10.1

1. How many tangents can a circle have?
Ans: We know that a circle has infinite number of points on its perimeter. Hence, there will be infinite number of tangents on a circle.

2. Fill in the blanks:
(i)A tangent to a circle intersects it in _________ point(s).
Ans: A tangent to a circle intersects it in exactly one point(s).

(ii) A line intersecting a circle in two points is called a _______.
Ans: A line intersecting a circle in two points is called a secant.

(iii)A circle can have _______ parallel tangents at the most.
Ans: A circle can have two parallel tangents at the most.

(iv)The common point of a tangent to a circle and the circle is called ______.
Ans: The common point of a tangent to a circle and the circle is called point of contact.

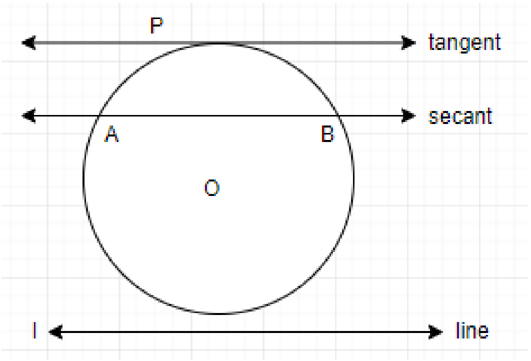
3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that $OQ = 12\text{ cm}$.
Length of PQ is: (A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$.
Ans: As, the tangent PQ is base and radius is the height of 5 cm. Also, OQ is the hypotenuse. Therefore, the length of PQ will be –

$$PQ = \sqrt{OQ^2 - OP^2}$$
$$\Rightarrow PQ = \sqrt{12^2 - 5^2}$$
$$\Rightarrow PQ = \sqrt{144 - 25}$$
$$\Rightarrow PQ = \sqrt{119}$$

Hence, option (D) is correct.

4. Draw a circle and two lines parallel to a given line such that one is tangent and the other, a secant to the circle.

Ans:



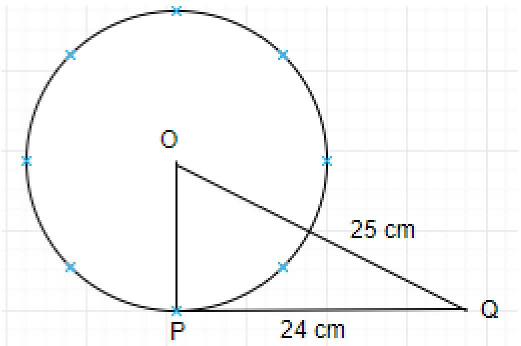
From the figure given above let us assume that a line l and a circle having centre O, comprises a line PT which is parallel to the line l and is a tangent to the circle. Similarly, AB is a secant parallel to the line l.

Exercise 10.2

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm.

Ans:



From the figure given above, let us assume that O is the centre of the circle and given we have PQ as the tangent to the circle of length 24 cm, and OQ is of length 25 cm.

Now, by using the Pythagoras theorem we have –

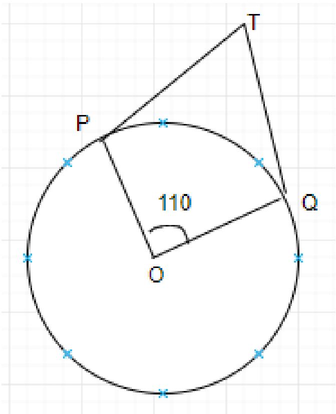
$$OP = \sqrt{OQ^2 - PQ^2}$$

$$\Rightarrow OP = \sqrt{25^2 - 24^2}$$

$$\Rightarrow OP = \sqrt{625 - 576}$$

$\Rightarrow OP = \sqrt{49}$
$\Rightarrow OP = 7\text{ cm}$.
Hence, option (A) is correct.

2. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to
(A) $60°$ (B) $70°$ (C) $80°$ (D) $90°$.



Ans:
As, from the given figure, we can observe that there are two tangents perpendicular to the radius of the circle as TP and TQ. Since, they are perpendicular to the radius hence, we have $\angle OPT = 90°$ and
$\angle OQT = 90°$.
Therefore, $\angle PTQ = 360° - \angle OPT - \angle OTQ - \angle POQ$
$\Rightarrow \angle PTQ = 360° - 90° - 90° - 110°$
$\Rightarrow \angle PTQ = 360° - 290°$
$\Rightarrow \angle PTQ = 70°$
Hence, option (B) is correct.

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of $80°$, then $\angle POA$ is equal to
(A) $50°$ (B) $60°$ (C) $70°$ (D) $80°$.
Ans:
As, we have two tangents PA and PB which will be perpendicular to the radius of the circle. Hence, we have $\angle PAO = 90°$ and
$\angle PBO = 90°$. Since, this is a quadrilateral, we will have sum of all interior angle equal to $360°$.
Therefore,
$\angle PAO + \angle PBO + \angle APB + \angle AOB = 360°$

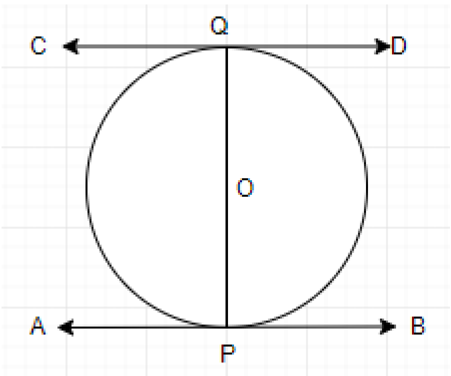
$\Rightarrow 90° + 90° + 80° + \angle AOB = 360°$
$\Rightarrow \angle AOB = 360° - 260°$
$\Rightarrow \angle AOB = 100°$.
Now, we know that $\angle POA$ is half of the $\angle AOB$.
Therefore,

$$\angle POA = \frac{\angle AOB}{2}$$

$$\Rightarrow \angle POA = \frac{100°}{2}$$

$\Rightarrow \angle POA = 50°$.
Hence, option (A) is correct.

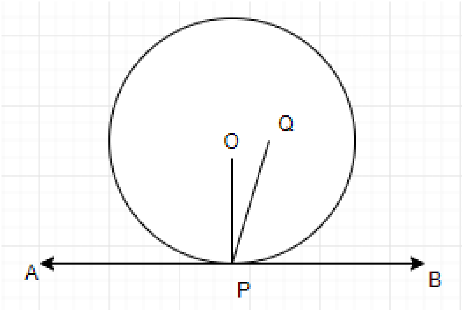
4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
Ans:



As, from the figure given above, let us assume that O is the centre of the circle and AB, CD are the two tangents at the ends of the diameter of the circle.
Now, we know that tangents are perpendicular to the radius of the circle.
Hence, $\angle CQO = 90°$
$\angle DQO = 90°$
$\angle APO = 90°$
$\angle BPO = 90°$.
Therefore, we can say that $\angle CPQ = \angle BQP$ because they are alternate angles. Similarly, $\angle AQP = \angle QPD$.
Hence, if the interior alternate angles are equal then the lines AB, CD should be parallel lines.
Hence, proved.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans:



Let us assume that the line perpendicular at the point of contact to the tangent of a circle do not passes through the centre O but passes through a point Q as shown in the figure above.

Hence, we have line $\mathrm{PQ} \perp \mathrm{AB}$.

$\Rightarrow \angle \mathrm{QPB} = 90°$

Also, we have $\angle \mathrm{OPB} = 90°$.

After comparing both the equations we get –

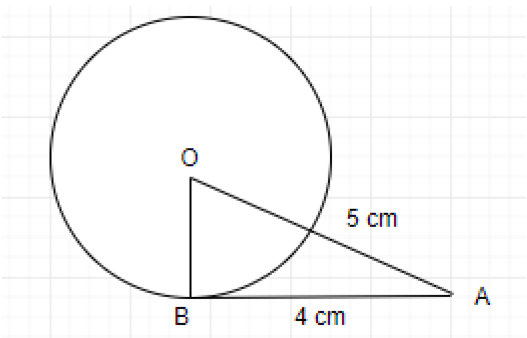
$\Rightarrow \angle \mathrm{QPB} = \angle \mathrm{OPB}$

But, from the figure drawn above we can observe that this is not the case as $\angle \mathrm{QPB} < \angle \mathrm{OPB}$.

Therefore, we can conclude that $\angle \mathrm{QPB} \neq \angle \mathrm{OPB}$. They can only be equal when these two-line segments QP and OP will be equal. This implies that the line perpendicular at the point of contact to the tangent of a circle passes through the centre O. Hence, proved.

6. The length of a tangent from a point $\mathbf{A}$ at distance $\mathbf{5\,cm}$ from the centre of the circle is $\mathbf{4\,cm}$. Find the radius of the circle.

Ans:



From the figure given above, let us assume that O is the centre of the circle and given we have AB as the tangent to the circle of length $4\,\mathrm{cm}$, and OA is of length $5\,\mathrm{cm}$.

Now, by using the Pythagoras theorem we have –

$$\mathrm{OB}=\sqrt{\mathrm{OA}^2-\mathrm{AB}^2}$$

$$\Rightarrow \mathrm{OB}=\sqrt{5^2-4^2}$$

$$\Rightarrow \mathrm{OB}=\sqrt{25-16}$$

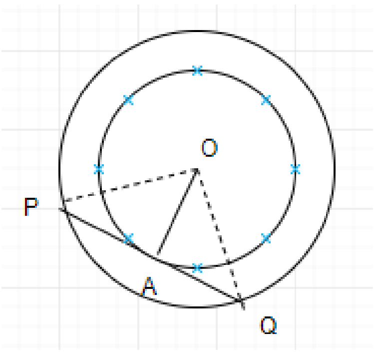
$$\Rightarrow \mathrm{OB}=\sqrt{9}$$

$$\Rightarrow \mathrm{OB}=3\,\mathrm{cm}.$$

Therefore, the radius of the circle will be $3\,\mathrm{cm}$.

7. Two concentric circles are of radii $5\,\mathrm{cm}$ and $3\,\mathrm{cm}$. Find the length of the chord of the larger circle which touches the smaller circle.

Ans:



From the figure given above, we can observe that the line segment PQ is the chord of the larger circle and a tangent to the smaller circle.

Therefore, $\mathrm{OA}\perp\mathrm{PQ}$.

Now, we can observe that the $\Delta\mathrm{OAP}$ forms a right-angled triangle.

Hence, by applying Pythagoras theorem –

$$\mathrm{AP}=\sqrt{\mathrm{OP}^2-\mathrm{OA}^2}$$

$$\Rightarrow \mathrm{AP}=\sqrt{5^2-3^2}$$

$$\Rightarrow \mathrm{AP}=\sqrt{25-9}$$

$$\Rightarrow \mathrm{AP}=\sqrt{16}$$

$$\Rightarrow \mathrm{AP}=4\,\mathrm{cm}.$$

Now, since the radius is perpendicular to the tangent therefore, we have $\mathrm{AP}=\mathrm{AQ}$.

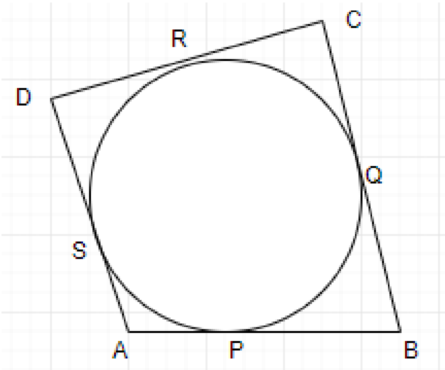
Hence, $\mathrm{PQ}=2\mathrm{AP}$.

$$\Rightarrow \mathrm{PQ}=2\times 4$$

$\Rightarrow PQ = 8\,\text{cm}$.

So, the length of the chord will be of $8\,\text{cm}$.

8. A quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.



Ans: From the figure given we can observe that the sides of the quadrilateral acts as tangents to the circle. As, the tangents drawn from any external point will have the same length. Therefore, we have –

$DR = DS$,

$CR = CQ$,

$BP = BQ$, and

$AP = AS$.

Now, we will add all the relations.

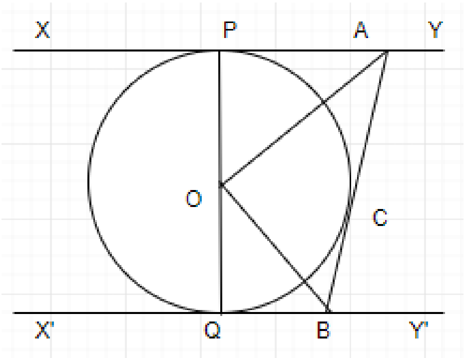
Hence, $DR + CR + BP + AP = DS + CQ + BQ + AS$

$\Rightarrow (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$

$\Rightarrow DC + AB = AD + BC$

Hence, proved.

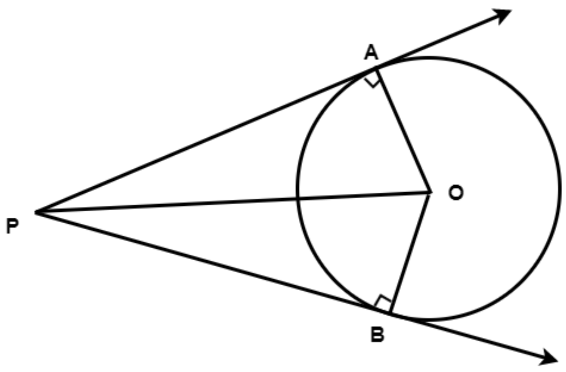
9. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$



Ans: From the figure given we can observe that AB, XY and $X'Y'$ are tangents to the circle and will be perpendicular to its radius.
Now, let us consider two triangles ΔOPA and ΔOCA, such that –
$OP = OC$,
$AP = AC$.
Hence, we have $\Delta OPA \cong \Delta OCA$ by the SSS congruence rule.
$\Rightarrow \angle POA = \angle COA$.
In the similar manner $\Delta OQB \cong \Delta OCB$.
$\Rightarrow \angle QOB = \angle COB$.
Now, we know that PQ is the diameter of the circle.
$\Rightarrow \angle POA + \angle COA + \angle COB + \angle QOB = 180°$
$\Rightarrow 2\angle COA + 2\angle COB = 180°$
$\Rightarrow \angle COA + \angle COB = 90°$
$\Rightarrow \angle AOB = 90°$.
Hence, proved.

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Ans:



Let us assume a circle with centre at O, which have two tangents PA and PB which are perpendicular to the radius of the circle as shown in the figure above.
Now, let us consider two triangles ΔOAP and ΔOBP, such that –
$PA = PB$, and
$OA = OB$.
Hence, we have $\Delta OAP \cong \Delta OBP$ by the SSS congruence criteria.
Therefore, $\angle OPA = \angle OPB$ and
$\angle AOP = \angle BOP$.
This implies that $\angle APB = 2\angle OPA$ and
$\angle AOB = 2\angle AOP$.

Hence, in the right-angled triangle ΔOAP, we have –

$\angle AOP + \angle OPA = 90°$

$2\angle AOP + 2\angle OPA = 180°$

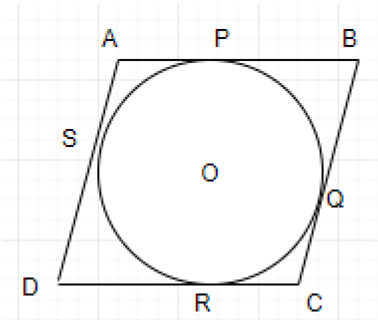
$\Rightarrow \angle AOB = 180° - \angle APB$

$\Rightarrow \angle AOP + \angle OPA = 180°$.

Hence, proved.

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans:



From the figure given above, we can observe that $ABCD$ is a parallelogram whose sides are tangent to the circle. This implies that $DR = DS$,

$CR = CQ$,

$BP = BQ$, and

$AP = AS$.

Now, we will add all the relations.

Hence, $DR + CR + BP + AP = DS + CQ + BQ + AS$

$\Rightarrow (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$

$\Rightarrow DC + AB = AD + BC$

As, the sides of a parallelogram are always parallel and equal in length.

Therefore, $AB = DC$ and

$AD = BC$.

$\Rightarrow 2AB = 2BC$

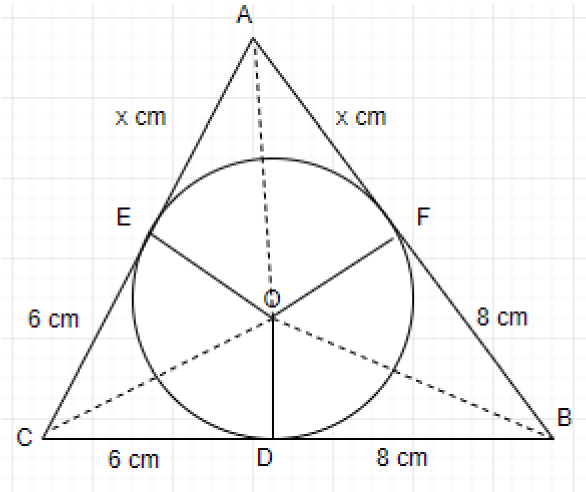
$\Rightarrow AB = BC$

Hence, all the sides of parallelogram are equal.

Therefore, we can conclude that it is a rhombus. Hence, proved.

12. A triangle ABC is drawn to circumscribe a circle of radius $4\,cm$ such that the Segments BD and DC into which BC is divided by the point of contact D are of lengths $8\,cm$ and $6\,cm$ respectively. Find the sides AB and AC.

Ans:



From the figure given above, we can observe that the sides of a triangle ABC are the tangents to the circle and are perpendicular to the radius of the circle.
Hence, in ΔABC
$CF = CD = 6\,\text{cm}$
Similarly, $BE = BD = 8\,\text{cm}$ and
$AE = AF = x\,\text{cm}$.
In ΔABE,
$AB = AE + BE$
$\Rightarrow AB = x + 8$
Similarly, $BC = 8 + 6 = 14$ and
$CA = 6 + x$.
Therefore, we have $2s = AB + BC + CA$
$\Rightarrow 2s = x + 8 + 14 + 6 + x$
$\Rightarrow 2s = 2x + 28$
$\Rightarrow s = x + 14$.
Now, we know that the area of a triangle can be calculated by using the formula
$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$.
Therefore,
$\text{Area of } \Delta ABC = \sqrt{(14+x)(14+x-6-x)(14+x-8-x)}$
$\Rightarrow \sqrt{x(14+x)(8)(6)}$
$\Rightarrow 4\sqrt{3(14x+x^2)}$.
Hence, Area of $\Delta OBC = \dfrac{1}{2} \times OD \times BC$
Area of $\Delta OBC = 28$
$\Rightarrow$ Area of $\Delta OCA = \dfrac{1}{2} \times OF \times AC$

Area of ΔOCA = 12+2x

$\Rightarrow$ Area of $\Delta OAB = \frac{1}{2} \times OE \times AB$

Area of ΔOAB = 16+2x

Therefore, the total area of the triangle ABC will be –

$= \text{Area } \Delta OBC + \text{Area } \Delta OCA + \text{Area } \Delta OAB$

$\Rightarrow 4\sqrt{3(14x + x^2)} = 28 + 12 + 2x + 16x + 2x$

$\Rightarrow \sqrt{3(14x + x^2)} = 14 + x$

$\Rightarrow 3(14x + x^2) = (14 + x)^2$

Hence, after further solving we have –

$(x + 14)(x - 7) = 0$

$\Rightarrow x = -14$ or

$\Rightarrow x = 7$

As, the side cannot be negative in nature, therefore, $x = 7\text{ cm}$.

Hence, $AB = 7 + 8$

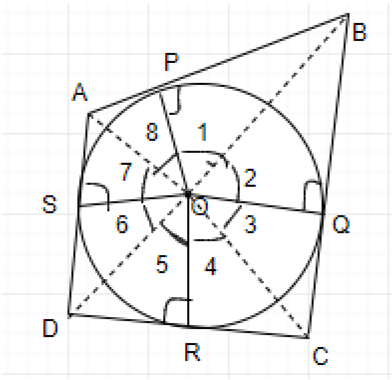
$\Rightarrow AB = 15\text{ cm}$ and

$CA = 6 + 7$

$\Rightarrow CA = 13\text{ cm}$.

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend Supplementary angles at the centre of the circle.

Ans:



From the figure given we can observe that the sides of the quadrilateral acts as tangents to the circle. Therefore, we have –

$\angle AOB + \angle COD = 180^\circ$ and

$\angle BOC + \angle DOA = 180^\circ$

Now, let us consider two triangles ΔOAP and ΔOAS,

Hence, we have $AP = AS$, and

$OP = OS$

$\Rightarrow \Delta OAP \cong \Delta OAS$ by the SSS congruence criteria.
Thus, $\angle POA = \angle AOS$
$\angle 1 = \angle 8$
Also,
$\angle 2 = \angle 3$,
$\angle 4 = \angle 5$,
$\angle 6 = \angle 7$
Therefore, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$
$\Rightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$
$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360°$
$\Rightarrow (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$
$\Rightarrow \angle AOB + \angle COD = 180°$
In the similar manner we can prove that $\angle BOC + \angle DOA = 180°$.
Therefore, we have proved that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.