mathematical reasoning

Chapter 14

Exercise 14.1

1. **Which of the following sentences are statements? Give reasons for your answer.**

(i) There are 35 days in a month.

Ans: This sentence is incorrect as a month can have a maximum of 31 days. Hence, it is a statement.

(ii) Mathematics is difficult.

Ans: For some people, mathematics can be easy and for some, it can be difficult. So, this sentence is subjective. Hence, it is not a statement.

(iii) The sum of 5 and 7 is greater than 10 .

Ans: The sum of 5 and 7 is 12, which is greater than 10. Therefore, this sentence is correct. Hence, it is a statement.

(iv) The square of a number is an even number.

Ans: This statement is sometimes correct and sometimes incorrect. For example, if we take the square of 2, it is 4 which is an even number whereas the square of 3 is 9 which is an odd number. Hence, it is not a statement.

(v) The sides of a quadrilateral have equal length.

Ans: This sentence is sometimes correct and sometimes incorrect. For example, rhombus and squares have sides of equal lengths whereas trapeziums and rectangles have sides of unequal lengths. Hence, it is not a statement.

(vi) Answer this question.

Ans: This is an order. Therefore, it is not a statement.

(vii) The product of (-1) and 8 is 8.

Ans: The product of (-1) and 8 is -8. Thus, the given statement is incorrect. Hence, it is a statement.

(viii) The sum of all interior angles of a triangle is $180°$.

Ans: It is a statement as the given sentence is correct.

(ix) Today is a windy day.

Ans: The day which is being referred to is not evident. Hence, the given sentence is not a statement.

(x) All real numbers are complex numbers.

Ans: All real numbers can be written as $aA - I + 0A - i$. Therefore, the given statement is always correct and hence it is a statement.

2. Give three examples of sentences which are not statements. Give reasons for the answers.

Ans: The three examples of sentences which are not statements, are as follows:

(i) She is a doctor.

It is not evident from the sentence as to whom 'she' is referred to. Hence, it is not a statement.

(ii) Geometry is difficult.

For some people, geometry can be easy and for some others, it can be difficult. Therefore, it is not a statement.

(iii) Where is he going?

This is a question in which it is not evident as to whom 'he' is referred to. Hence, it is not a statement.

Exercise 14.2

1. **Write the negations of the following statements:**

(i) Chennai is the capital of Tamil Nadu.

Ans: Chennai is not the capital of Tamil Nadu.

(ii) $\sqrt{2}$ is not a complex number.

Ans: $\sqrt{2}$ is not a complex number.

(iii) All triangles are not equilateral triangles.

Ans: All triangles are equilateral triangles.

(iv) The number 2 is greater than 7

Ans: The number 2 is not greater than 7.

(v) Every natural number is an integer.

Ans: Every natural number is not an integer.

2. **Are the following pair of statements negations of each other?**

(i) The number x is not a rational number.
The number x is not an irrational number.

Ans: The negation of the first statement is 'the number x is a rational number'. This is because if the number is not an irrational number, then it is a rational number. This is same as the second statement.

(ii) The number x is a rational number.
The number x is an irrational number.

Ans: The negation of the first statement is 'the number x is not a rational number'. This means that the number is an irrational number which is same as the second statement.
Therefore, the given statements are negations of each other.

3. **Find the component statements of the following compound statements and check whether they are true or false.**

(i) Number 3 is prime or it is odd.

Ans: The component statement are as follows:
a: Number 3 is prime.

b: Number 3 is odd.
Both the above statements are true.

(ii) All integers are positive or negative.

Ans: The component statement are as follows:
a: All integers are positive.
b: All integers are negative.
Both the above statements are false.

(iii) 100 is divisible by x, 5, and 7.

Ans: The component statement are as follows:
a: 100 is divisible by 3.
b: 100 is divisible by 11.
c: 100 is divisible by 5.

Here, the statements, a and b, are false and statement c is true.

Exercise 14.3

1. For each of the following compound statements first identify the connecting words and then break it into component statements.

(i) All rational numbers are real and all real numbers are not complex.

Ans: Here, the connecting word is ‘and’.
The component statements are as follows.
a: All rational numbers are real.
b: All real numbers are not complex.

(ii) Square of an integer is positive or negative.

Ans: Here, the connecting word is ‘or’.
The component statements are as follows.
a: Square of an integer is positive.
b: Square of an integer is negative.

(iii) The sand heats up quickly in the Sun and does not cool down fast at night.

Ans: Here, the connecting word is 'and'.

The component statements are as follows.

a: The sand heats up quickly in the sun.

b: The sand does not cool down fast at night.

(iv) $x = 2$ and $x = 3$ are the roots of the equation.

Ans: Here, the connecting word is 'and'.

The component statements are as follows.

a: $x = 2$ is a root of the equation $3x^2n - xn - 10 = 0$

b: $x = 3$ is a root of the equation $3x^2n - xn - 10 = 0$

2. Identify the quantifier in the following statements and write the negation of the statements.

(i) There exists a number which is equal to its square.

Ans: The quantifier is 'There exists'.

The negation of this statement is as follows.

There does not exist a number which is equal to its square.

(ii) For every real number x, x is less than $x+1$.

Ans: The quantifier is 'For every'.

The negation of this statement is as follows.

There exist a real number x for which x is not less than $x+1$.

(iii) There exists a capital for every state in India.

Ans: The quantifier is 'There exists'.

The negation of this statement is as follows.

There exists a state in India whose capital does not exists.

3. Check whether the following pair of statements is negation of each other. Give reasons for the answer.

(i) $x + y = y + x$ is true for every real number x and y.

(ii) There exists real number x and y for which $x + y = y + x$.

Ans: The negotiation of statement (i) is:

There exists real number x and y for which $x + y \neq y + x$ that is not the same as statement (ii).

Therefore, the given statements are not negotiation of each other.

4. State whether the 'or' used in the following statements is exclusive or inclusive. Give reasons for your answer.

(i) Sun rises or Moon sets.

Ans: It is not possible for the Sun to rise and the moon to set together. Hence, the 'or' here is exclusive.

(ii) To apply for a driving license, you should have a ration card or a passport.

Ans: Since a person can have both a ration card and a passport to apply for a driving license. So, the 'or' here is inclusive.

(iii) All integers are positive or negative.

Ans: All integers cannot be both positive and negative. Hence, the 'or' here is exclusive.

Exercise 14.4

1. Rewrite the following statement with 'if-then' in five different ways conveying the same meaning.

If a natural number is odd, then its square is also odd.

Ans: The given statements can be written in five different ways as follows.

(i) A natural number is odd implies that its square is odd.

(ii) A natural number is odd only if its square is odd.

(iii) If the square of a natural number is not odd, then the natural number is not odd.

(iv) For a natural number to be odd, it is necessary that its square is odd.

(v) For the square of a natural number to be odd, it is sufficient that the number is odd.

2. Write the contrapositive and converse of the following statements.

(i) If x is a prime number, then x is odd.

Ans: Contrapositive: If x is not odd, then x is not a prime number.

Converse: If x is odd, then x is a prime number.

(ii) It the two lines are parallel, then they do not intersect in the same plane.

Ans: Contrapositive: If two lines intersect in the same plane, then they are not parallel.

Converse: If two lines do not intersect in the same plane, then they are parallel.

(iii) Something is cold implies that it has low temperature.

Ans: Contrapositive: If something does not have low temperature, then it is not cold.

Converse: If something has low temperature, then it is cold.

(iv) You cannot comprehend geometry if you do not know how to reason deductively.

Ans: Contrapositive: If you know how to reason deductively, then you can comprehend geometry.

Converse: If you do not know how to reason deductively, then you cannot comprehend geometry.

(v) x is an even number implies that x is divisible by 4.

Ans: Contrapositive: If x is not divisible by 4, then x is not an even number.

Converse: If x is divisible by 4, then x is an even number.

3. Write each of the following statement in the form 'if-then'.

(i) You get a job implies that your credentials are good.

Ans: If you get a job, then your credentials are good.

(ii) The Banana trees will bloom if it stays warm for a month.

Ans: If the Banana tree stays warm for a month, then it will bloom.

(iii) A quadrilateral is a parallelogram if its diagonals bisect each other.

Ans: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(iv) To get A+ in the class, it is necessary that you do the exercises of the book.

Ans: If you want to get an A+ in the class, then you have to do all the exercises of the book.

4. Given statements in (a) and (b). Identify the statements given below as contrapositive or converse of each other.

(a) If you live in Delhi, then you have winter clothes.

(i) If you do not have winter clothes, then you do not live in Delhi.

Ans: This is the contrapositive of the given statement (a).

(ii) If you have winter clothes, then you live in Delhi.

Ans: This is the converse of the given statement (a).

(b) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

(i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

Ans: This is the contrapositive of the given statement (b).

(ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Ans: This is the converse of the given statement (b).

Exercise 14.5

1. Show that the statement

p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0' is true by

Ans: Given the statement

p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0.

Let q: x is a real number such that $x^3 + 4x = 0$

r: x is 0.

(i) direct method

Ans: To show that statement p is true, we assume that q, is true and the show that r is true.

Let q be true.

$x^3 + 4x = 0$

$x^3 + 4x = 0$

Hence, $x = 0$ or $x^2 + 4 = 0$

Since x is real, it is 0.

Therefore, the given statement p is true.

(ii) method of contradiction

Ans: To show that statement p is true by the method of contradiction, we assume that p is not true.

Let x be a real number such that $x^3 + 4x = 0$ and let x be not 0.

Hence, $x^3 + 4x = 0$

$x(x^2 + 4) = 0$

$x = 0$ or $x^2 + 4 = 0$

$x = 0$ or $x^2 = -4$

However, since x is real, it is 0, which is a contradiction since we have assumed that x is not 0.

Therefore, the given statement p is true.

(iii) method of contrapositive

Ans: To show that statement p is true by contrapositive method, we assume that r is false and then show that q must be false.

Let r be false.

This implies that x is not 0.

We know that $x^2 + 4$ will always be positive.

$x \neq 0$ implies that the product of any positive real number with x is not zero.

Let us consider the product of x with $x^2 + 4$

So, $x(x^2 + 4) \neq 0$

$x^3 + 4x \neq 0$

This shows that statement q is false.

Thus, it has been proved that ~ r implies ~ q.

Therefore, the given statement p is true.

2. Show that the statement 'For any real numbers a and b, $a^2 = b^2$ implies that $a = b$' is not true by giving a counter-example.

Ans: Let us take an example.

Let $a = 1$ and $b = -1$

Therefore, $a^2 = (1)^2 = 1$

and $b^2 = (-1)^2 = 1$

$a^2 = b^2$ but $a \neq b$ $(1 \neq -1)$

Hence $a, b \in R$ and $a^2 = b^2$

However, $a \neq b$

Thus, it can be concluded that the given statement is false.

3. Show that the following statement is true by the method of contrapositive.

p: If x is an integer and x^2 is even, then x is also even.

Ans: Given the statement p: If x is an integer and x^2 is even, then x is also even.

Let q: x is an integer and x^2 is even.

r: x is even.

To prove that p is true by contrapositive method, we assume that r is false, and then prove that q is also false.

Let r be false, then x is not even.

To prove that q is false, it has to be proved that x is not an integer or x^2 is not even.

x is not even implies that x^2 is also not even.

Therefore, q is false.

Thus, the given statement p is true.

4. By giving a counter example, show that the following statements are not true.

(i) p: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

Ans: The given statement is of the form ‘if q then r’.

q: All the angles of a triangle are equal.

r: The triangle is an obtuse-angled triangle.

The given statement p has to be proved false. For this purpose, it has to be proved that if q,

then ~ r.

To prove this, the angles of a triangle are required such that

none of them is an obtuse angle.

We know that the sum of all angles of a triangle is 180°.

Therefore, if all the three angles are equal, then each of them is of measure 60°, which is not an obtuse angle.
In an equilateral triangle, the measure of all angles is equal.
However, the triangle is not an obtuse-angled triangle.
Thus, it can be concluded that the given statement p is false.

(ii) q: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

Ans: The given statement is as follows.
q: The equation $x^2 - 1 = 0$, i.e., the root $x = 1$, lies between 0.
and 2.
This statement has to be proved false. To show this, use a counter example.
Consider $x^2 - 1 = 0$
$x^2 = 1$
$x^2 = \pm 1$
One root of the equation $x^2 - 1 = 0$, i.e., the root lies between
0 and 2.
Thus, the given statement is false.

5. Which of the following statements are true and which are false? In each case give a valid reason for saying so.

(i) p: Each radius of a circle is a chord of the circle.

Ans: The given statement p is false.
According to the definition of chord, it should intersect the circle at two distinct points.

(ii) q: The centre of a circle bisects each chord of the circle.

Ans: The given statement q is false.
If the chord is not the diameter of the circle, then the centre will not bisect that chord.

(iii) r: Circle is a particular case of an ellipse.

Ans: The equation of an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$x^2 + y^2 = 1$, which is an equation of a circle

Therefore, circle is a particular case of an eclipse.

Thus, statement r is true.

(iv) s: If x and y are integers such that $x > y$, then $-x < -y$.

Ans: $x > y$

$\Rightarrow -x < -y$ (By a rule of inequality)

Thus, the given statement s is true.

(v) t: $\sqrt{11}$ is a rational number.

Ans: 11 is a prime number and we know that the square root of any prime number is an irrational number.

Therefore, $\sqrt{11}$ is an irrational number.

Thus, the given statement t is false.

Miscellaneous Exercise

1. Write the negation of the following statements:

(i) p: For every positive real number x, the number x – 1 is also positive.

Ans: The negation of statement p is as follows.

There exists a positive real number x, such that $x-1$ is negative.

(ii) q: All cats scratch.

Ans: The negation of statement q is as follows.

There exists a cat that does not scratch.

(iii) r: For every real number x, either $x > 1$ or $x < 1$.

Ans: The negation of statement r is as follows.

There exists a real number x, such that neither $x > 1$ nor $x < 1$.

(iv) s: There exists a number x such that $0 < x < 1$.

Ans: The negation of statement s is as follows.

There does not exist a number x, such that $0 < x < 1$.

2. State the converse and contrapositive of each of the following statements:

(i) p: A positive integer is prime only if it has no divisors other than 1 and itself.

Ans: Statement p can be written as follows.

If a positive integer is prime, then it has no divisors other than 1 and itself.

The converse of the statement is as follows.

If a positive integer has no divisors other than 1 and itself, then it is prime.

The contrapositive of the statement is as follows.

If positive integer has divisors other than 1 and itself, then it is not prime.

(ii) q: I go to a beach whenever it is a sunny day.

Ans: The given statement can be written as follows.

If it is a sunny day, then I go to a beach.

The converse of the statement is as follows.

If I go to a beach, then it is a sunny day.

The contrapositive of the statement is as follows.

If I don't go to a beach, then it is not a sunny day.

(iii) r: If it is hot outside, then you feel thirsty.

Ans: The converse of statement r is as follows.

If you feel thirsty, then it is hot outside.

The contrapositive of statement r is as follows.

If you do not feel thirsty then it is not hot outside.

3. Write each of the statements in the form 'if p, then q'.

(i) p: It is necessary to have a password to log on to the server.

Ans: Statement p can be written as follows.

If you log on to the server, then you have a password.

(ii) q: There is traffic jam whenever it rains.

Ans: Statement q can be written as follows.

If it rains, then there is a traffic jam.

(iii) r: You can access the website only if you pay a subscription fee.

Ans: If you can access the website, then you pay a subscription fee.

4. Re write each of the following statements in the form 'p if and only if q'.

(i) p: If you watch television, then your mind is free and if your mind is free, then you watch television.

Ans: You watch television if and only if your mind is free.

(ii) q: For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.

Ans: You get an A grade if and only if you do all the homework regularly.

(iii) r: If a quadrilateral is equiangular, then it is a rectangle and if a quadrilateral is a rectangle, then it is equiangular.

Ans: A quadrilateral is equiangular if and only if it is a rectangle.

5. Given below are two statements

p: 25 is a multiple of 5.

q: 25 is a multiple of 8.

Write the compound statements connecting these two statements with 'And' and 'Or'. In both cases check the validity of the compound statement.

Ans: The compound statement with 'And' is '25 is a multiple of 5 and 8'.

This is a false statement, since 25 is not a multiple of 8.

The compound statement with 'Or' is '25 is a multiple of 5 or 8'.

This is a true statement, since 25 is not a multiple of 8 but it is a multiple of 5.

6. Check the validity of the statements given below by the method given against it.

(i) p: The sum of an irrational number and a rational number is irrational (by contradiction method).

Ans: The given statement is as follows.

p: the sum of an irrational number and a rational number is irrational.

Let us assume that the given statement, p, is false. That is, we assume that the sum of an irrational number and a rational number is rational.

Therefore, $\sqrt{p}+\frac{q}{r}=\frac{s}{t}$ is irrational when $\sqrt{p}$ is irrational and q, r, s, t are integers.

This implies $\frac{s}{t}-\frac{q}{r}$ is a rational number and $\sqrt{p}$ is an irrational number.

This is a contradiction. Therefore, our assumption is wrong.

Therefore, the sum of an irrational number and a rational number is rational.

Thus, the given statement is true.

(ii) q: If n is a real number with $n>3$, then $n^2>9$ (by contradiction method).

Ans: The given statement, q is as follows.

If n is a real number with $n>3$, then $n^2>9$.

Let us assume that n is a real number with $n>3$, but $n^2>9$ is false. That is, $n^2<9$.

Then, $n>3$ and n is a real number.

Squaring both the sides, we obtain

$n^2>(3)^2$

$\Rightarrow n^2>9$

which is a contradiction, since we have assumed that $n^2<9$.

Thus, the given statement is true. That is, if n is a real number with $n>3$, then $n^2>9$.

7. Write the following statement in five different ways, conveying the same meaning.

p: If a triangle is equiangular, then it is an obtuse angled triangle.

Ans: The given statement can be written in five different ways as follows.

(i) A triangle is equiangular implies that is an obtuse-angled triangle.

(ii) A triangle is equilateral only if it an obtuse-angled triangle.

(iii) For a triangle to be equiangular, it is necessary that the triangle is an obtuse-angled triangle.

(iv) For a triangle to be an obtuse-angled triangle, it is sufficient that the triangle is equiangular.

(v) If a triangle is not an obtuse-angled triangle, then the triangle is not equiangular.