ELECTRICITY
Difference Between Series and Parallel Circuits

Combination of Resistances (Resistors):

Apart from potential difference, current in circuit depend or resistance of the circuit. So, in the electrical circuits of radio, television and other similar things, it is usually necessary to combine two or more resistances to get the required current in the circuit. We can combine the resistances lengthwise (called series) or we can put the resistances parallel to one another. Thus, the resistances can be combined in two ways :

(i) series combination

(ii) parallel combination

(i) Series combination of resistors:

Consider three resistors of resistances R_1, R_2 and R_3 connected in series to cell of potential difference V as shown in figure. Since the three resistors are connected in series therefore the current I through each of them is same.

Then by Ohm's law the potential drop across each resistor is given by $V_1 = IR_1$, V_2 and $V_3 = IR_3$.

Since V is the total potential in the circuit therefore by conservation of energy we have

$$V = V_1 + V_2 + V_3$$

Substituting for V_1, V_2 and V_3 in above equation we have,

$$V = IR_1 + IR_2 + IR_3 \quad \text{........... (i)}$$

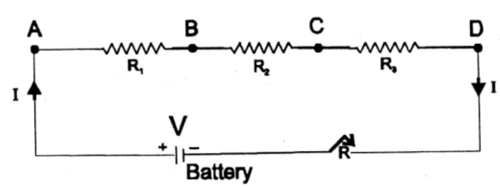
If R_S is the equivalent resistance of the series combination, then by Ohm's law we have

$$V = IR_s \quad \text{........... (ii)}$$

Therefore, from equations (i) and (ii) we have

$$IR_s = IR_1 + IR_2 + IR_3$$

Hence $R_s = R_1 + R_2 + R_3$



Series comination of resistances

Thus in series combination the equivalent resistance is the sum of the individual resistances. For more resistors, the above expression would have been-

$R_s = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots$

NOTE :

In a circuit, if the resistors are connected in series :

(A) The current is same in each resistor of the circuit :

(B) The resistance of the combination of resistors is equal to sum of the individual resistors.

(C) The total voltage across the combination is equal to the sum of the voltage drop across the individual resistors.

(D) The equivalent resistance is greater than that of any individual resistance in the series combination.

(ii) Parallel combination of resistors:

Consider two resistors R_1 and R_2 connected in parallel as shown in figure. When the current I reached point 'a', it splits into two parts I_1 going through R_1 and I_2 going through R_2. If R_1 is greater than R_2, then I_1 will be less than I_1 i.e., the current will tend to take the path of least resistance.

Since charge must be conserved, therefore the current I that enters point 'a' must be equal to the current that leaves that point. Therefore, we have

$$I = I_1 + I_2 \qquad \ldots\ldots\ldots(i)$$

Since the resistors are connected in parallel therefore the potential across each must be same, hence by Ohm's law we have

$I_1 = \frac{V}{R_1}$ and $I_2 = \frac{V}{R_2}$ substituting in equation (i) we have,

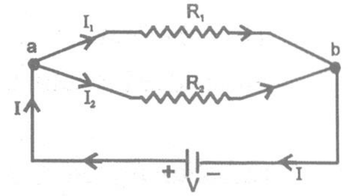
$$I = \frac{V}{R_1 + \frac{V}{R_2}} \qquad \ldots\ldots\ldots(ii)$$

Let R_P be the equivalent resistance of the parallel combination, they by Ohm's law we have,

$$I = \frac{V}{R_P} \qquad \ldots\ldots\ldots(iii)$$

Hence from equations (ii) and (iii) we have,

$$\frac{V}{R_P}=\frac{V}{R_1}+\frac{V}{R_2} \; or \qquad \frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}$$



An extension of this analysis to three or more resistors in parallel gives the following general expression

$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+..........$$

NOTE:

(A) The sum of the reciprocals of the individual resistance is equal to the reciprocal of equivalent resistance, R_P.

(B) The currents in various resistors are inversely proportional to the resistances, higher the resistance of a branch, the lower will be the current through it. The total current is the sum of the currents flowing in the different branches.

(C) The voltage across each resistor of a parallel combination is the same and is also equal to the voltage across the whole group considered as unit.

NOTE: For n equal residences $\frac{R_s}{R_p}=n^2$

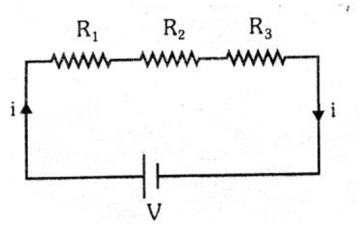
EQUIVALENT RESISTANCE IN SERIES CONCESSION:

Figure (a) shows three resistors of resistances R_1, R_2 and R_3 connected in series. The cell connected across the combination maintains a potential difference V across the combination. The current through the cell is i. The same current i flows through each resistor.

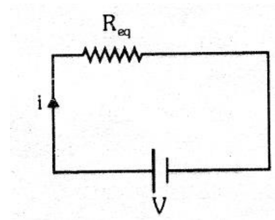
Let us replace the combination of resistors by a single resistor R_{eq} such that current does not change, i.e., it remains i. This resistance is called the **equivalent resistance** of the combination, and its value is given by

Ohm's law as $R_{eq} = V / i$

Thus $V = iR_{eq}$.



(a)



(b)

The potential differences V_1, V_2 and V_3 across the resistors R_1, R_2 and R_3 respectively are given by Ohm's law as: $V_1 = iR_1$, $V_2 = iR_2$, $V_3 = iR_3$

Since the resistors are in series, $V = V_1 + V_2 + V_3$

Substituting the values of the potential differences in the above equation,

$$iR_{eq} = iR_1 + iR_2 + iR_3$$

or $$iR_{eq} = i(R_1 + R_3)$$

or $$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

Similarly, for n resistors connected in series,

Equivalent resistance of resistors in series: $\boxed{\mathbf{Req = R_1 + R_2 + R_3 + \ldots\ldots\ldots + R_n}}$

PARALLEL CONNECTION OF RESISTORS

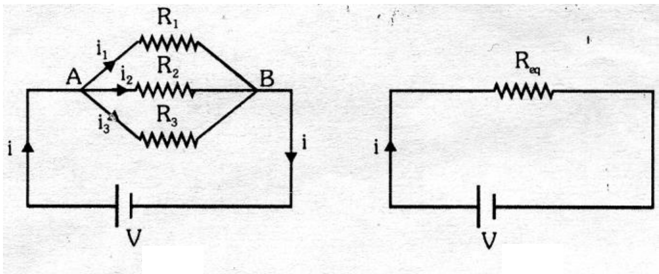
The total current flowing into the combination is equal to the sum of the currents passing through the individual resistors.

$$i = i_1 + i_2 + i_3$$

If resistors are connected in such a way that the same potential difference gets applied to each of them, they are said to be connected in parallel.

EQUIVALENT RESISTANCE IN PARALLEL CONNECTION

Figure (a) shows three resistors of resistances R_1, R_2 and R_3 connected in parallel across the points A and B. The cell connected across these two points maintains a potential difference V across each resistor. The current through the cell is i. It gets divided at A in to three parts i_1, i_2 and i_3. which flow through R_1, R_2 and R_3 respectively.



(a) (b)

Let us replaced the combination of resistors by an equivalent resistor R_{eq} such that the current i in the circuit does not change (Fig). The equivalent resistance is given by Ohm's law as $R_{eq} = V/I$.

Thus,

$$i = \frac{V}{R_{eq}} \text{ i}$$

The currents i_1, i_2 and i_3 through the resistor R_1, R_2 and R_3 respectively are given by Ohm's law as

$$i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3}$$

Since the resistors are in parallel,

$$i = i_1 + i_2 + i_3$$

Substituting the values of the currents in above equation,

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Similarly, if there are n resistors connected in parallel , their equivalent resistance R_{eq} is given by

Equivalent Resistance of resistors in parallel : $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$

For two resistances R_1 and R_2 connected in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \quad \text{or} \quad R = \frac{R_1 R_2}{R_1 + R_2}$$

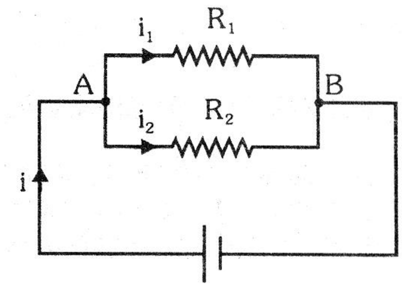
The equivalent resistance in parallel connection is less than each of the resistances.

When a resistance is joined parallel to a comparatively smaller resistance, the equivalent resistance is very close to the value of the smaller resistance.

NOTE : If a resistor connected in series with others is removed or fails, the current through each resistor becomes zero. On the other hand, if a resistor connected in parallel with others fails or is removed, the current continues to flow through the other resistors.

DISTRIBUTION OF CURRENT IN TWO RESISTORS IN PARALLEL

Consider the circuit in fig. The resistors R_1 and R_2 are connected in parallel. The current i gets distributed in the two resistors.



$$i = i_1 + i_2 \quad \text{...(i)}$$

Applying Ohm's law to the resistor R_1

$$V_A - V_B = R_1 i_1 \quad \text{...(ii)}$$

And applying Ohm's law to the resistor R2

$$V_A - V_B = R_2 i_2 \quad \text{...(iii)}$$

From (ii) and (iii), $R_1 i_1 = R_2 i_2$ or $i_2 = \frac{R_1}{R_2} i_1$

Substituting for i_2 in (i), we have

$$i = i_1 + \frac{R_1}{R_2} i_1 = i_1\left(1 + \frac{R_1}{R_2}\right) = i_1 \frac{R_1 + R_2}{R_2}$$

or $$i_1 = \frac{R_2}{R_1 + R_2} i$$

Similarly, $$i_2 = \frac{R_1}{R_1 + R_2} i$$

Thus, $$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

The current through each branch in parallel combination of resistors is inversely proportional to its resistance

DEVICES IN SERIES AND PARALLEL

You must have seen tiny bulbs strung together for decorating buildings during festivals like Diwali, and occasions like marriages, etc. These bulbs are connected in series, and the mains voltage is applied to the combination. The potential difference (V) of the mains gets divided across the bulbs ($V = V_1 + V_2 + V_3 + \ldots$). So, a small potential difference exists across each bulb, close to that required to make the bulb work. However, the same current flows all the bulbs. So, if one bulb goes bad. The current through it strops. and this stops the current through the rest of the bulbs as well. To make the chain of lights work, we have to find and replace the defective bulb. This problem does not occur with the lights in our house. That is because **in houses, lights, fans, etc., are connected in parallel.** In parallel connection, the same mains voltage gets applied to each device, but the current through each is different. If one of them goes bad, the current in the other branches of the parallel connection does not stop. Another advantage of parallel connection is that, unlike series connection, each device can draw a different current, as per its requirement.