INTRODUCTION TO TRIGONOMETRY

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES :

Let Δ ABC be a right angle triangle at $\angle C = 90^\circ$

so $\angle A + \angle B = 90^\circ$

So $\angle A$ & $\angle B$ are complement angle of each other

$\angle B = 90^\circ - \angle A$

$\sin A = \frac{a}{c} = \cos B = \cos(90^\circ - A)$

$\cos A = \frac{b}{c} = \sin B = \sin(90^\circ - A)$

$\tan A = \frac{a}{b} = \cot B = \cot(90^\circ - A)$

$\cot A = \frac{b}{a} = \tan B = \tan(90^\circ - A)$

$\sec A = \frac{c}{b} = \text{cosec } B = \text{cosec}(90^\circ - A)$

$\text{cosec } A = \frac{c}{a} = \sec B = \sec(90^\circ - A)$

so $\sin(90^\circ - A) = \cos A$ $\cos(90^\circ - A) = \sin A$

$\tan(90^\circ - A) = \cot A$ $\cot(90^\circ - A) = \tan A$

$\sec(90^\circ - A) = \text{cosec } A$ $\text{cosec}(90^\circ - A) = \sec A$

Ex.1 Without using trigonometric tables, evaluate the following :

(i) $\frac{\cos 37^\circ}{\sin 53^\circ}$ (ii) $\frac{\sin 41^\circ}{\cos 49^\circ}$ (iii) $\frac{\sin 30^\circ 17'}{\cos 59^\circ 43'}$

Sol. (i) We have

$$\frac{\cos 37^\circ}{\sin 53^\circ} = \frac{\cos(90^\circ - 53^\circ)}{\sin 53^\circ} = \frac{\sin 53^\circ}{\sin 53^\circ} = 1$$

[∵ $\cos(90^\circ - \theta) = \sin\theta$]

(ii) We have,

$$\frac{\sin 41^\circ}{\cos 49^\circ} = \frac{\sin(90^\circ - 49^\circ)}{\cos 49^\circ} = \frac{\cos 49^\circ}{\cos 49^\circ} = 1$$

[∵ $\sin(90^\circ - \theta) = \cos\theta$]

(iii) We have,

$$\frac{\sin 30^\circ 17'}{\cos 59^\circ 43'} = \frac{\sin(90^\circ - 59^\circ 43')}{\cos 59^\circ 43'} = \frac{\cos 59^\circ 43'}{\cos 59^\circ 43'} = 1.$$

Ex.2 Without using trigonometric tables evaluate the following :

(i) $\sin^2 25^\circ + \sin^2 65^\circ$ (ii) $\cos^2 13^\circ - \sin^2 77^\circ$

Sol. (i) We have

$$\sin^2 25^\circ + \sin^2 65^\circ = \sin^2(90^\circ - 65^\circ) + \sin^2 65^\circ$$

$$= \cos^2 65^\circ + \sin^2 65^\circ = 1$$

[∵ $\sin(90^\circ - \theta) = \cos\theta$]

(ii) We have,

$\cos^2 13º – \sin^2 77º = \cos^2(90º – 77º) – \sin^2 77º$

$= \sin^2 77º – \sin^2 77º = 0$

$[\because \cos(90º – \theta) = \sin\theta]$

Ex.3 Without using trigonometric tables, evaluate the following :

(i) $\dfrac{\cot 54^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot 70^\circ} - 2$

(ii) sec 50º sin 40° + cos 40º cosec 50º

Sol. (i) We have,

$\dfrac{\cot 54^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot 70^\circ} - 2$

$= \dfrac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\cot(90^\circ - 20^\circ)} - 2$

$= \dfrac{\tan 36^\circ}{\tan 36^\circ} + \dfrac{\tan 20^\circ}{\tan 20^\circ} - 2 = 1 + 1 - 2 = 0$

(ii) We have,

$\sec 50º \sin 40º + \cos 40º \operatorname{cosec} 50º$

$= \sec(90º – 40º) \sin 40º + \cos 40º \operatorname{cosec}(90º – 40º)$

$= \operatorname{cosec} 40º \sin 40º + \cos 40º \sec 40º$

$= \dfrac{\sin 40^\circ}{\sin 40^\circ} + \dfrac{\cos 40^\circ}{\cos 40^\circ} = 1 + 1 = 2$

Ex.4 If $\tan 2\theta = \cot(\theta + 6^\circ)$, where 2θ and $\theta + 6^\circ$ are acute angles, find the value of θ.

Sol. We have,

$\tan 2\theta = \cot(\theta + 6^\circ)$

$\Rightarrow \cot(90^\circ - 2\theta) = \cot(\theta + 6^\circ)$

$\Rightarrow 90^\circ - 2\theta = \theta + 6^\circ \Rightarrow 3\theta = 84^\circ$

$\Rightarrow \theta = 28^\circ$