COORDINATE GEOMETRY

AREA OF TRIANGLE

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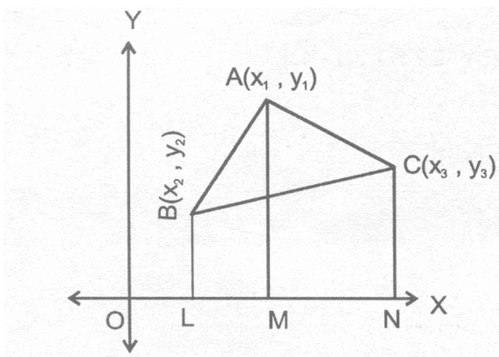
Let ABC be any triangle whose vertices are $A(x_1, y_1)$ $B(x_2, y_3)$. Draw BL, AM and CN perpendicular from B,A and C respectively, to the X-axis. ABLM, AMNC and BLNC are all trapeziums.



Area of ΔABC = Area of trapezium ABLM + Area of trapezium AMNC - Area of trapezium BLNC We know that, Area of trapezium $=\frac{1}{2}$ (Sum of parallel sides) (distance b/w them)

Therefore

Area of $\Delta ABC = \frac{1}{2}(BL + AM)(LM) + \frac{1}{2}(AM + CN) MN - \frac{1}{2}(BL + CN)(LN)$

Area of $\Delta ABC = \frac{1}{2}(y_2 + y_1) x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$

Area of $\Delta ABC = \frac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y) + x_3(y_1 - y_2)]\right|$

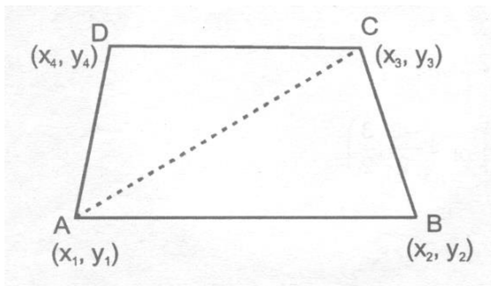
(a) Condition for collinearity :

Three points **A (x_1, y_1) B(x_2, y_2) and C(x_3, y_3)** are collinear if **Area of ΔABC = 0.**

AREA OF QUADRILATERAL :

Let the vertices of Quadrilateral ABCD are **$A(x_1,y_1)$, $B(x_2,y_2$, $C(x_3,y_3)$** and **$D(x_4, y_4)$**

So, Area of quadrilateral ABCD = Area of Δ ABC + Area of Δ ACD



Ex.1 The vertices of Δ ABC are (-2, 1), (5, 4) and (2, -3) respectively. Find the area of triangle.

Sol. A(-2, 1), B(-2, 1) and C(2, -3) be the vertices of triangle.

So, x_1 = - 2, y_1 = 1 ; x_2 = 5, y_2 = 4; x_3 = 2 y_3 = -3

Area of Δ ABC $=\frac{1}{2}|[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$=\frac{1}{2}|[(-2)(4+3)+(5)(-3-1)+2(1-4)]$

$=\frac{1}{2}|[-14+(-20)+(-6)]$

$=\frac{1}{2}|-40|$

= 20 Sq. unit.

Ex.2 The area of a triangle is 5. Two of its vertices area (2, 1) and (3, -2). The third vertex lies on y = x + 3. Find the third vertex.

Sol. Let the third vertex be (x_3, y_3) area of triangle

$=\frac{1}{2}|[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

As $x_1 = 2$ $y_1 = 1$; $x_2 = 3$, $y_2 = -2$; Area of $\Delta = 5$ sq. unit

$\Rightarrow$ $5 = \frac{1}{2}|2(-2-y_3)+3(y_3-1)+x_3(1+2)|$

$\Rightarrow$ $10 = |3x_3 + y_3 - 7|$

$\Rightarrow$ $3x_3 + y_3 - 7 = \pm 10$

Taking positive sign

$3x_3 + y_3 - 7 = 10$ $\Rightarrow$ $3x\therefore + y_3 = 17$ .....(i)

Taking negative sing

$\Rightarrow$ $3x_3 + y_3 - 7 = -10$

$\Rightarrow$ $3x\therefore + y_3 = -3$ .....(ii)

Given that (x_3, y_3) lies on $y = x + 3$

So, $-x\therefore + y_3 = 3$ ....(iii)

Solving eq. (i) & (iii)

$x_3 = \frac{7}{2}$, $y_3 = \frac{13}{2}$

Solving eq. (ii) & (iii)

$x_3 = \frac{-3}{2}$, $y_3 = \frac{3}{2}$

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

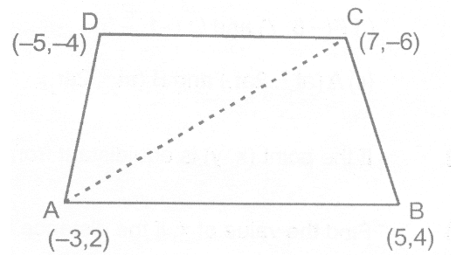
Ex.3 Find the area of quadrilateral whose vertices, taken in order, are (-3, 2), B(5, 4), (7, -6) and D(-5, -4).

Sol. Area of quadrilateral = Area of ΔABC + Area of Δ ACD

So, Area of ΔABC $= \frac{1}{2}|(-3)(4+6)+5(-6-2)+7(2-4)|$

$= \frac{1}{2}|-30-40-14|$

$= \frac{1}{2}|-84| = 42$ Sq unit



Area of Δ ACD = $\frac{1}{2}|-3(-6+4)+7(-4-2)+(-5)(2+6)|$

$= \frac{1}{2}|+6-42-40| = \frac{1}{2}|-76| = 38$ Sq unit

So, Area of quadrilateral ABCD = 42 + 38 = 80 Sq. units.