TRIANGLE

BPT OR THALES THEOREM

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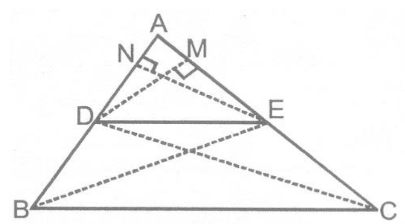
Statement : If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, then the other two sides are divided in the same ratio.

Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD and draw DM $\perp$AC and EN $\perp$AB.

Proof : Area of ΔADE $(= \frac{1}{2}$ base $\times$ height$) = \frac{1}{2} AD \times EN$.

Area of ΔADE is denoted as are (ADE)

So, $ar(ADE) = \frac{1}{2} DB \times EN$

And $ar(BDE) = \frac{1}{2} DB \times EN$,

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} AD \times EN}{\frac{1}{2} DB \times EN} = \frac{AD}{DB}$ ....(i)

Similarly, $ar(ADE = \frac{1}{2} AE \times DM$ and $ar(DEC = \frac{1}{2} EC \times DM$.

And $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} EC \times DM} = \frac{AE}{EC}$ ......(ii)

Note that Δ BDE and Δ DEC are on the same base DE and between the two parallel lines BC and DE.

So, ar(BDE) = ar(DEC) .....(iii)

Therefore, from (i), (ii) and (iii), we have :

$\frac{AD}{DB} = \frac{AE}{EC}$ **Hence Proved.**

COROLLARY

If in a ΔABC, a line DE || BC, intersects AB in D and AC in E, then

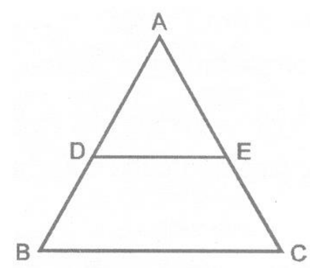
(i) $\frac{DB}{AD} = \frac{EC}{AE}$

(ii) $\frac{AB}{AD} = \frac{AC}{AE}$

(iii) $\frac{AD}{AB} = \frac{AE}{AC}$

(iv) $\frac{AB}{DB} = \frac{AC}{EC}$

(v) $\frac{DB}{AB} = \frac{EC}{AC}$



Ex.1: In DABC, DE is parallel to BC and intersects AB and AC at D and E respectively, then

(i) $\frac{AB}{DC} = \frac{AC}{EC}$ (ii) $\frac{AB}{AD} = \frac{AC}{AE}$

Sol. **Proof :** (i) By proportionality Theorem $\frac{AD}{DB} = \frac{AE}{EC}$ On adding 1 to both sides

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \frac{AD+DB}{DB}=\frac{AE+EC}{EC}$$

$$\Rightarrow \frac{AB}{DB}=\frac{AC}{EC}$$

(ii) $\frac{AD}{DB}=\frac{AE}{EC}$ (By basic proportionality Theorem)

Taking inverse and then adding 1 to both sides

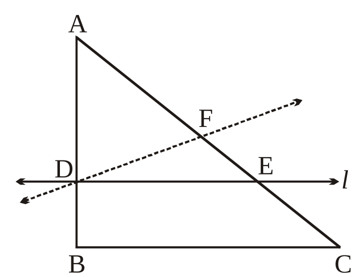
$$1+\frac{DB}{AD}=1+\frac{EC}{AE}$$

$$\frac{AD+DB}{AD}=\frac{AE+EC}{AE}$$

$$\frac{AB}{AD}=\frac{AC}{AE}$$

CONVERSE OF BASIC PROPORTIONALITY THEOREM :

Statement : If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third line.



Given : A triangle ABC and line l intersecting AB at D and AC at E such that $\frac{AD}{DB}=\frac{AE}{EC}$.

To Prove : DE || BC

Proof : Let us suppose that DE is not parallel to BC. Then, through D there must be some other line DF (let) parallel to BC. since DF || BC, by basic proportionality theorem, we get

$$\frac{AD}{DB}=\frac{AF}{FC} \qquad .....(1)$$

But $\frac{AD}{DB}=\frac{AE}{EC}$ (given) .....(2)

From (1) and (2), $\frac{AF}{FC}=\frac{AE}{EC}$ On adding 1 to both sides

$$\frac{AF}{FC}+1=\frac{AE}{EC}+1$$

$$\frac{AF+FC}{FC}=\frac{AE+EC}{EC}$$

$$\frac{AC}{FC}=\frac{AC}{EC}$$

Hence, FC = EC

But this is impossible unless the points F and E coincide, i.e. DF and DE are coincident lines.

Hence, DE || BC.

Ex.2 LM || AB. If AL = x – 3, AC = 2x, BM = x – 2, BC = 2x + 3. find the value of x?

Sol. In D ABC we have

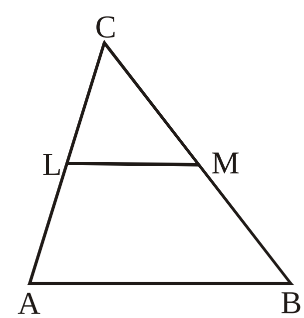
LM || AB

$$\frac{AL}{LC}=\frac{BM}{MC}$$

$$\frac{AL}{AC-AL}=\frac{BM}{BC-BM}$$

$$\frac{x-3}{2x-(x-3)}=\frac{x-2}{(2x+3)-(x-2)}$$

$$\frac{x-3}{x+3}=\frac{x-2}{x+5}$$

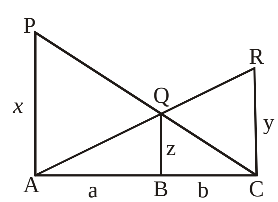


(x – 3)(x + 5) = (x – 2)(x + 3)

$x^2 + 2x - 15 = x^2 + x - 6$

x = 9

Ex.3 In the given figure PA, QB and RC each is perpendicular to AC such that PA = x, RC = y, QB = z, AB = a and BC = b. Prove that $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$



Sol. PA ⊥ AC and QB ⊥ AC

$\Rightarrow$ QB || PA

Thus in D PAC, QB || PA

so Δ QBC ~ Δ PAC

$$\frac{QB}{PA}=\frac{BC}{AC}$$

$\Rightarrow \quad \frac{z}{x}=\frac{b}{a+b}$ ..... (i) (by the property of similar triangle)

In Δ RAC, QB || RC, so Δ QAB ~ Δ RAC

$$\frac{QB}{RC}=\frac{AB}{AC}$$

$\Rightarrow \quad \frac{z}{y}=\frac{a}{a+b}$ ..... (ii) (by the property of similar triangle)

Now from (i) and (ii) we get

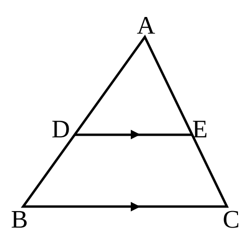
$$\frac{z}{x}+\frac{z}{y}=\left(\frac{b}{a+b}+\frac{a}{a+b}\right)=1$$

$$\Rightarrow \frac{z}{x}+\frac{z}{y}=1$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$

Ex.4 D and E are points on the sides AB and AC respectively of a ΔABC such that DE || BC.

Find the value of x, when



(i) AD = 4 cm, DB = (x – 4) cm, AE = 8 cm and EC = (3x – 19) cm

(ii) AD = (7x – 4) cm, AE = (5x – 2) cm, DB = (3x + 4) cm and EC = 3x cm.

Sol. (i) In ΔABC, DE || BC

$\therefore \frac{AD}{DB}=\frac{AE}{EC}$ (By thales theorem)

$$\frac{4}{x-4} = \frac{8}{3x-19}$$

4(3x – 19) = 8(x – 4)

12x – 76 = 8x – 32

4x = 44

x = 17

(ii) In ΔABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By thales theorem)

$$\frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$

$21x^2 - 12x = 15x^2 - 6x + 20x - 8$

$6x^2 - 26x + 8 = 0$

$3x^2 - 13x + 4 = 0$

$(x - 4)(3x - 1) = 0 \Rightarrow x = 4, 1/3$