ARITHMETIC PROGRESSION

INTRODUCTION OF ARITHMETIC PROGRESSION

INTRODUCTION :

In practical life, we must have observed many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the grains on a maize cob, the spirals on a pineapple on a pipe cone.

In our day-to-day life we see patterns of geometric figures on clothes, picture, posters. They make the learners motivated to form such new pattern.

For example, $\boxed{\rightarrow}$ $\boxed{\leftarrow}$ $\boxed{\rightarrow}$ $\boxed{\leftarrow}$ $\boxed{?}$

Solution:

Likewise number patterns are also faced by learners. In their study, number pattern play an important role in the field of mathematics.

Ex. (i) 2, 4, 6, 8, 10 then next number is 12.

(ii) $4, \frac{1}{2}, \frac{1}{16}, \frac{1}{128}$ next number is . $\frac{1}{1024}$

Idea on A.P. was given by mathematician Carl Friedrich Gauss, who was a young boy, stunned his teacher by adding up

$1 + 2 + 3 + ...+ 99 + 100$ within a few minutes. Here's how he did it:

He realised that adding the first and last numbers, 1 and 100, gives, 101; and adding the second and second last numbers, 2 and 99, gives 101, as well as $3 + 98 = 101$ so on .

Thus he concluded that there are 50 sets of 101. So the series is :

$$50(1 + 100) = 5050.$$

In this chapter, you will study only Arithmetic Progression (A.P.) and Geometric Progression (G.P.).

SEQUENCE :

The number patterns or arrangement of numbers according to definite rule or a set of rules is called a Sequence.

The various numbers occurring in a sequence are called its terms. The n^{th} term of the sequence is denoted by x_n. The n^{th} term is also called the general term of the sequence.

For example,

(i) The number represent a sequence written according to the rule $x_n = n^2, n \in N$.

(ii) The number represent a sequence written according to the rule $x_n = 2n - 1$, $n \in N$.

(iii) The numbers represent a sequence of prime numbers.

In every sequence it is not always possible to write a specific formula.

Sequence is a set of terms which may be real, complex and an algebric expression arranged in a define order according to certain rule.

Ex. 1, 2, 3............

$(x + 1), (2x + 2), (3x + 3),$............

SERIES :

If is a sequence, then the expression $x_1 + x_2 + x_3 + ...$ is called the series associated with the given sequence.

PROGRESSION :

Those sequence whose terms follow certain patterns are called Progressions.

In this chapter, you will study two types of progressions

(i) Arithmetic Progression (A.P.)

(ii) Geometric Progression (G.P.)

ARITHMETIC PROGRESSION (A.P.) :

A sequence is called an arithmetic progression, if the difference of a term and previous term is always same.

The difference is called the common difference of arithmetic progression.

The sequence is called an arithmetic progression (A.P.),

if $d = x_2 - x_1 = x_3 - x_2 = x_n - x_{n-1} = ...$

It is a sequence whose terms decrease or increase by a fix/constant number. This constant number is called common difference of A.P. and it generally denoted by 'd'.

$$[d = a_{n+1} - a_n]$$

If 'a' is the first term and 'd' is the common difference, then an AP can be written as

$$a + (a + d) + (a + 2d) + (a + 3d) + ...$$

Note: If a,b,c, are in AP $\Leftrightarrow 2b = a + c$

Ex.1 Write the first three terms in each of the sequences defined by the following -

(i) $a_n = 3n + 2$ (ii) $a_n = n^2 + 1$

Sol.(i) We have, $a_n = 3n + 2$

Putting n = 1, 2 and 3, we get

$a_1 = 3 \times 1 + 2 = 3 + 2 = 5,$

$a_2 = 3 \times 2 + 2 = 6 + 2 = 8,$

$a_3 = 3 \times 3 + 2 = 9 + 2 = 11$

Thus, the required first three terms of the sequence defined by $a_n = 3n + 2$ are 5, 8, and 11.

(ii) We have,

$a_n = n^2 + 1$

Putting n = 1, 2, and 3 we get

$a_1 = 1^2 + 1 = 1 + 1 = 2$

$a_2 = 2^2 + 1 = 4 + 1 = 5$

$a_3 = 3^2 + 1 = 9 + 1 = 10$

Thus, the first three terms of the sequence defined by $a_n = n^2 + 1$ are 2, 5 and 10.

Ex.2 Write the first five terms of the sequence defined by $a_n = (-1)^{n-1} . 2^n$

Sol. $a_n = (-1)^{n-1} \times 2^n$

Putting n = 1, 2, 3, 4, and 5 we get

$a_1 = (-1)^{1-1} \times 2^1 = (-1)^0 \times 2 = 2$

$a_2 = (-1)^{2-1} \times 2^2 = (-1)^1 \times 4 = -4$

$a_3 = (-1)^{3-1} \times 2^3 = (-1)^2 \times 8 \times 8$

$a_4 = (-1)^{4-1} \times 2^4 = (-1)^3 \times 16 = -16$

$a_5 = (-1)^{5-1} \times 2^5 = (-1)^4 \times 32 = 32$

Thus the first five term of the sequence are 2, –4, 8, –16, 32.

Ex.3 Find the common difference of the following A.P. : 1,4,7,10,13,16 ......

Sol. 4 - 1 = 7 - 4 = 10 - 7 = 13 - 10 = 16 - 13 = 3 (constant).

$\therefore$ Common difference (d) = 3.

EX. 4 Find the A.P. whose 1st term is 2 & common difference is 3.

Sol. Given : First term (a) = 2 & Common difference (d) = 3.

A.P. is 2, 5, 8, 11, 14,.......

Remarks:

The common difference ‘d’ should be independent of n.