QUADRATIC EQUATION

Solution of Quadratic Equation by Quadratic Formula

Solving a quadratic equation by using quadratic formula Procedure

Let $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, be the given quadratic equation.

Find (discriminant) $D = b^2 - 4ac$. Three cases arise :

Case I. If $D > 0$, then the given quadratic equation has two real and different roots. These roots are given by

$$x = \frac{-b \pm \sqrt{D}}{2a} \text{ i.e. } x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Case II. If $D = 0$, then the given quadratic equation has two equal real roots i.e. it has one real root which is repeated twice. This repeated root is given by $x = -\frac{b}{2a}$.

Case III. If $D < 0$, then the given equation has no real roots. Since we are concerned only with real roots, so in this case we may say that the given equation has no roots

Ex. 1 Solve the quadratic equation $x^2 - 7x - 5 = 0$.

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, we find that $a = 1$, $b = -7$ and $c = -5$.

Therefore, $D = (-7)^2 - 4 \times 1 \times (-5) = 49 + 20 = 69 > 0$

Since D is positive, the equation has two roots given by $\frac{7+\sqrt{69}}{2}, \frac{7-\sqrt{69}}{2}$

$\Rightarrow$ $x = \frac{7+\sqrt{69}}{2}, \frac{7-\sqrt{69}}{2}$ are the required solutions.

Ex. 2 For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square.

Sol. The given equation is a perfect square, if its discriminate is zero

i.e. $(2k + 4)^2 - 4(4 - k)(8k + 1) = 0$

$\Rightarrow \quad 4(k + 2)^2 - 4(4 - k)(8k + 1) = 0 \Rightarrow 4[4(k + 2)^2 - (4 - k)(8k + 1)] = 0$

$\Rightarrow \quad [(k^2 + 4k + 4) - (-8k^2 + 31k + 4)] = 0 \Rightarrow 9k^2 - 27k = 0$

$\Rightarrow \quad 9k(k - 3) = 0 \Rightarrow k = 0 \text{ or } k = 3$

Hence, the given equation is a perfect square, if k = 0 or k = 3.

Ex. 3 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, show that $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$.

Sol. Since the roots of the given equations are equal, so discriminant will be equal to zero.

$\Rightarrow \quad b^2(c - a)^2 - 4a(b - c) . c(a - b) = 0$

$\Rightarrow \quad b^2(c^2 + a^2 - 2ac) - 4ac(ba - ca - b^2 + bc) = 0,$

$\Rightarrow \quad a^2b^2 + b^2c^2 + 4a^2c^2 + 2b^2ac - 4ac^2bc - 4abc^2 = 0 \Rightarrow (ab + bc - 2ac)^2 = 0$

$\Rightarrow \quad ab + bc - 2ac = 0$

$\Rightarrow \quad ab + bc = 2ac$

$\Rightarrow \quad \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c}.$

Ex. 4 If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

Sol. If the roots of the given equation are equal, then discriminant is zero i.e.

$(c - a)^2 - 4(b - c)(a - b) = 0 \Rightarrow c^2 + a^2 - 2ac + 4b^2 - 4ab + 4ac - 4bc = 0$

$\Rightarrow \quad c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow \quad (c + a - 2b)^2 = 0$

$\Rightarrow \quad c + a = 2b$