Quadratic Equations

Problem Based on Quadratic Equation

Applications of quadratic equations

Quadratic equations are used to solve a number of applied problems that arise in our daily life. Due to the
wide variety of applied problems, there is no single technique that works in all cases. However, the following
general suggestions should prove helpful:

(i) Read the statement of the problem carefully and determine what quantity (or quantities) must be found.

(ii) Represent the unknown quantity (or quantities) by a variable (or variables).

(iii) Identify the relationships existing in the problem and determine which expressions are equal and write an equation (or equations).

(iv) Form the quadratic equation and solve the resulting equation.

(v) Interpret the solution of the equation.

Note. Check the answers obtained, by determining whether they fulfill the conditions of the original problem. It may happen that out of the two roots of the quadratic equation, only one satisfies the conditions of the problem, reject the other root.

ALGORITHM

The method of problem solving consist of the following three steps :

Step (i) Translating the word problem into symbolic language (mathematical statement) which means identifying relationship existing in the problem and then forming the quadratic equation.

Step (ii) Solving the quadratic equation thus formed.

Step (iii) Interpreting the solution of the equation, which means translating the result of mathematical statement into verbal language.

REMARKS

★ Two consecutive odd natural numbers be 2x - 1, 2x + 1 where $x \in N$

★ Two consecutive even natural numbers be 2x, 2x + 2 where $x \in N$

★ Two consecutive even positive integers be 2x, 2x + 2 where $x \in Z^+$

★ Consecutive multiples of 5 be 5x, 5x + 5, 5x + 10 .....

Ex. 1 The sum of the squares of two consecutive positive integers is 545. Find the integers.

Sol. Let x be one of the positive integers. Then the other integer is x + 1, $x \in Z^+$

Since the sum of the squares of the integers is 545, we get

$$x^2 + (x + 1)^2 = 545$$

or $2x^2 + 2x - 544 = 0$

or $x^2 + x - 272 = 0$

$$x^2 + 17x - 16x - 272 = 0$$

or $x(x + 17) - 16(x + 17) = 0$

or $(x - 16)(x + 17) = 0$

Here, x = 16 or x = -17 But, x is a positive integer. Therefore, reject x = - 17 and take **x = 16.** Hence, two consecutive positive integers are 16 and (16 + 1), i.e., 16 and 17.

Ex. 2 The length of a hall is 5 m more than its breath. If the area of the floor of the hall is 84 m^2, what are the length and the breadth of the hall ?

Sol. Let the breadth of the hall be x metres.

Then the length of the ball is (x + 5) metres.

The area of the floor = $x(x + 5)$ m^2

Therefore, $x(x + 5) = 84$

or $x^2 + 5x - 84 = 0$

or $(x + 12)(x - 7) = 0$

This given $x = 7$ or $x = -12$.

Since, the breadth of the hall cannot be negative, we reject $x = -12$ and take

$x = 7$ only.

Thus, breadth of the hall = 7 metres, and length of the hall = (7 + 5), i.e., 12 metres.

Ex. 3 Out of group of swans $\frac{7}{2}$ times the square root of the total number are playing on the shore of a tank. The two remaining ones are playing, in deep water. What is the total number of swans ?

Sol. Let us denote the number of swans by x.

Then, the number of swans playing on the shore of the tank $= \frac{7}{2}\sqrt{x}$.

There are two remaining swans.

Therefore, $x = \frac{7}{2}\sqrt{x} + 2$

or $x - 2 = \frac{7}{2}\sqrt{x}$

or $(x-2)^2 = \left(\frac{7}{2}\right)^2 x$

or $4(x^2 - 4x + 4) = 49x$

or $4x^2 - 65x + 16 = 0$

or $4x^2 - 64x - x + 16 = 0$

or $4x(x - 16) - 1(x - 16) = 0$

or $(x - 16)(4x - 1) = 0$

This gives $x = 16$ or $x = \frac{1}{4}$

We reject $x = \frac{1}{4}$ and take $x = 16$.

Hence, the total number of swans is 16.

Ex. 4 The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol. Let the length of the shorter side b x cm. Then, the length of the longer side = (x + 5) cm. Since the triangle is right-angled, the sum of the squares of the sides must be equal to the square of the hypotenuse (Pythagoras Theorem).

$x^2 + (x + 5)^2 = 235^2$

or $x^2 + x^2 + 10x + 25 = 625$

or $2x^2 + 10x - 600 = 0$

or $x^2 + 5x - 300 = 0$

or $(x + 20)(x - 15) = 0$

This gives $x = 15$ or $x = -20$

We reject $x = -20$ and take $x = 15$.

Thus, length of shorter side = 15 cm.

Length of longer side = (15 + 5) cm, i.e., 20 cm.