POLYNOMIALS

DIVISION ALGORITHM OF POLYNOMIALS

WORKING RULE TO DIVIDE A POLYNOMIAL BY ANOTHER POLYNOMIAL

Step 1: First arrange the term of dividend and the divisor in the decreasing order of their degrees.

Step 2 : To obtain the first term of quotient divide the highest degree term of the dividend by thehighest degree term of the divisor.

Step 3 : To obtain the second term of the quotient, divide the highest degree term of the new dividend obtained as remainder by the highest degree term of the divisor.

Step 4 : Continue this process till the degree of remainder is less than the degree of divisor.

Division Algorithm for Polynomial

If p(x) and g(x) are any two polynomials with $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that $p(x) = q(x) \times g(x) + r(x)$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

The result is called Division Algorithm for polynomials.

Dividend = Quotient × Divisor + Remainder

Ex.1 Divide $3x^3 + 16x^2 + 21x + 20$ by $x + 4$.

Sol.

Quotient: $3x^2 + 4x + 5$

$x+4 \,\overline{)\, 3x^3 + 16x^2 + 21x + 20}$ First term of q(x) $= \dfrac{3x^3}{x} = 3x^2$

$3x^3 + 12x^2$

$- \quad -$

$4x^2 + 21x + 20$ Second term of q(x) $= \dfrac{4x^2}{x} = 4x$

$4x^2 + 16x$

$- \quad -$

$5x + 20$ Third term of q(x) $= \dfrac{5x}{x} = 5$

$5x + 20$

$- \quad -$

0

Quotient = $3x^2 + 4x + 5$

Remainder = 0

Ex.2 Apply the division algorithm to find the quotient and remainder on dividing p(x) by g(x) as given below :

$p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

Sol. We have,

$p(x) = x^3 - 3x^2 + 5x - 3$ and $g(x) = x^2 - 2$

$$\begin{array}{r|l} & x - 3 \\ \hline x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\ & x^3 \qquad\ - 2x \\ & - \qquad\quad + \\ \hline & - 3x^2 + 7x - 3 \\ & - 3x^2 \qquad + 6 \\ & + \qquad\quad - \\ \hline & \qquad 7x - 9 \end{array}$$

First term of quotient is $\frac{x^3}{x^2} = x$

Second term of quotient is $\frac{-3x^2}{x^2} = -3$

We stop here since

degree of $(7x - 9)$ < degree of $(x^2 - 2)$

So, quotient = $x - 3$, remainder = $7x - 9$

Therefore,

Quotient × Divisor + Remainder

$= (x - 3)(x^2 - 2) + 7x - 9$

$= x^3 - 2x - 3x^2 + 6 + 7x - 9$

$= x^3 - 3x^2 + 5x - 3$ = Dividend

Therefore, the division algorithm is verified.

Ex.3 On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

Sol. $p(x) = x^3 - 3x^2 + x + 2$

$q(x) = x - 2$ and $r(x) = -2x + 4$

By Division Algorithm, we know that

$p(x) = q(x) \times g(x) + r(x)$

Therefore,

$x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$\Rightarrow x^3 - 3x^2 + x + 2 + 2x - 4 = (x - 2) \times g(x)$

$\Rightarrow g(x) = \dfrac{x^3 - 3x^2 + 3x - 2}{x - 2}$

On dividing $x^3 - 3x^2 + 3x - 2$ by $x - 2$,

we get g(x)

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\ \hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\ \hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\ \hline
 & 0
\end{array}
$$

First term of quotient is $\dfrac{x^3}{x} = x$

Second term of quotient is $\dfrac{-x^2}{x} = -x$

Third term of quotient is $\dfrac{x}{x} = 1$

Hence, $g(x) = x^2 - x + 1$.

Ex.4 If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k & a.

Sol.

$$
\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + x^2k \\
 & - \quad + \quad - \\ \hline
 & -4x^3 + x^2(16 - k) - 25x + 10 \\
 & -4x^3 + x^2(8) - 4xk \\
 & + \quad - \quad + \\ \hline
 & x^2[8 - k] + x[4k - 25] + 10 \\
 & x^2[8 - k] - 2x[8 - k] + k(8 - k) \\
 & - \quad + \quad - \\ \hline
 & x[4k - 25 + 16 - 2k] + 10 - 8k + k^2
\end{array}
$$

According to questions, remainder is x + a

$\therefore$ coefficient of x = 1

$\Rightarrow 2k - 9 = 1$

$\Rightarrow k = (10/2) = 5$

Also constant term = a

$\Rightarrow$ $k^2 - 8k + 10 = a$ Þ $(5)^2 - 8(5) + 10 = a$

$\Rightarrow$ $a = 25 - 40 + 10$

$\Rightarrow$ $a = -5$

$\therefore$ $k = 5, a = -5$