POLYNOMIALS

BASIC INTRODUCTION OF POLYNOMIALS

INTRODUCTION :

Algebra is that branch of mathematics which treats the relation of numbers.

CONSTANT AND VARIABLE:

In algebra, two types of symbols are used: constants and variable (literals).

Constant :

It is a symbol whose value always remains the same, whatever the situation be.

For example: 5, -9, $\frac{3}{8}$, π , $\frac{7}{15}$, etc.

Variable :

It is a symbol whose value changes according to the situation.

For example : x, y, z, ax, a + x, 5y, -7x, etc.

Algebraic Expressions :

- (a) An algebraic expression is a collection of terms separated by plus (+) or minus (-) sign. For example : 3x + 5y, 7y 2x, 2x ay + az, etc.
- (b) The various parts of an algebraic expression that are separated by '+' or '-' sign are called terms.

SN.	ALGEBRAIC EXPRESSION	NO. OF TERMS	TERMS
(1)	-32x	1	-32x
(2)	2x+3y	2	2x and 3y
(3)	Ax-5y+cz	3	Ax,-5y and cz
(4)	$\frac{3}{x} + \frac{y}{7} - \frac{xy}{8} + 9$	4	$\frac{3}{x}, \frac{y}{7}, -\frac{xy}{8}$ and 9

For example :

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Types of Algebraic Expressions :

(i) Monomial:

An algebraic expression having only one term is called a monomial.

For ex. 8y, -7xy, $4x^2$, abx, etc. 'mono' means 'one'.

(ii) Binomial :

An algebraic expression having two terms is called a binomial.

For ex. 8x + 3y, 8x + 3, 8 + 3y, a + bz, 9 - 4y, $2x^2 - 4z$, $6y^2 - 5y$, etc. 'bi' means 'two'.

(iii) Trinomial:

An algebraic expression having three terms is called a trinomial.

For ex. ax - 5y + 8z, $3x^2 + 4x + 7$, $9y^2 - 3y + 2x$, etc. 'tri means 'three'.

(iv) Multinomial :

An algebraic expression having two or more terms is called a multinomial.

FACTOR AND COEFFICIENT :

Factor :

Each combination of the constants and variables, which form a term, is called a factor.

For examples :

(i) 7, x and 7x are factors of 7x, in which7 is constant (numerical) factor and x is variable (literal) factor.

(ii) In $-5x^2y$, the numerical factor is -5 and literal factors are : x, y, xy, x^2 and x^2y .

Coefficient :

Any factor of a term is called the coefficient of the remaining term.

For example :

- (i) In 7x; 7 is coefficient of x
- (ii) In $-5x^2y$; 5 is coefficient of $-x^2y$; -5 is coefficient of x^2y .

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Ex. 1 Write the coefficient of :

- (i) $x^2 in 3x^3 5x^2 + 7$
- (ii) xy in 8xyz
- (iii) $-y \text{ in } 2y^2 6y + 2$
- (iv) $x^0 in 3x + 7$

- (ii) 8z
- (iii) 6
- (iv) Since $x^0 = 1$, Therefore

$$3x + 7 = 3x + 7x^0$$

coefficient of x^0 is 7.

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An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$; where a_0 , a_1, a_2, \dots, a_n are real numbers and all the indices of variable x are non-negative integers, is called a polynomial in variable x and the highest indices n is called the degree of the polynomial, if $a_n^{-1} 0$. Here, a_0, a_1x, a_2x^2, \dots and a_nx^n are called the terms of the polynomial and $a_0, a_1, a_2, \dots, a_n$ are called various coefficients of the polynomial f(x). A polynomial in x is said to be in standard form when the terms are written either in increasing order or in decreasing order of the indices of x in various terms.

$$x^{2}-a^{2}$$
, $ax^{2}+bx+c$, $x^{3}+3x^{2}+3x+1$, $y^{3}-7y+6$

etc. are the polynomials written in their standard form.

Ex.2: Which of the following expressions are polynomials?

(1)
$$\frac{1}{x^2} + \frac{1}{x^{-1}} + \frac{1}{2}$$

(2) $\frac{1}{x}(x-1)(x-2)$
(3) $\frac{(x^2+x+1)(x+1)}{(1+x)}$
(4) $x^2 + \frac{1}{x^2}$
Sol: (i) $\frac{1}{x^2} + \frac{1}{x^{-1}} + \frac{1}{2} = x^2 + x^1 + \frac{1}{2}x^0$ [It is a polynomial]
(ii) $\frac{1}{x}(x-1)(x-2) = x^{-1}(x^1-1)(x^1-2)$
 $= x^{-1}(x^2-3x+2)$
 $= x^{-3} + 2x^{-1}$
 $= x^1 - 3x^0 + 2x^{-1}$ [It is not a polynomial]
(iii) $\frac{(x^2+x+1)(x+1)}{(1+x)} = x^2 + x + 1 = x^2 + x^1 + 1 \cdot x^0$ [It is a polynomial]
(iv) $x^2 + \frac{1}{x^2} = x^2 + x^{-2}$ [It is not a polynomial]
Ex.3: Rewrite the following polynomials in the standard form :

(i) $x - 7 + 8x^2 + 9x^3$ (ii) $-5x^2 + 6 - 3x^3 + 4x$

Sol: (i)
$$9x^3 + 8x^2 + x - 7$$
 is the standard form of $x - 7 + 8x^2 + 9x^3$.

(ii) $-3x^3 - 5x^2 + 4x + 6$ is the standard form of $-5x^2 + 6 - 3x^3 + 4x$.