PROBABILITY

NUMERICAL BASED ON COIN AND DICE

Ex.1 A die is thrown, Find the probability of

(i) prime number (ii) multiple of 2 or 3 (iii) a number greater than 3

Sol. In a single throw of die any one of six numbers 1,2,3,4,5,6 can be obtained.

Therefore, the tome number of elementary events associated with the random experiment of throwing a die is 6.

(i) Let A denote the event "Getting a prime no". Clearly, event A occurs if any one of 2,3,5 comes as out come.

$\therefore$ Favorable number of elementary events $= 3$

Hence, P (Getting a prime no.) $= \frac{3}{6} = \frac{1}{2}$

(ii) An multiple of 2 or 3 is obtained if we obtain one of the numbers 2,3,4,6 as out comes

$\therefore$ Favorable number of elementary events $= 4$

Hence, P (Getting multiple of 2 or 3) $= \frac{4}{6} = \frac{2}{3}$

(iii) The event "Getting a number greater than 3" will occur, if we obtain one of number 4,5,6 as an out come.

$\therefore$ Favorable number of out comes $= 3$

Hence, required probability $= \frac{3}{6} = \frac{1}{2}$

Ex.2 Two unbiased coins are tossed simultaneously. Find the probability of getting

(i) two heads (ii) at least one head (iii) at most one head.

Sol. If two unbiased coins are tossed simultaneously, we obtain any one of the following as an out come : HH, HT, TH, TT

$\therefore$ Total number of elementary events $= 4$

(i) Two heads are obtained if elementary event HH occurs.

$\therefore$ Favorable number of events $= 1$

Hence, P (Two heads) $= \frac{1}{4}$

(ii) At least one head is obtained if any one of the following elementary events happen :

HH, HT, TH

$\therefore$ favorable number of events $= 3$

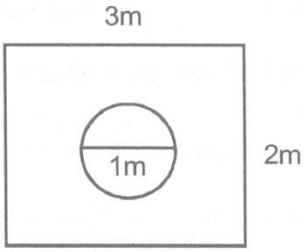
Hence P (At least one head) $= \frac{3}{4}$

(iii) If one of the elementary events HT, TH, TT occurs, than at most one head is obtained

$\therefore$ favorable number of events $= 3$

Hence, P (At most one head) $= \frac{3}{4}$

Ex.3 A die is drop at random on the rectangular region as shown in figure. What is the probability that it will land inside the circle with diameter 1m ?



Sol. Area of rectangular region $= 3\text{m} \times 2\text{m} = 6\text{m}^2$

Area of circle $= \pi r^2$

$$= \pi \times \left(\frac{1}{2}\right)^2$$

$$= \frac{\pi}{4}\text{m}^2$$

$\therefore$ Probability that die will land inside the circle

$$= \frac{\pi/4}{6}$$

$$= \frac{\pi}{24}$$

Ex.4 Two dice are thrown at a time. Find the probability of the following -

(i) these numbers shown are equal;

(ii) the difference of numbers shown is 1.

Sol. The sample space in a throw of two dice

$s = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$.

total no. of cases n (s) = 6 × 6 = 36.

(i) Here E_1 = the event of showing equal number on both dice

= {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) }

∴ n (E_1) = 6

∴ P (E_1) = $\frac{n(E_1)}{n(s)} = \frac{6}{36} = \frac{1}{6}$

(ii) Here E_2 = the event of showing numbers whose difference is 1.

= {(1, 2) (2, 1) (2, 3) (3, 2) (3, 4) (4, 3) (4, 5) (5, 4) (5, 6) (6, 5)}

∴ n (E_2) = 10 ∴p (E_2) = $\frac{n(E_2)}{n(s)} = \frac{10}{36} = \frac{5}{18}$

Ex.5 Three coins are tossed together -

(i) Find the probability of getting exactly two heads,

(ii) Find the probability of getting at least two tails.

Sol. The sample space in tossing three coins

S = (H, T) × (H, T) × (H, T)

∴ Total no. of cases n (s) = 2 × 2 × 2 = 8

(i) Here E_1 = the event of getting exactly two heads

= {HHT, HTH, THH}

∴ n (E_1) = 3 ∴ P (E_1) = $\frac{n(E_1)}{n(s)} = \frac{3}{8}$

(ii) E_2 = {HTT, THT, TTH, TTT}

∴ n (E_2) = 4,

$\therefore \quad P(E_2) = \frac{n(E_2)}{n(s)} = \frac{4}{8} = \frac{1}{2}$

Ex.6 Find the probability of throwing (a) 3, (b) an even number with an ordinary six faced die.

Sol. (a) There are 6 possible ways in which the die can fall and there is only one way of throwing 3.

$\therefore$ The required probability

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}} = \frac{1}{6}$

(b) Total number of outcomes of throwing a die = 6.

Number of outcomes of falling even number i.e. 2, 4, 6 = 3.

The required probability $= \frac{3}{6} = \frac{1}{2}$

Ex.7 Find the probability of getting a number less than 5 in a single throw of a die.

Sol. There are 4 numbers which are less than 5, i.e. 1, 2, 3 and 4.

Number of such favourable outcomes = 4.

$\because$ The number marked on all the faces of a die are 1, 2, 3, 4, 5 or 6

$\therefore$ Total number of possible outcomes = 6

$\therefore$ P(a number less than 5) $= \frac{4}{6} = \frac{2}{3}$

Ex.8 Two coins are tossed simulataneously. Find the probability of getting

(i) two heads

(ii) at least one head

(iii) no head

Sol. Let H denotes head and T denotes tail.

$\therefore$ On tossing two coins simultaneously, all

the possible outcomes are

(i) The probability of getting two heads = P(HH)

$$= \frac{\text{Event of occurence of two heads}}{\text{Total number of possible outcom}} = \frac{1}{4}$$

(ii) The probability of getting at least one head = P(HT or TH or HH)

$$= \frac{\text{Event of occurence of at least one head}}{\text{Total number of possible outcomes}} = \frac{3}{4}$$

(iii) The probability of getting no head = P(TT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcom}} = \frac{1}{4}$$

Ex.9 On tossing three coins at a time, find -

(i) All possible outcomes.

(ii) events of occurence of 3 heads, 2 heads, 1 head and 0 head.

(iii) probabilty of getting 3 heads, 2 heads, 1 head and no head.

Sol. Let H denotes head and T denotes tail. On tossing three coins at a time,

(i) All possible outcomes = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}. These are the 8 possible outcomes.

(ii) An event of occurence of 3 heads = (HHH) = 1

An event of occurence of 2 heads = {HHT, HTH, THH} = 3

An event of occurence of 1 head = {HTT, THT, TTH} = 3

An event of occurence of 0 head = {TTT} =1

(iii) Now, probability of getting 3 heads = P (HHH)

$$= \frac{\text{Event of occurence of 3 heads}}{\text{Total number of possible outcom}} = \frac{1}{8}$$

Simultaneously, probability of getting 2 heads = P(HHT or THH or HTH)

$$= \frac{\text{Event of occurence of 2 heads}}{\text{Total number of possible outcom}} = \frac{3}{8}$$

Probability of getting one head = P (HTT or THT or TTH)

$$= \frac{\text{Event of occurence of 1 head}}{\text{Total number of possible outcom}} = \frac{3}{8}$$

Probability of getting no head = P (TTT)

$$= \frac{\text{Event of occurence of no head}}{\text{Total number of possible outcom}} = \frac{1}{8}$$

Ex.10 Three unbiased coins are tossed together. Find the probability of getting :

(i) All heads, (ii) Two heads

(iii) One head (iv) At least two heads.

Sol. Elementary events associated to random experiment of tossing three coins are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$\therefore$ Total number of elementary events = 8.

(i) The event "Getting all heads" is said to occur, if the elementary event HHH occurs i.e. HHH is an outcome. Therefore,

$\therefore$ Favourable number of elementary events $= 1$

Hence, required probability $= \frac{1}{8}$

(ii) The event "Getting two heads" will occur, if one of the elementary events HHT, THH, HTH occurs.

$\therefore$ Favourable number of elementary events $= 3$

Hence, required probability $= \frac{3}{8}$

(iii) The events of getting one head, when three coins are tossed together, occurs if one of the elementary events HTT, THT, TTH happens.

$\therefore$ Favourable number of elementary events $= 3$

Hence, required probability $= \frac{3}{8}$

(iv) If any of the elementary events HHH, HHT, HTH and THH is an outcome, then we say that the event "Getting at least two heads" occurs.

$\therefore$ Favourable number of elementary events $= 4$

Hence, required probability $= \frac{4}{8} = \frac{1}{2}$

Ex.11 A piggy bank contains hundred 50 p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) will be a 50 p coin ?

(ii) will not be a Rs. 5 coin ?

Sol. Number of 50 Rs coins = 100

Number of 1 Rs coins = 50

Number of 2 Rs coins = 20

Number of 5 Rs coins = 10

(i) The number of favourable outcomes of 50 p coin to fall = 100

Total number of coins = 100 + 50 + 20 + 10 = 180

Total number of possible outcomes = 180

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(50\text{ p}) = \frac{100}{180} = \frac{5}{9}$$

(ii) Number of favourable outcomes of 5 Rs coin to not fall = 180 – 10 = 170

$$P = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{not Rs. 5}) = \frac{170}{180} = \frac{17}{18}$$