SURFACE AREAS AND VOLUMES

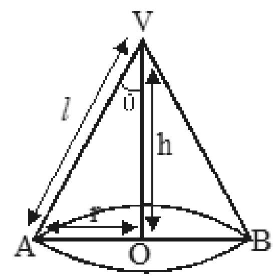
CONE, CUBE, SPHERE AND HEMI SPHERE

Right circular cone

For a right circular cone of height h, slant height *l* and radius of base r, we have,

(i) $l^2 = r^2 + h^2$

(ii) Curved surface area = p r *l* sq units



(iii) Total surface area

= Curved surface area + area of the base

= p r *l* + p r^2

= p r (l+r) sq units

(iv) Volume = p r^2h

Sphere

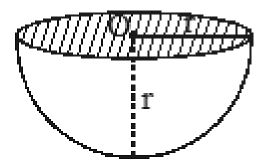
For a sphere of radius r, we have

(i) Surface area = 4 p r^2

(ii) Volume = p r^3

Hemisphere

For hemisphere of radius r, we have -



(i) Surface area = $2 \text{ p } r^2$

(ii) Total surface area = $2 \text{ p } r^2 + \text{p } r^2$

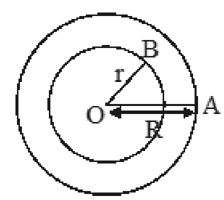
$= 3 \text{ p } r^2$

(iii) Volume = $\text{p } r^3$

Spherical shell

If R and r are respectively the outer and inner radii of a spherical shell, then,

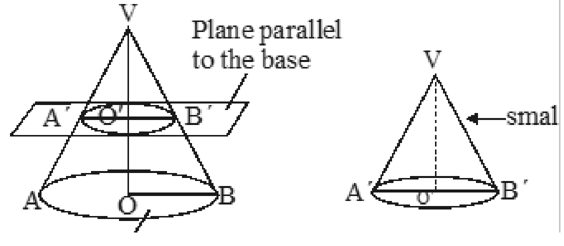
(i) Outer surface area = $4 \text{ p } R^2$



(ii) Volume of material = $\text{p } (R^3 - r^3)$

FRUSTUM OF A RIGHT CIRCULAR CONE

If we take a cone and cut it by a plane parallel to the base of the cone, then the portion between the plane and the base of the cone is called the frustum of the cone.



Height

The height or thickness of a frustum is the perpendicular distance between its two circular bases.

Here, OO′ is the height of the frustum.

OO′ = VO – VO′

The height of the frustum ABB′A′ is equal to the difference between the heights of the cones VAB and VA′B′.

Slant height

The slant height of a frustum of a right circular cone is the length of the line segment Joining the extremities of two parallel radii, drawn in the same direction, of two circular bases.

Slant height of the frustum ABB′A′ = AA′ = BB′

AA′ = VA – VA′ and BB′ = VB – VB′

Thus the slant height of the frustum equals the difference between the slant heights of the cones VAB and VA′ B′.

Volume and surface area of a frustum of a right circular cone

Let h be the height, *l* be the slant height and r_1 and r_2 the radii

of the circular bases of the frustum AB B′ A′ such that $r_1 > r_2$.

Let the height of the cone V AB be h_1 and its slant height be l_1

i.e. VO = h_1 and

VA = VB = l_1.

VA´ = VA –AA´ = $l_1 - l$.

and VO´ = VO – OO´ = h_1 – h.

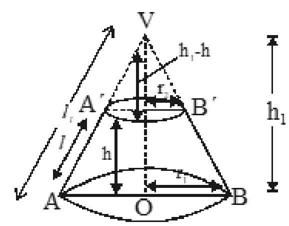
Thus, the volume of the frustum of the cone is given by

Thus, curved surface area of the frustum = $p(r_1+r_2)l$.

Total surface area of the frustum = curved surface area + surface area of circular bases.

$$= p(r_1 + r_2)\, l + pr_1^2 + pr_2^2$$

If h be the height & l be slant height of the frustum and r_1 & r_2 ($r_1 > r_2$) be radii of the two ends then



(a) Base area : Top base area = pr_2^2

Bottom base area = pr_1^2

(b) Slant height of the frustum, l =

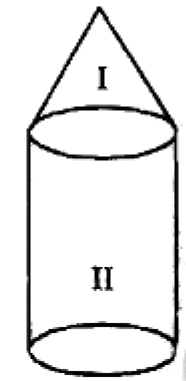
(c) Curved surface area = $p\, l\, (r_1+r_2)$

(d) Total surface area = $pl\,(r_1 + r_2) + pr_1^2 + pr_2^2$

$= p\,[l(r_1 + r_2) + r_1^2 + r_2^2]$

(e) Volume of the frustum = $ph\,(r_1^2 + r_2^2 + r_1 r_2)$

SURFACE AREA AND VOLUME OF COMBINATION OF SOLIDS



In this section we shall find the surface area and volume of solids which are combination of two or more solids.

Consider a circus tent shown in the given figure.

It consists of two parts I and II. Part-I is in the shape of a cone and Part-II is in the form of a cylinder.

$\therefore$ Total surface area of the circus tent = Curved surface area of part-I i.e., cone + Curved Surface area of part-II i.e., Cylinder.

VOLUME OF COMBINATION OF SOLIDS

In the previous section, we have studied about the surface area of solidsmade of two or more solids. There we find that while calculating the surface area of a solid, some surface areas are not included. But, here we will find the total volume of a solid which is also the actual volume of two or more combined solid.

CONVERSION OF SOLID FROM ONE SHAPE TO ANOTHER

Some solids like candle, clay etc. can be changed into any shape. But the volume of the both solid shapes are same. For example, if a candle which is generally in the shape of a cylinder can be changed into any shape, but the volume remains same.

If a solid is transformed into a number of small identical solids of same or a different shape, then

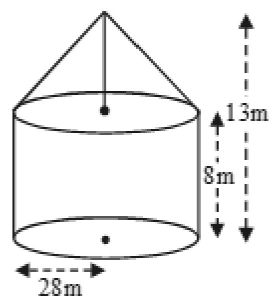
Number of small items $= \dfrac{\text{Volume of larger object}}{\text{Volume of a smaller object}}$

Ex. 1 A circus tent is in the shape of a cylinder, upto a height of 8 m, surmounted by a cone of the same radius 28 m. If the total height of the tent is 13 m, find:

(i) total inner curved surface area of the tent.

(ii) cost of painting its inner surface at the rate of ₹ 3.50 per m^2.

Sol. According to the given statement, the rough sketch of the circus tent will be as shown:



(i) For the cylindrical portion :

$r = 28$ and $h = 8$ m

$\therefore$ Curved surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 28 \times 8 \text{ m}^2 = 1408 \text{ m}^2$

For conical portion :

$r = 28$ m and $h = 13$ m – 8 m = 5 m

$\therefore \lambda^2 = h^2 + r^2 \Rightarrow \lambda^2 = 5^2 + 28^2 = 809$

$\Rightarrow \lambda = \sqrt{809}\text{ m} = 28.4$ m

$\therefore$ Curved surface area $= \pi r\lambda$

$= \frac{22}{7} \times 28 \times 28.4 \text{ m}^2 = 2499.2 \text{ m}^2$

$\therefore$ Total inner curved surface area of the tent.

= C.S.A. of cylindrical portion + C.S.A. of the conical portion

$1408 \text{ m}^2 + 2499.2 \text{ m}^2 = 3907.2 \text{ m}^2$

(ii) Cost of painting the inner surface

$= 3907.2 \times$ ₹ 3.50

$=$ ₹ 13675.20

Ex. 2 A cylinder and a cone have same base area. But the volume of cylinder is twice the volume of cone. Find the ratio between their heights.

Sol. Since, the base areas of the cylinder and the cone are the same.

$\Rightarrow$ their radius are equal (same).

Let the radius of their base be r and their heights be h_1 and h_2 respectively.

Clearly, volume of the cylinder $= \pi r^2 h_1$

and, volume of the cone $= \frac{1}{3}\pi r^2 h_2$

Given :

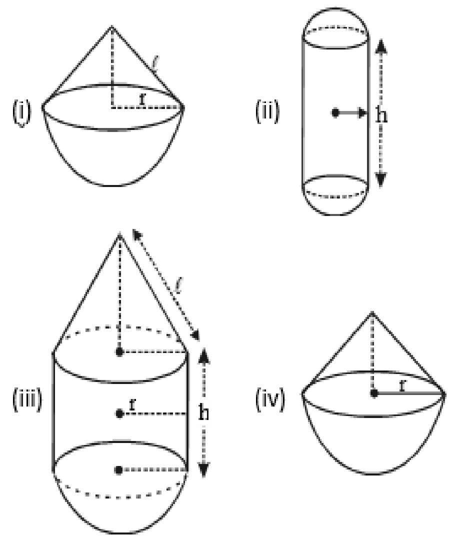
Volume of cylinder $= 2 \times$ volume of cone

$\Rightarrow \pi r^2 h_1 = 2 \times \frac{1}{3}\pi r^2 h_2$

$\Rightarrow h_1 = \frac{2}{3}h_2 \Rightarrow \frac{h_1}{h_2} = \frac{2}{3}$

i.e., $h_1 : h_2 = 2 : 3$

Ex. 3 Find the formula for the total surface area of each figure given bellow :



Sol. (i)Required surface area

$= \text{C.S.A. of the hemisphere} + \text{C.S.A. of the cone}$

$= 2\pi r^2 + \pi r\lambda = \pi r\,(2r + \lambda)$

(ii) Required surface area

$= 2 \times \text{C.S.A. of a hemisphere} + \text{C.S.A. of the cylinder}$

$= 2 \times 2\pi r^2 + 2\pi rh = 2\pi r\,(2r + h)$

(iii) Required surface area

$= \text{C.S.A. of the hemisphere}$

$+ \text{C.S.A. of the cylinder} + \text{C.S.A. of the cone}$

$= 2\pi r^2 + 2\pi rh + \pi r\lambda = \pi r\,(2r + 2h + \lambda)$

(iv) If slant height of the given cone be λ

$= \lambda^2 = h^2 + r^2$

$\Rightarrow \lambda = \sqrt{h^2 + r^2}$

And, required surface area

$= 2\pi r^2 + \pi r\lambda = \pi r\,(2r + \lambda)$

$= \pi r \left(2r + \sqrt{h^2 + r^2}\right)$

Ex. 4 The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Sol. Let the original radius be r.

$\Rightarrow$ Original surface area of the sphere $= 4\pi r^2$

Increase radius = r + 25% of r

$= r + \frac{25}{100}r = \frac{5r}{4}$

$\Rightarrow$ Increased surface area

$= 4\pi\left(\frac{5r}{4}\right)^2 = \frac{25\pi r^2}{4}$

Increased in surface area

$= \frac{25\pi r^2}{4} - 4\pi r^2 \quad = \frac{25\pi r^2 - 16\pi r^2}{4} = \frac{9\pi r^2}{4}$

and, percentage increase in surface area $= \frac{\text{Increase in area}}{\text{Original area}} \times 100\%$

$$= \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100\% = \frac{9}{16} \times 100\% \quad = 56.25\%$$

Alternative Method :

Let original radius = 100

$\Rightarrow$ Original C.S.A. = $\pi(100)^2 = 10000\pi$

Increased radius = 100 + 25% of 100 = 125

$\Rightarrow$ Increased C.S.A. = $\pi(125)^2 = 15625\pi$

Increase in C.S.A. = $15625\pi - 10000\pi \ = 5625\pi$

$\therefore$ Percentage increase in C.S.A.

$$= \frac{\text{Increase in C.S.A}}{\text{Original C.S.A}} \times 100\%$$

$$= \frac{5625\pi}{10000\pi} \times 100\% = 56.25\%$$

If the radius increases by 25%, the diameter also increases by 25%.

Conversely, if diameter decreases by 20%, the radius also decreases by 20%