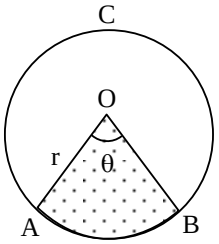
AREAS RELATED TO CIRCLE

SECTOR AND AREA OF SECTOR

SECTOR OF A CIRCLE



Minor sector :

A sector of a circle is called a minor sector if the minor arc of the circle is a part of its boundary In Fig. sector OAB is the minor sector.

Major sector :

A sector of a circle is called a major sector if the major arc of the circle is a part of its boundary. In Fig. sector OACB is the major sector.

Following are some important points to remember:

(i) A minor sector has an angle θ, subtended at the centre of the circle, whereas a major sector has no angle.

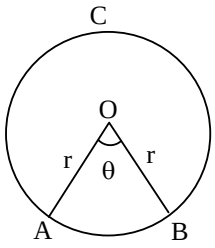
(ii) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(iii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(iv) The boundary of a sector consists of an arc of the circle and the two radii.

AREA OF A SECTOR

If the arc subtends an angle of θ at the centre, then its arc length is $\frac{\theta}{180} \times \pi r$



Hence, the arc length λ of a sector of angle θ in a circle of radius r is given by

$$\lambda = \frac{\theta}{180} \times \pi r \qquad \text{....(i)}$$

If the arc subtends an angle θ, then area of thecorresponding sector is

$$\frac{\pi r^2 \theta}{360}$$

Thus, the area A of a sector of angle θ in a circle of radius r is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{\theta}{360} \times \text{(Area of the circle)} \qquad \text{....(ii)}$$

Some useful results to remember:

(i) Angle described by minute hand in 60 minutes = 360º

Angle described by minute hand in one minute $= \left(\frac{360}{60}\right)^{\circ} = 6º$

Thus, minute hand rotates through an angle of 6º in one minute.

(ii) Angle described by hour hand in 12 hours = 360º

$\therefore$ Angle described by hour hand in one hour

$= \left(\frac{360}{12}\right)^{\circ} = 30º$

Ex.1 A sector is cut from a circle of radius 21 cm. The angle of the sector is 150º.Find the length of its arc and area.

Sol. The arc length l and area A of a sector of angle θ in a circle of radius r are given by

$l = \frac{\theta}{360} \times 2\pi r$ and $A = \frac{\theta}{360} \times \pi r^2$ respectively.

Here, r = 21 cm and $\theta = 150$

$\therefore l = \left\{\frac{150}{360} \times 2 \times \frac{22}{7} \times 21\right\}$ cm = 55 cm

and $A = \left\{\frac{150}{360} \times \frac{22}{7} \times (21)^2\right\} cm^2 = \frac{1155}{2} cm^2$

$= 577.5\ cm^2$

Ex.2 Find the area of the sector of a circle whose radius is 14 cm and angle of sector is 45º.

Sol. We know that the area A of a sector of angle in a circle of radius r is given by

$$A = \frac{\theta}{360} \times \pi r^2$$

Here, $r = 14$ cm and $\theta = 45$

$$\therefore A = \left\{\frac{45}{360} \times \frac{22}{7} \times (14)^2\right\} cm^2$$

$$= \left\{\frac{1}{8} \times \frac{22}{7} \times 14 \times 14\right\} cm^2$$

$$= 77 \text{ cm}^2$$

Ex.3 In Fig. there are shown sectors of two concentric circles of radii 7 cm and 3.5 cm. Find the area of the shaded region. (Use $\pi = 22/7$).

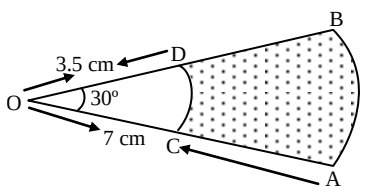
Sol. Let A_1 and A_2 be the areas of sectors OAB and OCD respectively. Then, A_1 = Area of a sector of angle 30º in a circle of radius 7 cm

$$\Rightarrow A_1 = \left\{\frac{30}{360} \times \frac{22}{7} \times 7^2\right\} cm^2$$

$$\left[\text{Using: } A = \frac{\theta}{360} \times \pi r^2\right]$$

$$\Rightarrow A_1 = 77/6 \text{ cm}^2$$

A_2 = Area of a sector of angle 30º in a circle of radius 3.5 cm.



$$\Rightarrow A_2 = \left\{\frac{30}{360}\times\frac{22}{7}\times(3.5)^2\right\}cm^2$$

$$\Rightarrow A_2 = \left\{\frac{1}{12}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right\}cm^2 = \frac{77}{24}cm^2$$

$\therefore$ Area of the shaded region

$$= A_1 - A_2 = \left(\frac{77}{6}-\frac{77}{24}\right)cm^2$$

$$= \frac{77}{24}\times (4-1)\ cm^2 = 77/8\ cm^2 = 9.625\ cm^2$$

Ex.4 A pendulum swings through an angle of 30º and describes an arc 8.8 cm in length. Find the length of the pendulum.

Sol. Here, $\theta = 30º$, l = arc = 8.8 cm

$$\therefore l = \frac{\theta}{360}\times 2\pi r$$

$$\Rightarrow 8.8 = \frac{30}{360}\times 2\times 22/7\times r$$

$$\Rightarrow r = \frac{8.8\times 6\times 7}{22}\ cm = 16.8\ cm$$