AREAS RELATED TO CIRCLE

AREA OF COMBINATION OF PLANE FIGURE

Area of Combination of Plane Figure

Ex.1 The inner and outer diameters of ring I of a dartboard are 32 cm and 34 cm respectively and those of rings II are 19cm and 21 cm respectively. What is the total area of these two rings ?

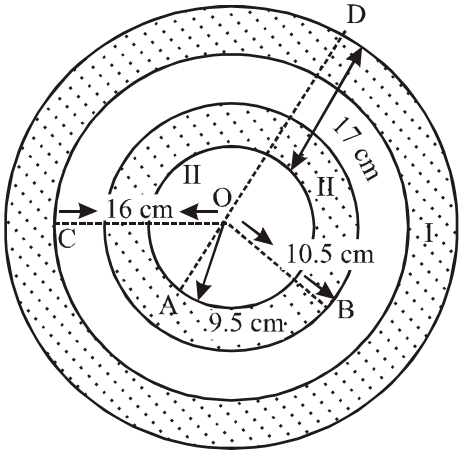
Sol. We have,

Area of ring I = $(\pi \times 17^2 - \pi \times 16^2)$ cm^2

$= \frac{22}{7} \times (17^2 - 16^2)$ cm^2

$= \frac{22}{7} \times (17 + 16)(17 - 16)$ cm^2

$= \frac{22}{7} \times 33$ cm^2



Area of ring II = $(\pi \times 10.5^2 - \pi \times 9.5^2)$ cm^2

$= \pi (10.5^2 - 9.5^2)$ cm^2

$= \frac{22}{7} \times (10.5 + 9.5)(10.5 - 9.5)$ cm^2

$= \frac{22}{7} \times 20$ cm^2

Hence,

Total are a of two rings $= \frac{22}{7} \times 33 + \frac{22}{7} \times 20$ cm^2

$= \frac{22}{7} \times (33 + 20)$ cm^2 = 166.57 cm^2

Ex.2 Find the area of the shaded region in Fig. where radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40^\circ$.

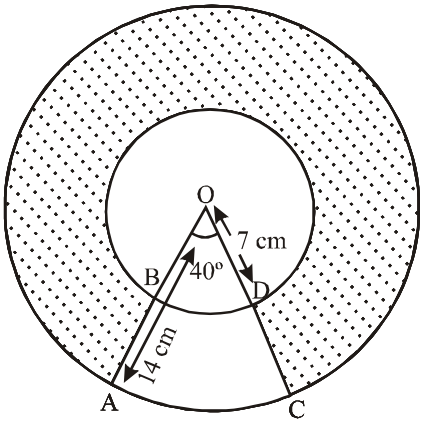
Sol. We have,

Area of the region ABDC = Area of sector AOC – Area of sector BOD

$$= \left(\frac{40}{360}\times\frac{22}{7}\times 14\times 14 - \frac{40}{360}\times\frac{22}{7}\times 7\times 7\right) \text{ cm}^2$$

$$= \left(\frac{1}{9}\times 22\times 14\times 2 - \frac{1}{9}\times 22\times 7\times 1\right) \text{ cm}^2$$

$$= \frac{22}{9} = \frac{154}{3} \times (28 - 7) \text{ cm}^2 = \text{ cm}^2$$



Area of the circular ring

$$= \left(\frac{22}{7}\times 14\times 14 - \frac{22}{7}\times 7\times 7\right)\text{cm}^2$$

$$= (22 \times 14 \times 2 - 22 \times 7 \times 1)\text{cm}^2$$

$$= 22 \times 21 \text{ cm}^2 = 462 \text{ cm}^2$$

Hence,

Required shaded area $= \left(462 - \frac{154}{3}\right)$ cm²

$$= \frac{1232}{3}\text{cm}^2 = 410.67 \text{ cm}^2$$

Ex.3 AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30^\circ$, find the area of the shaded region.

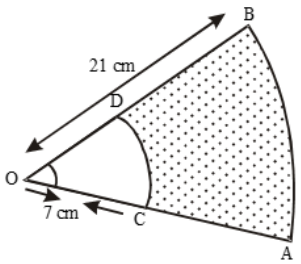
Sol. We have,

Shaded area = Area of sector OAB – Area of sector OCD

$\Rightarrow$ Shaded area

$$= \left(\frac{30}{360}\times\frac{22}{7}\times 21\times 21 - \frac{30}{360}\times\frac{22}{7}\times 7\times 7\right) \text{ cm}^2$$

$$= \frac{30}{360}\times\frac{22}{7}\times (21 \times 21 - 7 \times 7) \text{ cm}^2$$

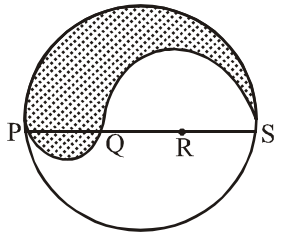


$$= \frac{11}{42}\times (21 + 7) \times (21 - 7) \text{ cm}^2$$

$= \frac{11}{42} \times 28 \times 14 \text{ cm}^2 = 102.67 \text{ cm}^2$

Ex.4 PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters as shown in Fig. Find the perimeter and area of the shaded region.

Sol. We have,



PS = Diameter of a circle of radius 6 cm = 12 cm

PQ = QR = RS = 12/3 = 4 cm

QS = QR + RS = (4 + 4) cm = 8 cm

Hence, required perimeter

= Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm

$= (\pi \times 6 + \pi \times 4 + \pi \times 2) \text{ cm} = 12\pi \text{ cm}$

Required area = Area of semi-circle with PS as diameter + Area of semi-circle with PQ as diameter – Area of semi-circle with QS as diameter.

$= \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$= \frac{1}{2} \times \frac{22}{7} (6^2 + 2^2 - 4^2)$

$= \frac{1}{2} \times \frac{22}{7} \times 24$

$= \frac{264}{7} = 37.71 \text{ cm}^2$