CIRCLE

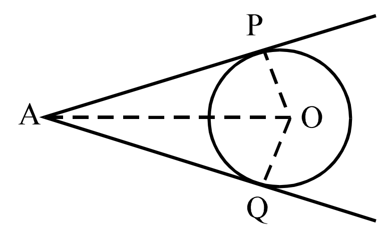
PROPERTIES OF TANGENT TO A CIRCLE (THEOREM 3 AND 4)

Theorem 3 :

The lengths of tangents drawn from an external point to a circle are equal.

Given : Two tangents AP and AQ are drawn from a point A to a circle with centre O.

To prove : AP = AQ



Construction : Join OP, OQ and OA.

Proof : AP is a tangent at P and OP is the radius through P.

$\therefore$ $OP \perp AP$.

Similarly, $OQ \perp AQ$.

In the right triangle OPA and OQA, we have

OP = OQ [radii of the same circle]

OA = OA [common]

$\therefore$ ΔOPA ΔOQA [by RHS–congruence]

Hence, AP = AQ.

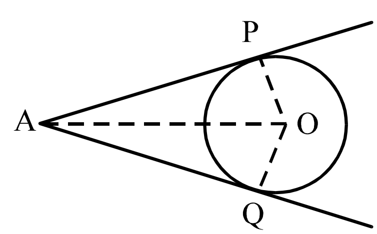
Theorem 4 :

If two tangents are drawn from an external point then

(i) They subtend equal angles at the centre, and

(ii) They are equally inclined to the line segment joining the centre to that point.

Given : A circle with centre O and a point A outside it. Also, AP and AQ are the two tangents to the circle.



To prove : $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

Proof : In ΔAOP and ΔAOQ, we have

AP = AQ [tangents from an external point are equal]

$OP = OQ$ [radii of the same circle]

$OA = OA$ [common]

$\therefore$ ΔAOP ΔAOQ [by SSS–congruence].

Result :

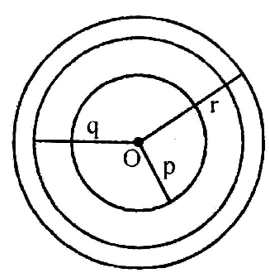
(i) If two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. $\angle OAQ = \angle OAP$ [By CPCT]

(ii) If two tangents are drawn to a circle from an external point, they are equally inclined to the segment, joining the centre to that point $\angle OAQ = \angle OAP$ [By CPCT]

IMPORTANT RESULTS FOR CIRCLE AND TANGENTS TO A CIRCLE

1. One and only one tangent can be drawn at any point on the circle.
2. If PAB is a secant to a circle intersecting it at A and B and PT is a tangent, then $PA \times PB = PT^2$.
3. The points of intersection of direct common tangents and transverse common tangents to two circles divide the line segment joining the two centres externally and internally respectively in the ratio of their radii.
4. If two chords AB and CD of a circle intersect each other at P outside the circle, then $PA \times PB = PC \times PD$.

CONGRUENT AND CONCENTRIC CIRCLES

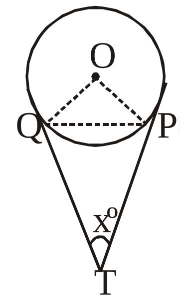


We say that two circles are congruent if they have the same radius. If two or more circles with the same centre are called concentric circles.

In the figure, circle (O,p), (O,q), (O,r) are three concentric circles.

Ex.1 Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

Sol. **Given :** A circle with centre O and an external point T from which tangents TP and TQ are drawn to touch the circle at P and Q.



To prove : $\angle PTQ = 2\angle OPQ$.

Proof : Let $\angle PTQ = x^\circ$. Then,

$\angle TQP + \angle TPQ + \angle PTQ = 180^\circ$

[$\because$ sum of the $\angle$s of a triangle is 180°]

$\Rightarrow \angle TQP + \angle TPQ = (180^\circ - x)$ ....(i)

We know that the lengths of tangent drawn from an external point to a circle are equal.

So, TP = TQ.

Now, TP = TQ

$\Rightarrow \angle TQP = \angle TPQ$

$= \frac{1}{2}(180^\circ - x) = \left(90^\circ - \frac{x}{2}\right)$

$\therefore \quad \angle OPQ = (\angle OPT - \angle TPQ)$

$= 90^\circ - \left(90^\circ - \frac{x}{2}\right) = \frac{x}{2}$

$\Rightarrow \angle OPQ = \frac{1}{2} \angle PTQ$

$\Rightarrow \angle PTQ = 2\angle OPQ$.

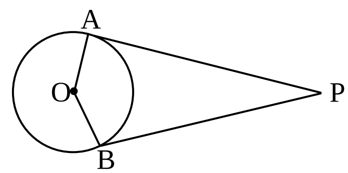
Hence, $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

Ex.2 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact to the centre.

Sol. **Given :** PA and PB are the tangent drawn from a point P to a circle with centre O. Also, the line segments OA and OB are drawn.

To Prove : $\angle APB + \angle AOB = 180°$

Proof : We know that the tangent to a circle is perpendicular to the radius through the point of contact.



$\therefore$ $PA \perp OA \Rightarrow \angle OAP = 90°$, and

$PB \perp OB \Rightarrow \angle OBP = 90°$.

$\therefore$ $\angle OAP + \angle OBP = 90°$.

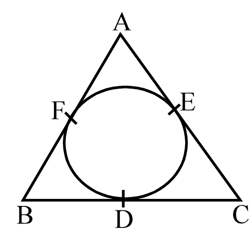
Hence, $\angle APB + \angle AOB = 180°$

[$\because$ sum of the all the angles of a quadrilateral is 360°]

Ex.3 In the given figure, the incircle of ΔABC touches the sides BC, CA and AB at D, E, F respectively.

Prove that $AF + BD + CE = AE + CD + BF$

$= \dfrac{1}{2}$ (perimeter of ΔABC)



Sol. We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ $AF = AE$ .... (i) [tangents from A]

$BD = BF$ ..... (ii) [tangents from B]

$CE = CD$ ..... (iii) [tangents from C]

Adding (i), (ii) and (iii), we get

$(AF + BD + CE) = (AE + BF + CD) = k$ (say)

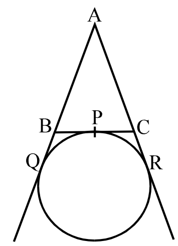
Perimeter of $\Delta ABC = (AF + BD + CE) + (AE + BF + CD)$

$= (k + k) = 2k$

$\therefore \quad k = \frac{1}{2}$ (perimeter of ΔABC).

Hence $AF + BD + CE = AE + CD + BF = \frac{1}{2}$ (perimeter of ΔABC)

Ex.4 A circle touches the side BC of a ΔABC at P, and touches AB and AC produced at Q and R respectively, as shown in the figure.



Show that $AQ = \frac{1}{2}$(perimeter of ΔABC)

Sol. We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore \quad AQ = AR$ .... (i) [tangents from A]

$BP = BQ$ .... (ii) [tangents from B]

$CP = CR$ .... (iii) [tangents from C]

Perimeter of ΔABC

$= AB + BC + AC$

$= AB + BP + CP + AC$

$= AB + BQ + CR + AC$ [using (ii) and (iii)]

$= AQ + AR$

$= 2AQ$ [using (i)].

Hence, $AQ = \frac{1}{2}$ (perimeter of ΔABC)

SUMMARY OF THE CHAPTER

BASIC CONCEPTS AND IMPORTANT RESUTLS

* **Circle**

  A circle is a closed plane figure consisting of all those points of the plane which are at a constant distance from a fixed point in that plane. The fixed point if called its centre and the constant distance is called its radius.

* **Intersection of a line and a circle :** The following three cases arise :

  Case I. A line may not intersect a circle.

  Case II. A line may intersect a circle in two points.

  Case III. A line may intersect a circle in one point.

  Thus, a line can intersect a circle atmost in two points.
* A line Which intersects a circle in two points is called a secant.
* A line which intersects a circle in one point is called a tangent to the circle. Thus, a line which intersects a circle in one point is called a tangent to the circle. Thus, a line which intersects a circle in one and only one point is a tangent to the circle. The common point is called point of contact.

  In the adjoining figure, the line l intersects the circle in only one point P. The line l is a tangent to the circle at P and P is the point of contact.
* Tangent to a circle is a special case of a secant,when the two ends of its corresponding chord coincide.
* The tangent at any point of a circle is perpendicular to the radius through the point of contact.
* If P is a point on a circle with centre C, then the line drawn through P and perpendicular to CP is the tangent to the circle at P.
* One and only one tangent can be drawn to a circle at a given point P on the circle with centre C because only one perpendicular can be drawn to CP through P.
* The tangents drawn at the ends of a diameter of a circle are parallel.

* The line segment joining the points of contact of two parallel tangens passes through the centre of the circle.

* Number of tangents to a circle from a point :

The following three cases arise :

Case I. There is no tangent to a circle passing through a point lying inside the circle.

Case II. There is one and only one tangent to a circle passing through a point lying on the circle.

Case III. There are exactly two tangents to a circle passing through a point lying outside the circle.

In the adjoining figure, P is an external point to a circle.

PA and PB are two tangents to the circle, A and B are points of contact. The length of the segment PA (or PB) is called the length of the tangent.