REAL NUMBERS

THE FUNDAMENTAL THEOREM OF ARITHMETIC

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Every composite number can be expressed as a product of primes, and this factorisation is unique, except for the order in which the prime factors occurs.

For example :

(i) $30 = 2 \times 3 \times 5, 30 = 3 \times 2 \times 5, 30 = 2 \times 5 \times 3$ and so on.

(ii) $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$ or $432 = 3^3 \times 2^4$.

(iii) $12600 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$

In general, a composite number is expressed as the product of its prime factors written in ascending order of their values.

e.g., (i) $6615 = 3 \times 3 \times 3 \times 5 \times 7 \times 7$

$= 3^3 \times 5 \times 7^2$

(ii) $532400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 11 \times 11$

Some Important Results:

(i) Let ‘p’ be a prime number and ‘a’ be a positive integer. If ‘p’ divides a^2, then ‘p’ divides ‘a’.

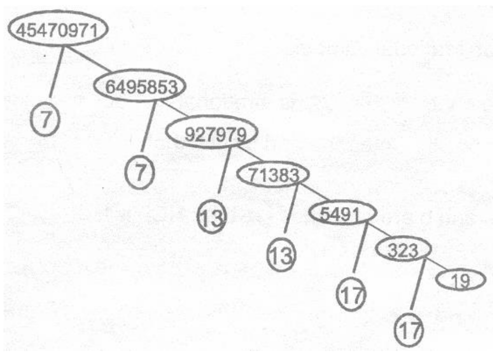
(ii) Let x be a rational number whose decimal expansion terminates. Then, x can be expressed in form $\frac{p}{q}$, where p and q are co-primes, and prime factorization of q is of the form $2^m \times 5^n$, where m,n are non-negative

(iii) Let x=$\frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^m \times 5^n$ where m, n are non-negative integers. Then, × has a decimal

expansion which is non- terminating repeating.

Ex.1 Determine the prime factors of 45470971.

Sol.



$\therefore$ $45470971 = 7^2 \times 13^2 \times 17^2 \times 19$.

Ex.2 Check whether 6^n can end with the digit 0 for any natural number.

Sol. Any positive integer ending with the digit zero is divisible by 5 and so its prime factorisations must contain the prime 5.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

$\Rightarrow$ The prime in the factorisation of 6^n is 2 and 3.

$\Rightarrow$ 5 does not occur in the prime factorisation of 6^n for any n.

$\Rightarrow$ 6^n does not end with the digit zero for any natural number n.

Ex.3 Find the LCM and HCF of 84, 90 and 120 by applying the prime factorisation method.

Sol. $84 = 2^2 \times 3 \times 7$, $90 = 2 \times 3^2 \times$ and $120 = 2^3 \times 3 \times 5$.

Prime factors	Least exponent
2	1
3	1
5	0
7	0

$\therefore$ HCF $= 2^1 \times 3^1 = 6$.

Common prime factors	Greatest exponent
2	3
3	2
5	1
7	1

$\therefore$ LCM $= 2^3 \times 3^3 \times 5^1 \times 7^1$

$= 8 \times 9 \times 5 \times 7$

$= 2520$.

Ex.4 In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that they can cover the distance in complete steps ?

Sol. Required minimum distance each should walk so, that they can cover the distance in complete step is the L.C.M. of 80 cm, 85 cm and 90 cm

$80 = 2^4 \times 5$

$85 = 5 + 17$

$90 = 2 \times 3^2 \times 5$

$\therefore \quad \text{LCM} = 2^4 \times 3^2 \times 5^1 \times 17^1$

$\text{LCM} = 16 \times 9 \times 5 \times 17$

$\text{LCM} = 12240 \text{ cm}, = 122 \text{ m } 40 \text{ cm}.$