Playing with Numbers

Introduction

Various types of numbers are –natural numbers, whole numbers, integers and rational numbers.

Natural numbers

Natural numbers are all of the whole numbers EXCEPT zero.

Example: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 …

Whole numbers

Whole numbers is the set of all counting numbers plus zero.

Example: 0, 1, 2, 3, 4, 5, 6……..

Integers

A number of the set of positive whole numbers, negative whole numbers and zero is called Integers.

Example:….-4,-3,-2,-1,0,1,2,3,4,5,…….

Rational numbers

A rational number is a number that can be written as a ratio.

Or

Rational number is written in $\frac{p}{q}$ form, where *p* and *q* are integers and *q* is a non-zero denominator.

Example...– $\frac{-1}{2}, 0, 1, \frac{7}{10}, \frac{12}{10}, 2, \frac{5}{2}, \ldots \ldots$

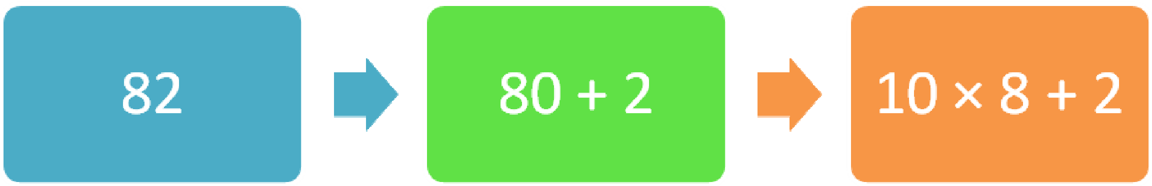
Numbers always have some interesting property.

Numbers in General Form

Usual form of numbers are 14, 234, 487, 97 etc.

Two Digit Number

68 write it as,



Similarly,

$$38 = 10 \times 3 + 8$$

$$96 = 10 \times 9 + 6$$

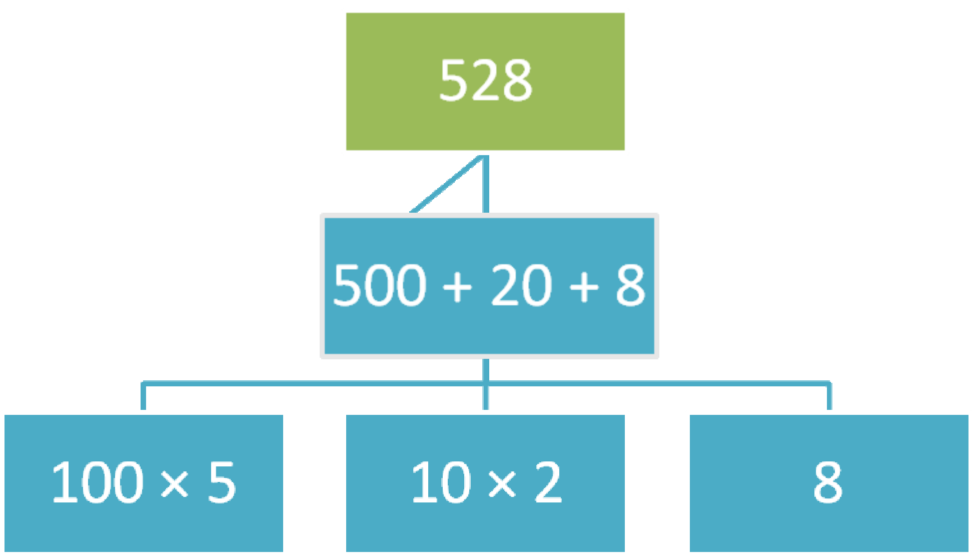
General Form of 2- Digit Number

In general, any two digit number **a***b* made of digits **a** and *b* can be written as,

$$ab = 10 \times \boldsymbol{a} + \boldsymbol{b} = \mathbf{10}\,\boldsymbol{a} + \boldsymbol{b}$$

$$ba = 10 \times \boldsymbol{b} + \boldsymbol{a} = \mathbf{10}\,\boldsymbol{b} + \boldsymbol{a}$$

Three Digit Number



Similarly,

$$497 = 100 \times \mathbf{4} + \mathbf{10} \times \mathbf{9} + \mathbf{1} \times \mathbf{7}$$

General Form of 3- Digit Number

In general a 3-digit number, abc made up of digits a, b, and c is written as

$$\mathbf{abc = 100 \times a + 10 \times b + 1 \times c}$$

$$\mathbf{= 100a + 10b + c}$$

In the same way,

$$\mathbf{cab = 100c + 10a + b}$$

$$\mathbf{bca = 111b + 10c + a}$$

Example: Write numbers in generalized form?

Solution:

(i) $74 = 10 \times 7 + 4$

$= 70 + 4$

$= 74$

(ii) $129 = 100 \times 1 + 10 \times 2 + 1 \times 9$

$= \mathbf{100} + \mathbf{20} + \mathbf{9}$

Example: Write in the usual form?

Solution:

(i) $10 \times 5 + 4 = 50 + 4$

$= 54$

(ii) $100 \times 7 + 10 \times 2 + 8 = 700 + 20 + 8$

$= 728$

$$\mathbf{100 \times \mathit{c} + 10 \times \mathit{a} + \mathit{b} = \boldsymbol{cab}}$$

Games with Numbers

Any number is the divisible by 11, by reversing the number and sum of the number is the multiple of 11.

The difference of numbers is the multiple of 9

Reversing the Digits-Two Digit Number

Divisibility by 11

Step 1 Two digit number ab = 10 $a + b$

Step 2 On reversing the digits, ba = 10$b + a$

Step 3 Adding two numbers,

$$\mathbf{(10a + b) + (10b + a) = 11a + 11b}$$

$$\mathbf{= 11(a+b)}$$

The sum is always a multiple of 11, so divisible by 11.

Divide the sum by 11; quotient is $(a + b)$ which is exactly the sum of the digits of ab.

To find a two digit number is divisible by 11 two digit numbers is **ab**

Step 1 reversing the number **ba** adding two numbers, **ab** + **ba**

Divisibility of 9

2-digit number, $ab = 10a + b$

Reversing the digits, $ba = 10b + a$

Subtracting, the smaller number from the larger one,

If $\boldsymbol{a > b}$,

$$10a + b - 10b + a = 10a + b - 10b - a$$

$$= 9a - 9b$$

$$= 9(a - b)$$

If $\boldsymbol{b > a}$,

$$10b + a - 10a + b = 10b + a - 10a - b$$

$= 9b - 9a$

$= 9(b - a)$

$a = b,\ a - b = 0\ and\ b - a = 0$

For Example:

2-digit number	64	------------ (a)
Reversing,	46	------------ (b)

Adding, a + b

$$\begin{array}{r} 64 \\ +\ 46 \\ \hline 110 \\ \hline \end{array}$$

Divided by 11,

110 ÷ 11 = 10 ----------- ---------no remainder

Subtracting, b-a

$$\begin{array}{r} 64 \\ -\ 46 \\ \hline 18 \\ \hline \end{array}$$

Divided by 9,

18 ÷ 9 = 2 ------------------- no remainder

Reversing the Digits-Three Digit Number

3 digit number $abc = 100a + 10b + c$

Reversing the digit, $cba = 100c + 10b + a$

Difference between the numbers $abc - cba$

If $a > b$ $99(a-c)$

If $c > a$ $99(c-a)$

Resulting number is divisible by 99

3-digit number: 456

Reversed number: 654

Difference:

$$\begin{array}{r} 654 \\ -\ 456 \\ \hline 198 \\ \hline \end{array}$$

Division: $198 \div 99 = 2$ (No remainder)

So, number is divisible by 99.

Forming Three-Digit Numbers With Given 3-Digit Number

3-digit number $abc = 100a + 10b + c$ $'c'$ shifted to 'left end'

Then $cab = 100c + 10a + b$ 'a' shifted to 'right end'

$then,$ $bca = 100b + 10c + a$

$$\mathbf{abc + cab + bca = 111(a+b+c)}$$

$$\mathbf{= 37 \times 3\ (a+b+c)}$$ which is divisible by 37.

For example:

3-digit number, 285

528 (5 to left end)

852 (8 to right end)

Adding,

$$\begin{array}{r} 285 \\ +\ 528 \\ +\ 852 \\ \hline 1665 \\ \hline \end{array}$$

1998 divided by 37,

$1665 \div 37 = 45$ --------- (no remainder)

285 is divisible by 37.

Sum of three reversible of the number is divided by 37.

Letters for Digits

Let us do some questions in which letters take the place of digits in an arithmetic sum or multiplication. So the problem is to find out the digits represented by the letters.

Rules for solving such puzzles:

- Each letter in the puzzle must stand for just one digit. Each digit must be represented by just one letter.
- The first digit of a number cannot be zero.

Let us solve some puzzles:

Example: Find Q in the addition:

$$
\begin{array}{r}
5\ 4\ Q \\
+\ 1\ Q\ 3 \\
\hline
7\ 4\ 2 \\
\hline
\end{array}
$$

Solution:

In this question we need to find out value of just letter i.e. Q.

Notice that from Q+3 we get 2 that is a number whose ones digit is 2.

For this to happen, the digit Q must be 9.So the puzzle can be solved a shown below:

$$
\begin{array}{r}
5\ 4\ 9 \\
+\ 1\ 9\ 3 \\
\hline
7\ 4\ 2 \\
\hline
\end{array}
$$

So, **Q=9**.

Example: Find A and B in the given question:

$$\begin{array}{cccc} & 2 & A & B \\ + & A & B & 1 \\ \hline & B & 1 & 8 \\ \hline \end{array}$$

Solution:

Let us start from ones place: B + 1 = 8, this means B = 7.

Now, see the tens place: A + 7 = 1, that is a number whose ones place is 1.For this to happen, the digit for A must be 4.

So the puzzle can be solved as shown below:

$$\begin{array}{cccc} & 2 & 4 & 7 \\ + & 4 & 7 & 1 \\ \hline & 7 & 1 & 8 \\ \hline \end{array}$$

So, **A = 4** and **B = 7**.

Example: Find P and Q in the given below:

$$\begin{array}{cccc} & & P & Q \\ \times & & P & 3 \\ \hline & 5 & 7 & Q \\ \hline \end{array}$$

Solution: We need to find out the value of P and Q.

Since the ones digit is Q × 3 = Q so Q can be equal to 0 or 5.

Let P = 1. Then PQ × P3 can be at the most equal to 15 × 13 = 195. So, P ≠ 1

Let P = 3, then PQ × P3 can be more than 30 × 33 = 990 and less than 35 × 33 = 1155. But the product here is 57Q which is less than 990. So P = 2.

Now the two possibilities are, 20×23 or 25×23.

The first possibility is not true, since $20 \times 23 = 460$.

Second possibility gives $25 \times 23 = 575$.

So, **P = 2** and **Q = 5**.

Tests of Divisibility

Divisibility by 10

A number is divisible by 10 only when its one's digit is 0.

Example:

1110, 220, 65780 are divisible by 10.

Divisibility by 5

A number is divisible by 5 if its one's digit is either 0 or 5.

Example: 15, 55, 70, 880, 3450 are divisible by 5.

Divisibility by 2

A number is divisible by 2 if its one's digit is either 0,2,4,6 or 8.

Example: 24, 770, 98356, 6, 808 are all divisible by 2.

Divisibility by 3

A number is divisible by 3 if sum of its digits is divisible by 3.

Example: 5763 is divisible by 3 because $5 + 7 + 6 + 3 = 21$ and 21 is divisible by 3.

So, 5763 is also divisible by 3.

Divisibility by 9

A number is divisible by 9 if sum of its digits is divisible by 9.

Example: 3478122 is divisible by 9 because $3 + 4 + 7 + 8 + 1 + 2 + 2 = 27$ which is divisible by 9.

So, 3478122 is also divisible by 9.