Practical Geometry

Constructing a Quadrilateral

We shall learn how to construct a unique quadrilateral by given the following measurements:

- When four sides and one diagonal are given.
- When two diagonals and three sides are given.
- When two adjacent sides and three angles are given.
- When three sides and two included angles are given.
- When other special properties are known.

When the lengths of four sides and a diagonal are given

Let us understand this construction through an example.

Example:

ABCD is a quadrilateral such that the diagonal BD = 7cm and the four sides measure AB = 5cm, BC = 4.5cm; CD = 6.5cm and AD = 4cm.

Solution:

A rough sketch will help us in visualising the quadrilateral. We draw this first and mark the measurements.

Steps of construction

Step: 1 Draw BD = 7cm, as AB = 5cm, so we take a compass and with B as centre and radius 5cm, draw an arc.

Step: 2 As AD = 4cm, so from D draw an arc of 4cm such that it intersect the previous arc. Mark the intersection point as A.

Step: 3 join AB and AD.

Step: 4 now we need to locate point C. As BC = 4.5cm. So, take B as centre and 4.5cm as radius draw an arc. Also, CD = 6.5cm. So, we take D as centre and 6.5 as radius and draw another arc. Mark this point of intersection as C.

Step: 5 join CB and CD.

So, ABCD is the required quadrilateral.







When two diagonals and three sides are given

When four sides and a diagonal were given, we first drew a triangle with the available data and then tried to locate the fourth point. The same technique is used here.

Let us understand this construction through an example.

Example:

Consider a quadrilateral PQRS such that the diagonals measure PR = 6.5cm, QS = 5.5cm and the three sides measure PQ = 4cm, QR = 5cm and RS = 4.5cm.

Solution:

We first draw a rough sketch of this quadrilateral. And label it with the given dimensions.

Steps of construction

Step: 1 Draw PR = 6.5cm. Now, as PQ = 4cm, so with P as centre draw an arc of radius 4cm.

Step: 2 As QR = 5cm, so from R draw an arc of radius 5cm such that it intersect the previous arc. Mark the intersection point as Q.

Step: 3 join QP and QR.

Step: 4 now we need to locate point S. As RS = 4.5cm. So, with R as centre and 4.5cm as radius draw an arc. Also, QS = 5.5cm. So, we take Q as centre and 5.5 as radius and draw an arc. Mark this point of intersection as S.

Step: 4 join SP and SR.

So, PQRS is the required quadrilateral.

When two adjacent sides and three angles are known

As before, we start with constructing a triangle and then look for the fourth point to complete the quadrilateral.

Example:

Construct a quadrilateral MIST where MI = 3.5 cm, IS = 6.5 cm, \angle M = 75°, \angle I = 105° and \angle S = 120°.

Solution:



Here is a rough sketch that would help us in deciding our steps of construction.

Steps of construction

Step: 1 Draw MI = 3.5 cm and making an angle of 105° at I which is produced to S. now, cut SI with 6.5 cm.

Step: 2 Make \angle ISY = 120° at S.

Step: 3 Make $\angle IMZ = 75^{\circ}$ at M. And extents the line MZ, it cuts the previous line SY.

Step: 4 Mark that point as T.

We get the required quadrilateral MIST.

When three sides and two included angles are given

When you draw a rough sketch, note carefully the "included" angles in particular.

Example:

Construct a quadrilateral ABCD, where AB = 4 cm, BC = 5 cm, CD = 6.5 cm and \angle B = 105° and \angle C = 80°.

Solution:

We first draw a rough sketch of this quadrilateral. And label it with the given dimensions.

Steps of construction

Step 1 Start with taking BC = 5 cm on B. Draw an angle of 105° along BX. Locate A = 4 cm away on this. We now have B, C and A.

Step 2 The fourth point D is on CY which is inclined at 80° to BC. So make \angle BCY = 80° at C on BC.

Step 3 D is at a distance of 6.5 cm on CY. With C as centre draw an arc of length 6.5 cm. It cuts CY at D.

Step 4 Complete the quadrilateral ABCD. Therefore, ABCD is the required quadrilateral.









Some Special Cases

Rectangle

Example: construct a rectangle HOME with length HO = 5cm and OM = 3cm.

Solution:

Steps of construction

Step: 1 draw a line segment HO of length 5cm.

Step: 2 now as all the angles of a rectangle measure 90°, so we draw a right angle at O. Now, OM measures 3cm.

Step: 3 with O as centre draw an arc of radius 3cm cutting the ray at M.

Step: 4 Again at M draw an angle of 90° . We know that opposite sides of a rectangle are equal, i.e. HO = ME = 5cm.



Step: 5 with M as centre we draw an arc of radius 5cm. Mark the point of intersection as E.

Step: 6 Join EH.

Hence, HOME is the required rectangle.

Square

Example: construct a square ROSE with side 4.5cm.

Solution:

We know that all the sides of a square are equal, so we have RO = OS = SE = ER = 4.5cm. Also, measure of each angle is 90 degrees. So, $\angle R = \angle O = \angle S = \angle E = 90^{\circ}$.

Step: 1 Draw a line segment RO which is equal to 4.5cm.

Step: 2 At O draw an angle of 90°.

Step: 3 as OS measures 4.5cm, so, with O as centre draw an arc of radius 4.5cm cutting the ray at S.

Step: 4 Again at S draw an angle of 90° . With radius 4.5cm, we draw an arc. This point is E.

Step: 5 Join ER. Hence, ROSE is the required square.



