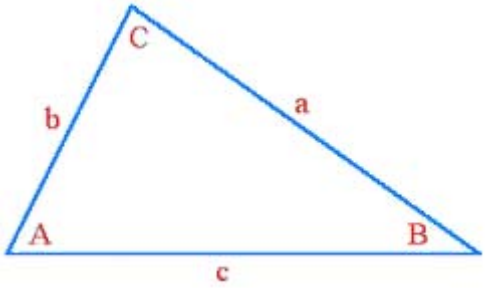
Triangle and its Properties

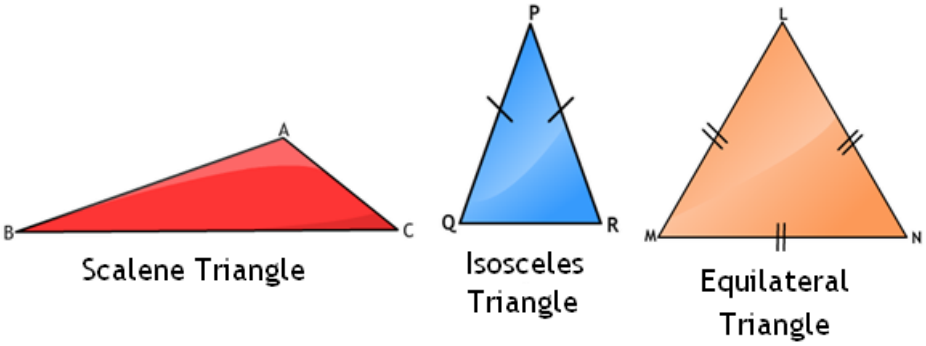
Introduction

A **Triangle** is a three-sided polygon. In fact, it is the polygon with the least number of sides. There are three sides, three vertices and three angles in a triangle.



Triangles can be classified on the basis of the sides and angles.

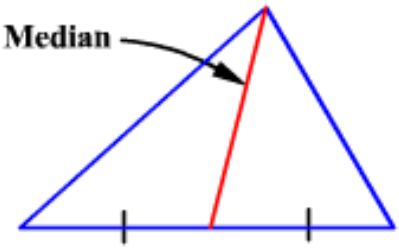
(i) **Based on Sides:** Scalene, Isosceles and Equilateral triangles.



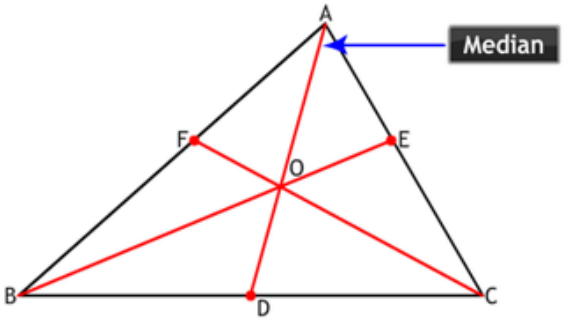
(ii) **Based on Angles:** Acute-angled, Obtuse-angled and Right-angled triangles.

Medians of a Triangle

The line segment AD, joining the mid point of BC to its opposite vertex A is called a **median of the triangle**.

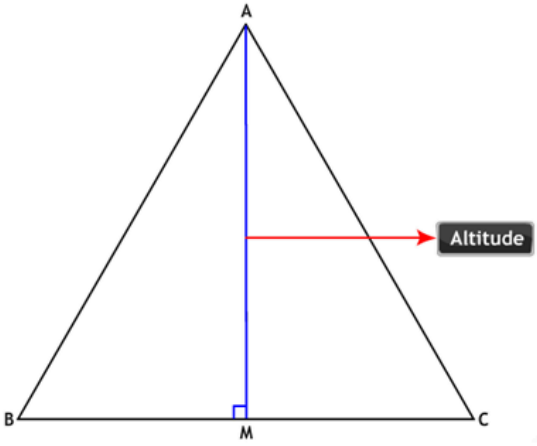


A median connects the mid point of a side of a triangle to its opposite vertex. There are 3 medians in a triangle.



Altitudes of a Triangle

An altitude has one end point at a vertex of the triangle and the other on the line containing the opposite side.



Exterior Angle of a Triangle and Its Property

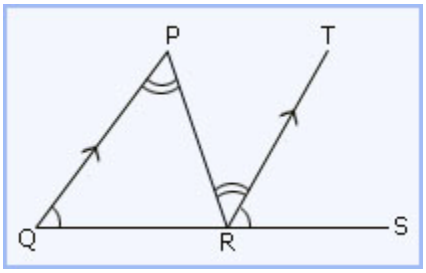
An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Theorem: If one side of a triangle is produced, then the exterior angle so formed is equal to the sum of its interior opposite angles.

Given: The triangle PQR whose side QR has been produced to S.

To prove: $\angle PRS = \angle Q + \angle P$

Construction: Draw RT || QP



Proof:

$\angle Q = \angle TRS$ . . . . . (i)

(∴Corresponding angles are equal, as RT || QP and QS is the transversal.)

$$\angle P = \angle PRT \ldots\ldots\ldots \text{(ii)}$$

(∴ Alternate interior angles are equal, as RT || QP and PR is the transversal.)

Adding (i) and (ii) we get:

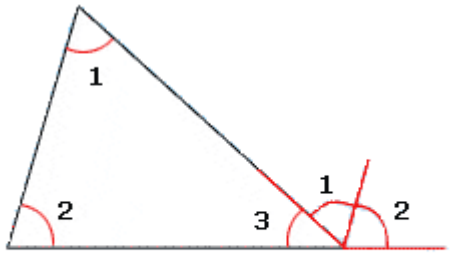
$$\angle Q + \angle P = \angle TRS + \angle PRT$$

$$= \angle PRS \quad (\therefore \angle TRS + \angle PRT = \angle PRS)$$

$$\angle PRS = \angle Q + \angle P$$ **Hence proved**

Angle Sum Property of a Triangle

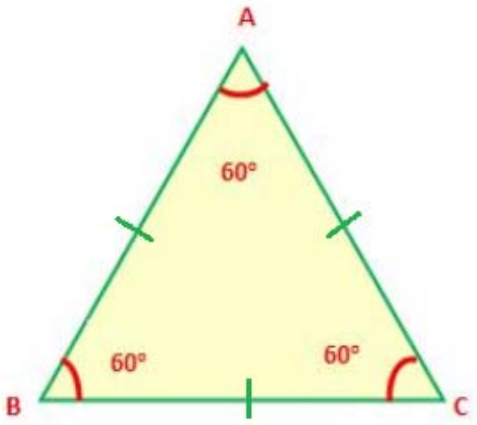
The total measure of the three angles of a triangle is 180°.



$$\angle 1 + \angle 2 + \angle 3 = 180^{o}$$

Two Special Triangles: Equilateral and Isosceles

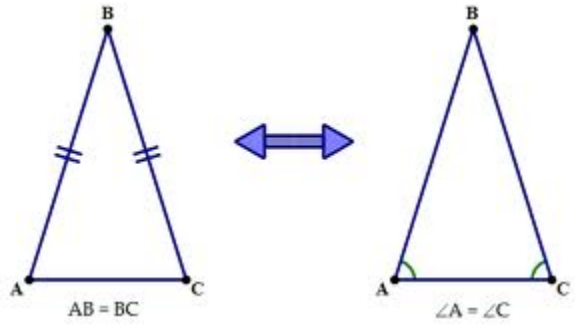
A triangle in which all the three sides are of equal lengths is called an **Equilateral triangle**.



In an **equilateral triangle**:

- All sides have same length.
- Each angle has measure 60°.

A triangle in which two sides are of equal lengths is called an **Isosceles triangle**.

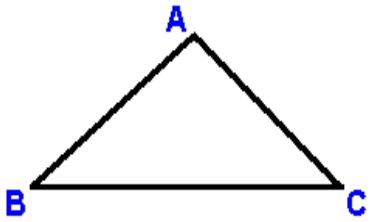


In an **isosceles triangle**:

- Two sides have same length.
- Base angles opposite to the equal sides are equal.

Sum of the Lengths of Two Sides of a Triangle

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



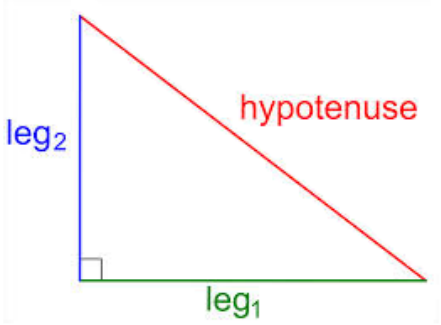
AB + BC > AC

BC + AC > AB

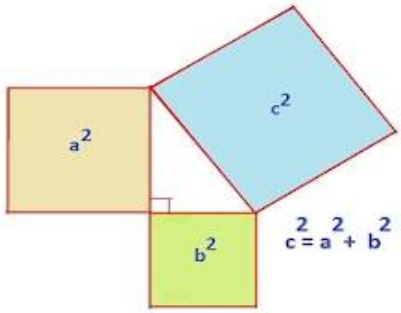
AC + AB > BC

Right-Angled Triangles and Pythagoras Property

In a right angled triangle, the side opposite to the right angle is called the **hypotenuse**; the other two sides are known as the **legs** of the right-angled triangle.



Pythagorean Theorem



The Pythagorean Theorem: The sum of the areas of the two squares on the legs ('a' and 'b') equals the area of the square on the hypotenuse (c).

In mathematics, the **Pythagorean Theorem** or **Pythagoras' Theorem** is a relation in Euclidean Geometry among the three sides of a right triangle (right-angled triangle). In terms of areas, it states:

In any right triangle, the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (the two sides that meet at a right angle).

The theorem can be written as an equation relating the lengths of the sides a, b and c, often called the **Pythagorean equation**:

$$c^2 = a^2 + b^2$$

Where c represents the length of the hypotenuse, and 'a' and 'b' represent the lengths of the other two sides.