Gravitational

Gravitation is a natural phenomenon by which objects with mass attract one another.

In everyday life, gravitation is most commonly thought of as the agency which lends weight to objects with mass.

It is responsible :

- for keeping the Earth and the other planets in their orbits around the Sun;
- for keeping the Moon in its orbit around the Earth,
- for the formation of tides;
- for convection (by which fluid flow occurs under the influence of a temperature gradient and gravity)
- for heating the interiors of forming stars and planets to very high temperatures; and
- for various other phenomena that we observe.

Modern physics

Modern physics describes Gravitation as a consequence of the curvature of space time which governs the motion of inertial objects.

Universal law of Gravitation

Every object in the universe attracts every other object with a force which is proportional to the product of their masses and inversely proportional to the square of the distance between them. The force is along the line joining the centres of two objects.

$$F = G\frac{Mm}{d^2}$$

If two objects A and B of masses M and m lie at a distance d from each other.

Let the force of attraction between two objects be F.

According to the universal law of gravitation,

- The force between two objects is directly proportional to the product of their masses. That is,

  $F \propto M \times m$

And the force between two objects is inversely proportional to the square of the distance between them, that is,

$$F \propto 1/d^2$$

Combining the Eqs. We get

$$F = G\frac{Mm}{d^2}$$

Where G is the constant of proportionality and is called the universal gravitation constant.

By multiplying crosswise, Eq. gives

$F \times d^2 = G M \times m$

$G = (F \times d^2)Mm$

The SI unit of G can be obtained by substituting the units of force, distance and mass in Eq. as

$N\ m^2\ kg^{-2}$.

The value of G was found out by Henry Cavendish (1731 – 1810) by using a sensitive balance. The accepted value of G is 6.673 × 10–11N $m^2\ kg^{-2}$.

importance of the Universal Law of Gravitation

The universal law of gravitation successfully explained several phenomena which were believed to be unconnected:

- the force that binds us to the earth;
- the motion of the moon around the earth;
- the motion of planets around the Sun; and
- the tides due to the moon and the Sun.

To Calculate the Value of g

To calculate the value of g, we should put the values of G, M and R in Eq. namely, universal gravitational constant,

$G = 6.7 \times 10^{-11}\ N\ m^2\ kg^{-2}$,

Mass of the earth, M = 6 × 1024 kg, and

Radius of the earth, $R = 6.4 \times 10^6$ m.

$F = ma$

$$F = G\frac{Mm}{d^2}$$

$$ma = G\frac{Mm}{d^2}$$

$$mg = G\frac{Mm}{d^2}$$

$$g = G\frac{M}{d^2}$$

$$g = \frac{6.7\times10^{-11}\,N\,m^2\,kg^{-2}\,6\times10^{24}\,kg}{(6.4\times10^6 m)^2}$$

$= 9.8\ m\ s^{-2}$

Thus, the value of acceleration due to gravity of the earth,g= 9.8 ms^{-2}

Kepler's Law

First law: All planets revolve around the sun with their respective elliptical orbit and is a foci.

Second law: The line joining the planet and the Sun sweep equal areas in equal intervals of time. Thus, if the time of travel from A to B is the same as that from C to D, then the areas OAB and OCD are equal.

Third law: The cube of the mean distance of a planet from the Sun is proportional to the square of its orbital period T. Or, r^3/T^2 = constant.

Motion of Objects under Gravitational Force of the Earth

- We know that an object experiences acceleration during free fall due to gravitational force.
- Gravitational acceleration experienced by an object is independent of its mass.
- This means that all objects hollow or solid, big or small should fall at the same rate.
- According to a story, Galileo dropped different objects from the top of the Leaning Tower of Pisa in Italy to prove the same.
- As g is constant near the earth, all the equations for the uniformly accelerated motion of objects become valid with acceleration a replaced by g

The equations are:

$v = u + at$

$S = ut + ½ at^2$

$v^2 = u^2 + 2as$

Where u and v are the initial and final velocities and s is the distance covered in time, t.

In applying these equations, we will take acceleration, a to be positive when it is in the direction of the velocity, that is, in the direction of motion. The acceleration, a will be taken as negative when it opposes the motion.

Example: A car falls off a ledge and drops to the ground in 0.5 s. Let g = 10 ms^{-2} (for simplifying the calculations).

(i) What is its speed on striking the ground?

(ii) What is its average speed during the 0.5 s?

(iii) How high is the ledge from the ground?

Solution:

Time, t = ½ second

Initial velocity, u = 0 ms^{-1}

Acceleration due to gravity, g = 10 ms^{-2}

Acceleration of the car, a = + 10 ms^{-2}

(i) speed v = at

$v = 10\ ms^{-2} \times 0.5\ s$

$= 5\ ms^{-1}$

(ii) Average speed = (u +v)/2

$= (0\ ms^{-1} + 5ms^{-1})/2 \quad = 2.5\ ms^{-1}$

(iii) Distance traveled,

$s = ½ at^2$

$= ½ \times 10\ ms^{-2} \times (0.5\ s)^2$

$= ½ \times 10\ ms^{-2} \times 0.25\ s^2$

$= 1.25\ m$

Thus,(i) Its speed on striking the ground = $5\ ms^{-1}$

(ii) Its average speed during the 0.5 s = $2.5\ ms^{-1}$

(iii) Height of the ledge from the ground = 1.25 m.

Mass

Mass refers to the degree of acceleration a body acquires when subject to a force: bodies with greater mass are accelerated less by the same force.

Weight

Weight is the force of gravity acting on a mass. Weight should be measured in Newton's and has a direction component (vector). This direction is normally downward due to gravity.

We know that the earth attracts every object with a certain force and this force depends on the mass (m) of the object and the acceleration due to the gravity (g).

The weight of an object is the force with which it is attracted towards the earth.

We know that

$$F = m \times a,$$

that is, $F = m \times g.$

The force of attraction of the earth on an object is known as the weight of the object. It is denoted by W.

$$W = m \times g$$

Weight of an Object on the Moon

The weight of an object on the earth is the force with which the earth attracts the object. In the same way, the weight of an object on the moon is the force with which the moon attracts that object. The mass of the moon is less than that of the earth. Due to this the moon exerts lesser force of attraction on objects.

Let the mass of an object be m. Let its weight on the moon be Wm. Let the mass of the moon be Mm and its radius be Rm. By applying the universal law of gravitation, the weight of the object on the moon will be

$G\times \frac{Mm \times m}{R^2 m}$

Let the weight of the same object on the earth be We. The mass of the earth is M and its radius is R.

From the above Eqs, we have

$We = G\ M \times \frac{m}{d^2}$

Substituting the values, we get

$W_m = 2.431 \times 10^{10} G \times m$

$W_e = 1.474 \times 10^{11} G \times m$

Dividing Eq., we get

$\frac{Wm}{We} = \frac{1}{6}$

Or, Weight of the Object on Moon/Weight of the Object on Earth = 1/6

Weight of the object on the moon = $\frac{1}{6}$ × its weight on the earth

Example

Example: An object weighs 10 N when measured on the surface of the earth. What would be its weight when measured on the surface of the moon?

Solution: We know, Weight of object on the moon = (1/6) × its weight on the earth.

That is, $W_m = \frac{We}{6} = \frac{10}{6} = 1.67$ N

Thus, the weight of object on the surface of the moon would be 1.67 N.

Thrust and Pressure

Thrust

Thrust is a reaction force described quantitatively by Newton's Second and Third Laws. When a system expels or accelerates mass in one direction the accelerated mass will cause a proportional but opposite force on that system.

Pressure

Pressure (symbol: p or sometimes P) is the force per unit area applied to an object in a direction perpendicular to the surface. Gauge pressure is the pressure relative to the local atmospheric or ambient pressure.

Pressure = $\frac{\text{Thrust}}{Area}$

The SI unit of pressure is N/m^2 or Nm^{-2}.

Example

A block of wood is kept on a tabletop. The mass of wooden block is 5 kg and its dimensions are 40 cm × 20 cm × 10 cm. Find the pressure exerted by the wooden block on the table top if it is made to lie on the table top with its sides of dimensions **(a) 20 cm × 10 cm and (b) 40 cm × 20 cm.**

Solution:

The mass of the wooden block = 5 kg

The dimensions = 40 cm × 20 cm × 10 cm

Here, the weight of the wooden block applies a thrust on the table top.

That is,

Thrust = F = m × g

= 5 kg × 9.8 ms^{-2}

= 49 N

Area of a side = length × breadth

= 20 cm × 10 cm

= 200 cm^2 = 0.02 ^{m2}

From Eq.

Pressure = 49/0.02 m^2 = 2450 Nm^{-2}

When the block lies on its side of dimensions 40 cm × 20 cm, it exerts the same thrust.

Area= length × breadth

= 40 cm × 20 cm = 800 cm^2 = 0.08 m^2

Or, Pressure = 49/0.08 m2 = 612.5 Nm^{-2}

The pressure exerted by the side 20 cm × 10 cm is 2450 Nm^{-2} and by the side 40 cm × 20 cm is 612.5 N m^{-2}.

Pressure in fluids

All liquids and gases are fluids. A solid exerts pressure on a surface due to its weight. Similarly, fluids have weight, and they also exert pressure on the base and walls of the container in which they are enclosed. Pressure exerted in any confined mass of fluid is transmitted undiminished in all directions.

Why objects float or sink when placed on the surface of water?

The nail sinks. The force due to the gravitational attraction of the earth on the iron nail pulls it downwards. There is an up thrust of water on the nail, which pushes it upwards. But the downward force acting on the nail is greater than the up thrust of water on the nail. So it sinks.

The cork floats while the nail sinks. This happens because of the difference in their densities.

Density

The density of a substance is defined as the mass per unit volume. The density of cork is less than the density of water. This means that the up thrust of water on the cork is greater than the weight of the cork. So it floats.

The density of an iron nail is more than the density of water. This means that the up thrust of water on the iron nail is less than the weight of the nail. So it sinks.

Therefore, objects of density less than that of a liquid float on the liquid. The objects of density greater than that of a liquid sink in the liquid.

Archimedes' Principle

Archimedes' principle, states that a body immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid.

- The principle applies to both floating and submerged bodies and to all fluids, i.e., liquids and gases.

Buoyancy

- The tendency of a liquid to exert an upward force on an object placed in it is called buoyancy.
- It explains not only the buoyancy of ships and other vessels in water but also the rise of a balloon in the air and the apparent loss of weight of objects underwater.
- In determining whether a given body will float in a given fluid, both weight and volume must be considered; that is, the relative density,
- Weight per unit of volume, of the body compared to the fluid determines the buoyant force. If the body is less dense than the fluid, it will float or, in the case of a balloon, it will rise. If the body is denser than the fluid, it will sink.

Archimedes' principle has many applications.

- It is used in designing ships and submarines.
- Lactometers, which are used to determine the purity of a sample of milk
- Hydrometers used for determining density of liquids, are based on this principle.

Relative density

Relative density also determines the proportion of a floating body that will be submerged in a fluid.

If the body is two thirds as dense as the fluid, then two thirds of its volume will be submerged, displacing in the process a volume of fluid whose weight is equal to the entire weight of the body. In the case of a submerged body, the apparent weight of the body is equal to its weight in air less the weight of an equal volume of fluid.

$$Relative\ density = \frac{\text{Density of substance}}{\text{Density of water}}$$