Sample Paper – II

Mathematical Methods in Engineering & Science

Attempt 5 Questions from Each Section **Max Marks-100**

Each Question Carries Equal Marks **Time: 3 Hrs.**

Section A

Question-1 Prove that the eigenvectors of a real matrix that correspond to the real eigenvalues can be chosen to be real.

Question - 2 Use Leverrier's method to compute the cofactors of the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$

Question - 3 Use unitary similarity transformations to upper triangularize the matrix

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}.$$

Question – 4 Prove the Triangle inequality $|V_1+V_2| \leq |V_1|+|V_2|$.

Question - 5 Prove the identity

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a} \cdot \mathbf{b})^2 = (|\mathbf{a}||\mathbf{b}|)^2.$$

Deduce that if $\mathbf{a}$ and $\mathbf{b}$ are unit vectors, then

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a} \cdot \mathbf{b})^2 = 1.$$

Question - 6 Show that

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}.$$

Deduce from this $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ is not necessarily equal to $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$. In fact, can you say when they are equal?

Question - 7

(a) Show that if A is an $m \times n$ matrix, x, y are $n \times 1$ matrices and α, β are numbers then $A(\alpha x + \beta y) = \alpha Ax + \beta Ay$.
(b) Using the matrix notation of a linear system, prove that, if a linear system has more than one solution then it must have an infinite number of solutions.

Section B

Question - 8

Find the Fourier series of the following functions:

(a) $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 2 & 0 < x < \pi \end{cases}, \quad f(x + 2\pi) = f(x),$

(b) $f(x) = \begin{cases} 1 & -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ -1 & \dfrac{\pi}{2} < x < \dfrac{3\pi}{2} \end{cases}, \quad f(x + 2\pi) = f(x),$

(c) $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}, \quad f(x + 2\pi) = f(x).$

Question - 9

Use Gauss-Jordan elimination to transform the following matrix first into row-echelon form and then into reduced row-echelon form

$$\begin{pmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{pmatrix}$$

Question - 10.

It can be shown that any four *noncoplanar* points (i.e. points that do not lie in the same plane) determine a sphere.[12] Find the equation of the sphere that passes through the points (0,0,0), (0,0,2), (1,−4,3) and (0,−1,3).

Question - 11.

Let S be the sphere with radius 1 centered at $(0,0,1)$, and let S^* be S without the "north pole" point $(0,0,2)$. Let (a,b,c) be an arbitrary point on S^*. Then the line passing through $(0,0,2)$ and (a,b,c) intersects the xy-plane at some point $(x,y,0)$, as in Figure 1.6.10. Find this point $(x,y,0)$ in terms of a, b and c.
(Note: Every point in the xy-plane can be matched with a point on S^*, and vice versa, in this manner. This method is called *stereographic projection*, which essentially identifies all of $\mathbb{R}^2$ with a "punctured" sphere.)

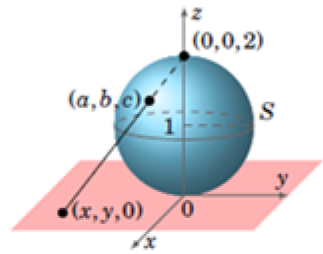

Figure 1.6.10

Question - 12

Show that the distance d between the points P_1 and P_2 with cylindrical coordinates (r_1,θ_1,z_1) and (r_2,θ_2,z_2), respectively, is

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1) + (z_2 - z_1)^2}.$$

Question - 13

Show that the distance d between the points P_1 and P_2 with spherical coordinates (ρ_1,θ_1,ϕ_1) and (ρ_2,θ_2,ϕ_2), respectively, is

$$d = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1 \rho_2[\sin\phi_1 \sin\phi_2 \cos(\theta_2 - \theta_1) + \cos\phi_1 \cos\phi_2]}.$$

Question - 14

We know that every vector in $\mathbb{R}^3$ can be written as a scalar combination of the vectors **i**, **j**, and **k**. Can every vector in $\mathbb{R}^3$ be written as a scalar combination of just **i** and **j**, i.e. for any vector **v** in $\mathbb{R}^3$, are there scalars m, n such that $\mathbf{v} = m\,\mathbf{i} + n\,\mathbf{j}$? Justify your answer.