3. Heat exchanger design

The great object to be effected in the boilers of these engines is, to keep a small quantity of water at an excessive temperature, by means of a small amount of fuel kept in the most active state of combustion...No contrivance can be less adapted for the attainment of this end than one or two large tubes traversing the boiler, as in the earliest locomotive engines.

The Steam Engine Familiarly Explained and Illustrated,
Dionysus Lardner, 1836

3.1 Function and configuration of heat exchangers

The archetypical problem that any heat exchanger solves is that of getting energy from one fluid mass to another, as we see in Fig. 3.1. A simple or composite wall of some kind divides the two flows and provides an element of thermal resistance between them. Direct contact heat exchangers are an exception to this configuration. Figure 3.2 shows one such arrangement in which steam is bubbled into water. The steam condenses and the water is heated at the same time. In other arrangements, immiscible fluids might contact each other or noncondensible gases might be bubbled through liquids.

Our interest here is in heat exchangers with a dividing wall between the two fluids. They come in an enormous variety of configurations, but most commercial exchangers reduce to one of three basic types. Figure 3.3 shows these types in schematic form. They are:

- *The simple parallel or counterflow configuration.* These arrangements are versatile. Figure 3.4 shows how the counterflow arrangement is bent around in a so-called Heliflow compact heat exchanger configuration.

- *The shell-and-tube configuration.* Figure 3.5 shows the U-tubes of a two-tube-pass, one-shell-pass exchanger being installed in the

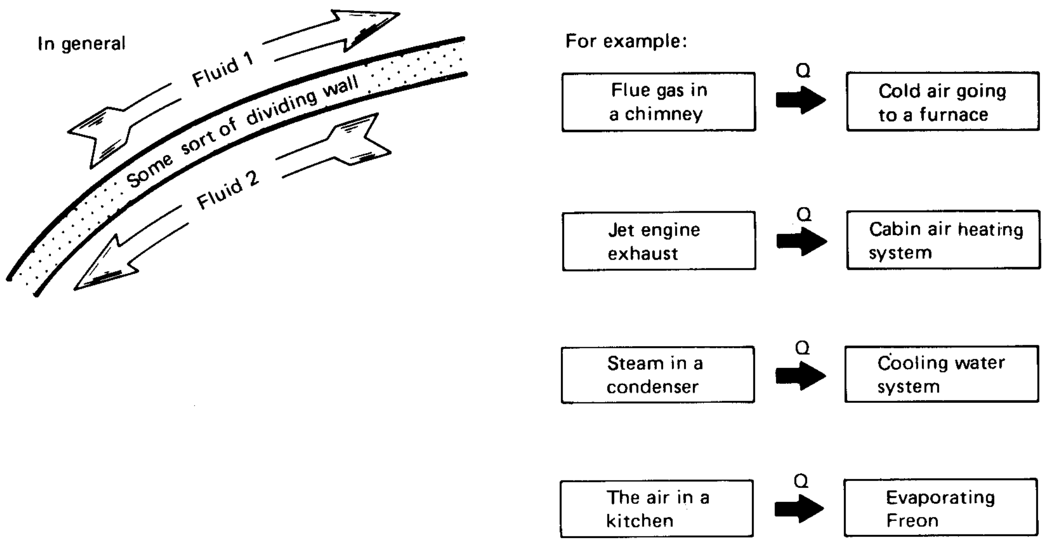


Figure 3.1 Heat exchange.

supporting baffles. The shell is yet to be added. Most of the really large heat exchangers are of the shell-and-tube form.

- *The cross-flow configuration.* Figure 3.6 shows typical cross-flow units. In Fig. 3.6a and c, both flows are *unmixed*. Each flow must stay in a prescribed path through the exchanger and is not allowed to "mix" to the right or left. Figure 3.6b shows a typical plate-fin cross-flow element. Here the flows are also unmixed.

Figure 3.7, taken from the standards of the Tubular Exchanger Manufacturer's Association (TEMA) [3.1], shows four typical single-shell-pass heat exchangers and establishes nomenclature for such units.

These pictures also show some of the complications that arise in translating simple concepts into hardware. Figure 3.7 shows an exchanger with a single tube pass. Although the shell flow is baffled so that it crisscrosses the tubes, it still proceeds from the hot to cold (or cold to hot) end of the shell. Therefore, it is like a simple parallel (or counterflow) unit. The kettle reboiler in Fig. 3.7d involves a divided shell-pass flow configuration over two tube passes (from left to right and back to the "channel header"). In this case, the isothermal shell flow could be flowing in any direction—it makes no difference to the tube flow. Therefore, this exchanger is also equivalent to either the simple parallel or counterflow configuration.

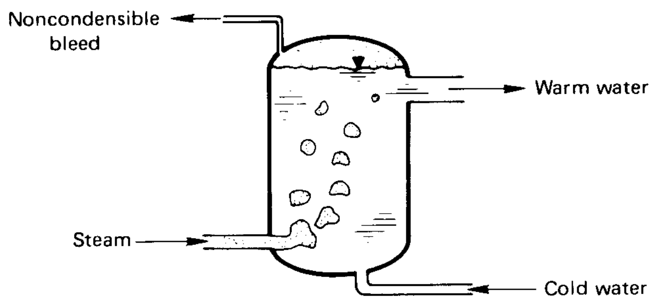


Figure 3.2 A direct-contact heat exchanger.

Notice that a salient feature of shell-and-tube exchangers is the presence of baffles. Baffles serve to direct the flow normal to the tubes. We find in Part III that heat transfer from a tube to a flowing fluid is usually better when the flow moves across the tube than when the flow moves along the tube. This augmentation of heat transfer gives the complicated shell-and-tube exchanger an advantage over the simpler single-pass parallel and counterflow exchangers.

However, baffles bring with them a variety of problems. The flow patterns are very complicated and almost defy analysis. A good deal of the shell-side fluid might unpredictably leak through the baffle holes in the axial direction, or it might bypass the baffles near the wall. In certain shell-flow configurations, unanticipated vibrational modes of the tubes might be excited. Many of the cross-flow configurations also baffle the fluid so as to move it across a tube bundle. The plate-and-fin configuration (Fig. 3.6b) is such a cross-flow heat exchanger.

In all of these heat exchanger arrangements, it becomes clear that a dramatic investment of human ingenuity is directed towards the task of augmenting the heat transfer from one flow to another. The variations are endless, as you will quickly see if you try Experiment 3.1.

Experiment 3.1

Carry a notebook with you for a day and mark down every heat exchanger you encounter in home, university, or automobile. Classify each according to type and note any special augmentation features.

The results of most of what follows in this chapter appear on the Internet in many forms. Information that we derive and present graphically

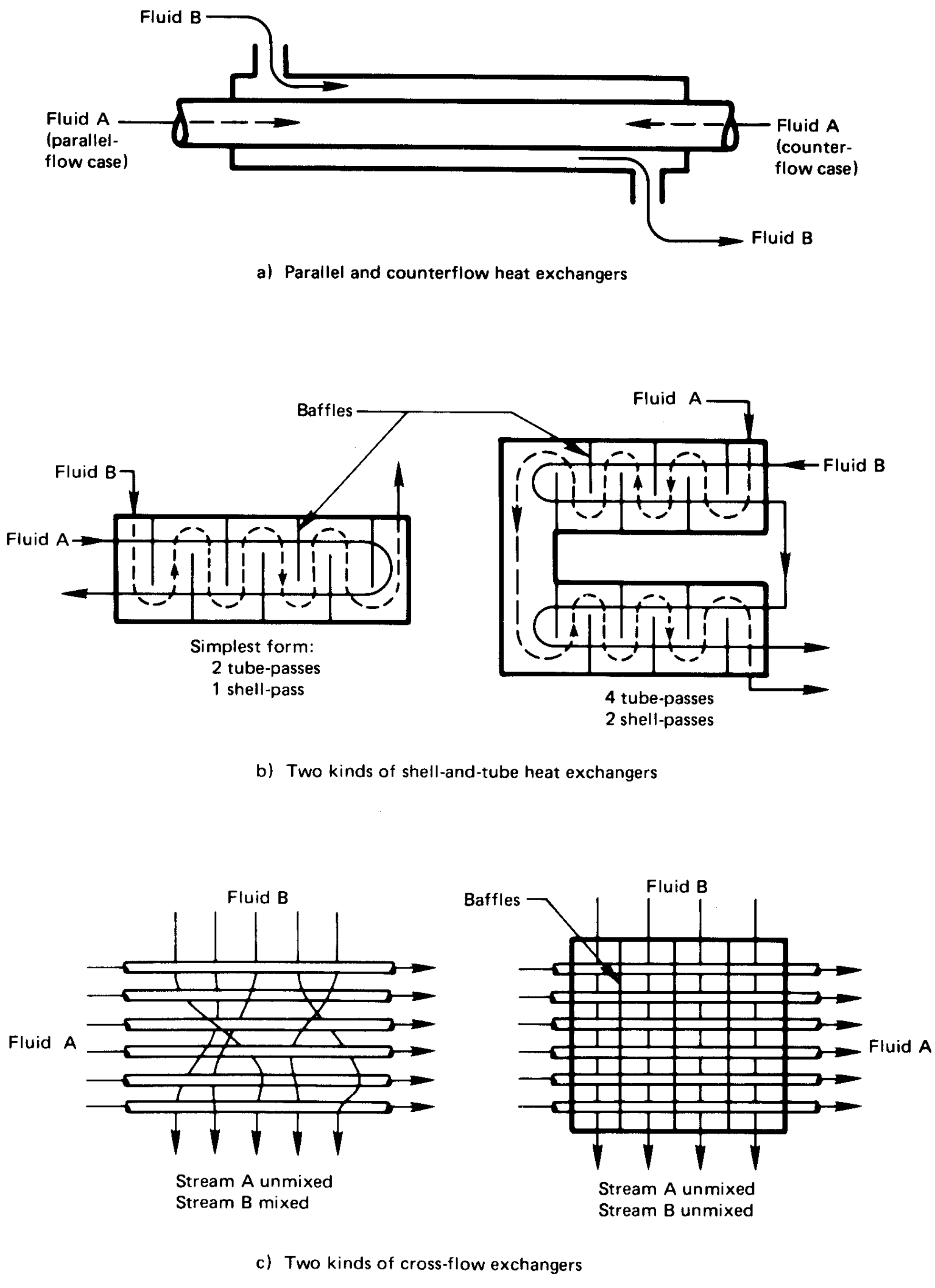


Figure 3.3 The three basic types of heat exchangers.

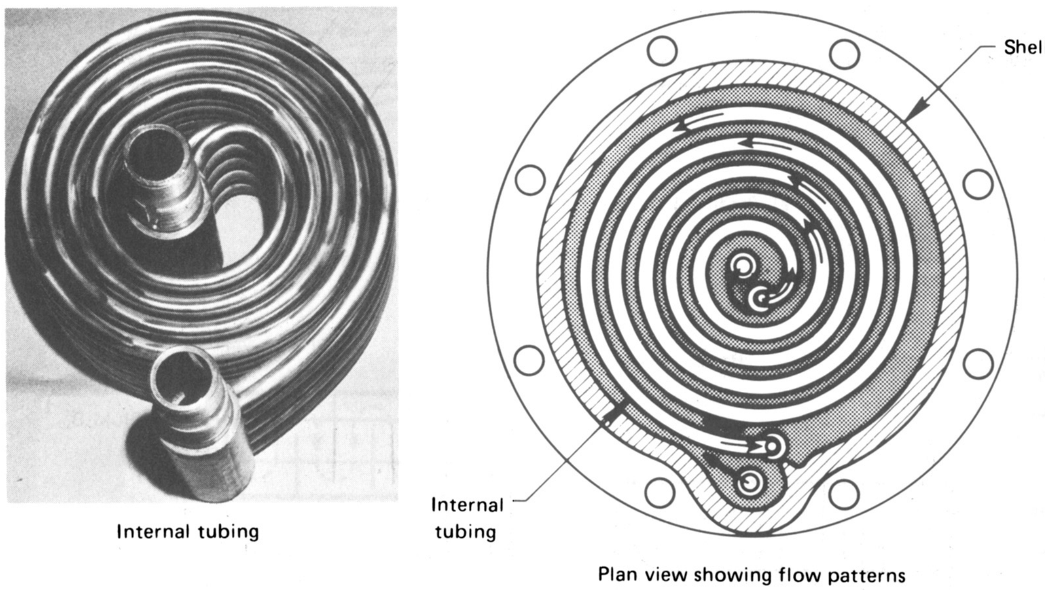


Figure 3.4 Heliflow compact counterflow heat exchanger. (Photograph coutesy of Graham Manufacturing Co., Inc., Batavia, New York.)

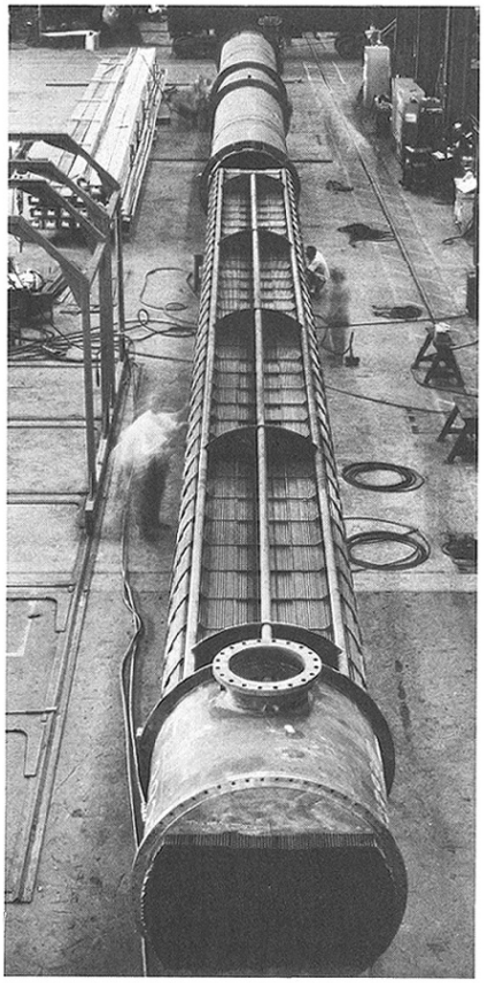
is buried in a variety of very effective canned routines for heat exchanger selection. But our job as engineers is not merely to select, it is also to develop new and better systems for exchanging heat. We must be able to look under the hood of those selection programs.

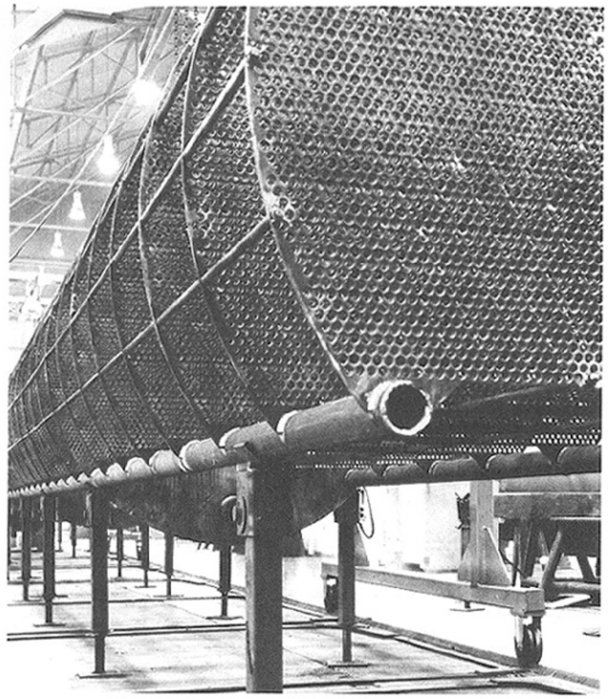
This under the hood analysis of heat exchangers first becomes complicated when we account for the fact that two flow streams change one another's temperature. We turn next, in Section 3.2, to the problem of predicting an appropriate mean temperature difference. Then, in Section 3.3 we develop a strategy for use when this mean cannot be determined initially.

3.2 Evaluation of the mean temperature difference in a heat exchanger

Logarithmic mean temperature difference (LMTD)

To begin with, we take U to be a constant value. This is fairly reasonable in compact single-phase heat exchangers. In larger exchangers, particu-

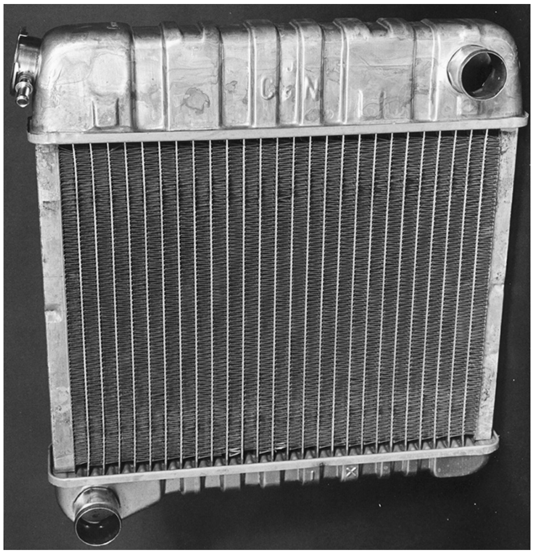




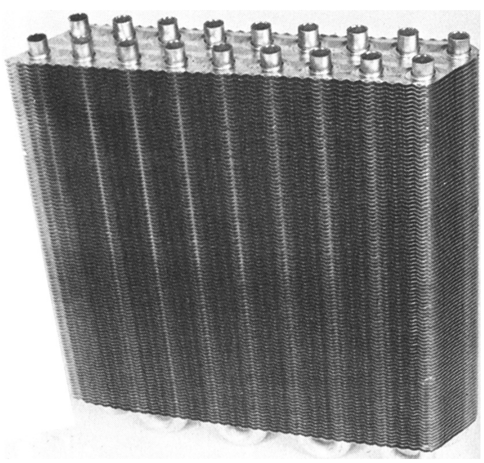
Above and left: A very large feed-water preheater. Tubes are shown withdrawn from the shell on the left. Inset above shows baffles before tubes are inserted. (Photos courtesy of Southwest Engineering Co., Subsidiary of Cronus Industries, Inc., Los Angeles, Calif.)

Below: Small "Swinglok" exchanger with tube-bundle removed from shell. (Photo courtesy of Graham Manufacturing Co. Inc., Batavia, New York.)

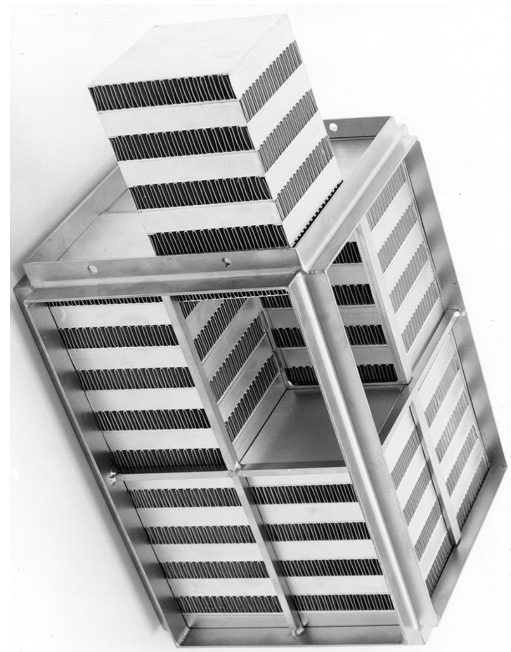
Figure 3.5 Typical commercial one-shell-pass, two-tube-pass heat exchangers.



a. A 1980 Chevette radiator. Cross-flow exchanger with neither flow mixed. Edges of flat vertical tubes can be seen.

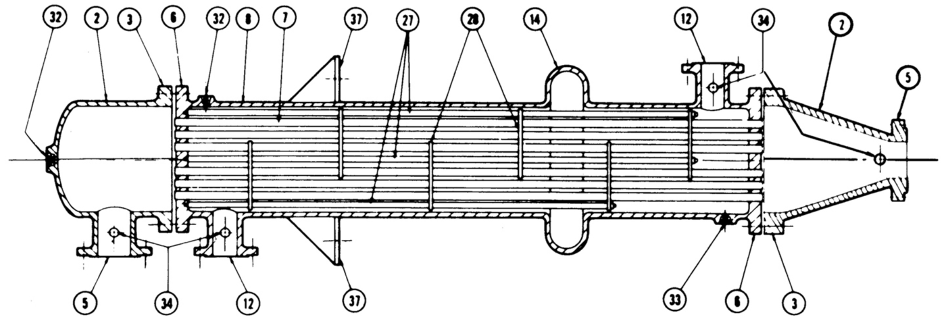


b. A section of an automotive air conditioning condenser. The flow through the horizontal wavy fins is allowed to mix with itself while the two-pass flow through the U-tubes remains unmixed.

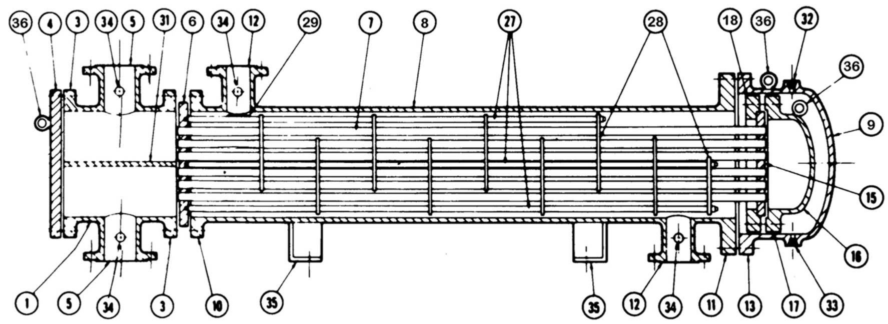


c. The basic 1 ft. $\times$ 1 ft.$\times$ 2 ft. module for a waste heat recuperator. It is a plate-fin, gas-to-air cross-flow heat exchanger with neither flow mixed.

Figure 3.6 Several commercial cross-flow heat exchangers. (Photographs courtesy of Harrison Radiator Division, General Motors Corporation.)



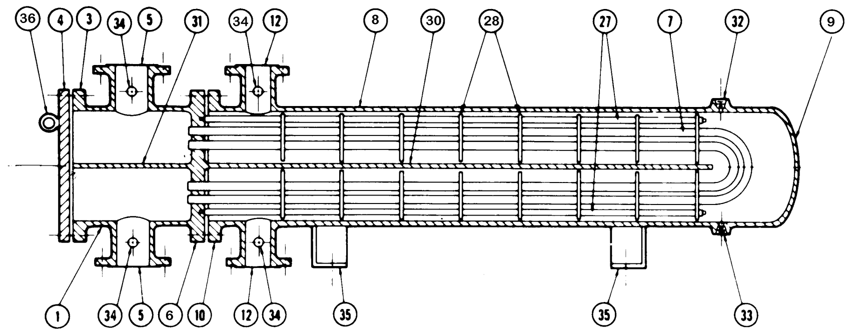
a) Single shell-pass, single tube-pass exchanger



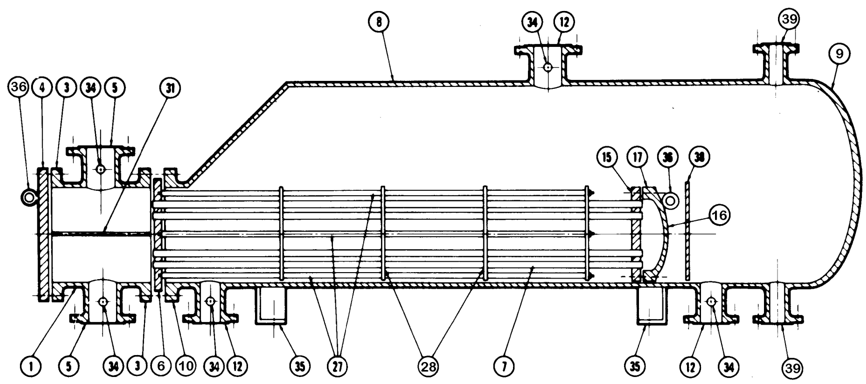
b) One shell-pass, two tube-pass exchanger

1. Stationary head-channel
2. Stationary head-bonnet
3. Stationary head-flange-channel or bonnet
4. Channel cover
5. Stationary head nozzle
6. Stationary tube sheet
7. Tubes
8. Shell
9. Shell cover
10. Shell flange-stationary head end
11. Shell flange-rear head end
12. Shell nozzle
13. Shell cover flange
14. Expansion joint
15. Floating tube sheet
16. Floating head cover
17. Floating head flange
18. Floating head backing device
19. Split shear ring
20. Slip-on backing flange
21. Floating head cover-external
22. Floating tube sheet skirt
23. Packing box
24. Packing
25. Packing gland
26. Lantern ring
27. Tie rods and spacers
28. Transverse baffles or support plates
29. Impingement plate
30. Longitudinal baffle
31. Pass partition
32. Vent connection
33. Drain connection
34. Instrument connection
35. Support saddle
36. Lifting lug
37. Support bracket
38. Weir
39. Liquid level connection

Figure 3.7 Four typical heat exchanger configurations (continued on next page). (Drawings courtesy of the Tubular Exchanger Manufacturers' Association.)



c) Two tube-pass, two shell-pass exchanger



d) One split shell-pass, two tube-pass, kettle type of exchanger

Figure 3.7 Continued

larly in shell-and-tube configurations and large condensers, U is apt to vary with position in the exchanger and/or with local temperature. But in situations in which U is fairly constant, we can deal with the varying temperatures of the fluid streams by writing the overall heat transfer in terms of a mean temperature difference between the two fluid streams:

$$Q = UA\,\Delta T_{\text{mean}} \tag{3.1}$$

Our problem then reduces to finding the appropriate mean temperature difference that will make this equation true. Let us do this for the simple parallel and counterflow configurations, as sketched in Fig. 3.8.

The temperature of both streams is plotted in Fig. 3.8 for both single-pass arrangements—the parallel and counterflow configurations—as a

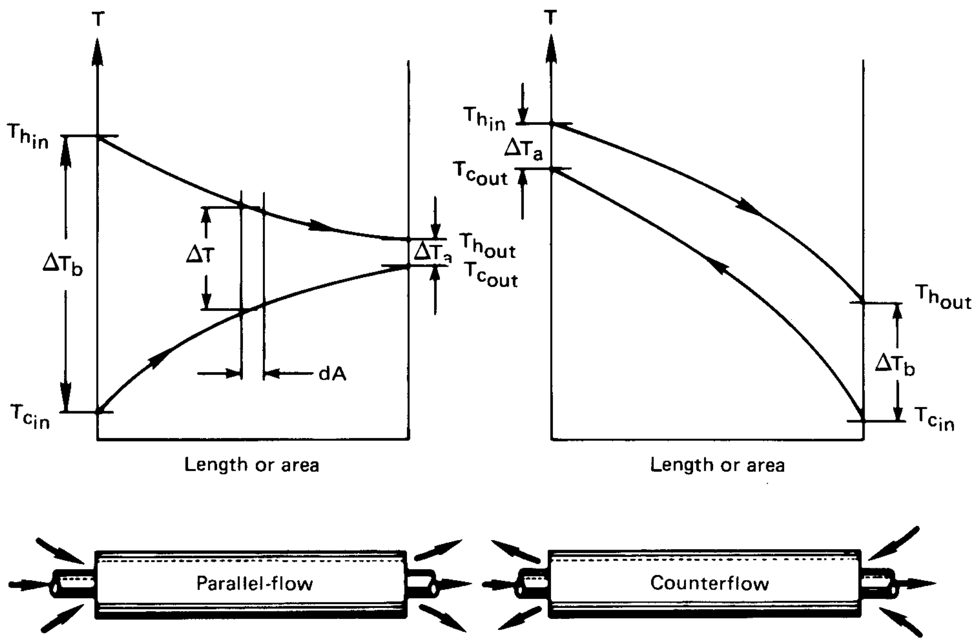


Figure 3.8 The temperature variation through single-pass heat exchangers.

function of the length of travel (or area passed over). Notice that, in the parallel-flow configuration, temperatures tend to change more rapidly with position and less length is required. But the counterflow arrangement achieves generally more complete heat exchange from one flow to the other.

Figure 3.9 shows another variation on the single-pass configuration. This is a condenser in which one stream flows through with its temperature changing, but the other simply condenses at uniform temperature. This arrangement has some special characteristics, which we point out shortly.

The determination of ΔT_{mean} for such arrangements proceeds as follows: the differential heat transfer within either arrangement (see Fig. 3.8) is

$$dQ = U\Delta T\, dA = -(\dot{m}c_p)_h\, dT_h = \pm(\dot{m}c_p)_c\, dT_c \tag{3.2}$$

where the subscripts h and c denote the hot and cold streams, respectively; the upper and lower signs are for the parallel and counterflow cases, respectively; and dT denotes a change from left to right in the

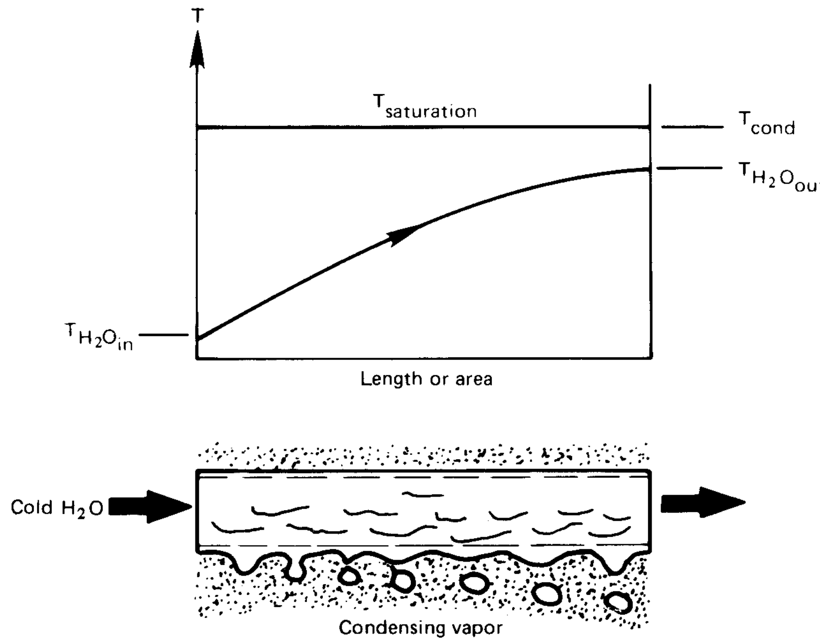


Figure 3.9 The temperature distribution through a condenser.

exchanger. We give symbols to the total heat capacities of the hot and cold streams:

$$C_h \equiv (\dot{m}c_p)_h \text{ W/K} \qquad \text{and} \qquad C_c \equiv (\dot{m}c_p)_c \text{ W/K} \tag{3.3}$$

Thus, for either heat exchanger, $\mp C_h dT_h = C_c dT_c$. This equation can be integrated from the lefthand side, where $T_h = T_{h_{\text{in}}}$ and $T_c = T_{c_{\text{in}}}$ for parallel flow or $T_h = T_{h_{\text{in}}}$ and $T_c = T_{c_{\text{out}}}$ for counterflow, to some arbitrary point inside the exchanger. The temperatures inside are thus:

$$\text{parallel flow:} \qquad T_h = T_{h_{\text{in}}} - \frac{C_c}{C_h}(T_c - T_{c_{\text{in}}}) \qquad = T_{h_{\text{in}}} - \frac{Q}{C_h} \tag{3.4a}$$

$$\text{counterflow:} \qquad T_h = T_{h_{\text{in}}} - \frac{C_c}{C_h}(T_{c_{\text{out}}} - T_c) \qquad = T_{h_{\text{in}}} - \frac{Q}{C_h} \tag{3.4b}$$

where Q is the total heat transfer from the entrance to the point of interest. Equations (3.4) can be solved for the local temperature differences:

$$\Delta T_{\text{parallel}} = T_h - T_c = T_{h_{\text{in}}} - \left(1 + \frac{C_c}{C_h}\right) T_c + \frac{C_c}{C_h} T_{c_{\text{in}}} \tag{3.5a}$$

$$\Delta T_{\text{counter}} = T_h - T_c = T_{h_{\text{in}}} - \left(1 - \frac{C_c}{C_h}\right) T_c - \frac{C_c}{C_h} T_{c_{\text{out}}} \tag{3.5b}$$

Substitution of these in $dQ = C_c \, dT_c = U \Delta T \, dA$ yields

$$\left.\frac{U dA}{C_c}\right|_{\text{parallel}} = \frac{dT_c}{\left[-\left(1 + \dfrac{C_c}{C_h}\right) T_c + \dfrac{C_c}{C_h} T_{c_{\text{in}}} + T_{h_{\text{in}}}\right]} \tag{3.6a}$$

$$\left.\frac{U dA}{C_c}\right|_{\text{counter}} = \frac{dT_c}{\left[-\left(1 - \dfrac{C_c}{C_h}\right) T_c - \dfrac{C_c}{C_h} T_{c_{\text{out}}} + T_{h_{\text{in}}}\right]} \tag{3.6b}$$

Equations (3.6) can be integrated across the exchanger:

$$\int_0^A \frac{U}{C_c} dA = \int_{T_{c\,\text{in}}}^{T_{c\,\text{out}}} \frac{dT_c}{[- - -]} \tag{3.7}$$

If U and C_c can be treated as constant, this integration gives

$$\begin{aligned}
\text{parallel:} \quad & \ln \left[\frac{-\left(1 + \dfrac{C_c}{C_h}\right) T_{c_{\text{out}}} + \dfrac{C_c}{C_h} T_{c_{\text{in}}} + T_{h_{\text{in}}}}{-\left(1 + \dfrac{C_c}{C_h}\right) T_{c_{\text{in}}} + \dfrac{C_c}{C_h} T_{c_{\text{in}}} + T_{h_{\text{in}}}} \right] = -\frac{UA}{C_c}\left(1 + \frac{C_c}{C_h}\right) \\
\text{counter:} \quad & \ln \left[\frac{-\left(1 - \dfrac{C_c}{C_h}\right) T_{c_{\text{out}}} - \dfrac{C_c}{C_h} T_{c_{\text{out}}} + T_{h_{\text{in}}}}{-\left(1 - \dfrac{C_c}{C_h}\right) T_{c_{\text{in}}} - \dfrac{C_c}{C_h} T_{c_{\text{out}}} + T_{h_{\text{in}}}} \right] = -\frac{UA}{C_c}\left(1 - \frac{C_c}{C_h}\right)
\end{aligned} \tag{3.8}$$

If U were variable, the integration leading from eqn. (3.7) to eqns. (3.8) is where its variability would have to be considered. Any such variability of U can complicate eqns. (3.8) terribly. Presuming that eqns. (3.8) are valid, we can simplify them with the help of the definitions of ΔT_a and ΔT_b, given in Fig. 3.8:

$$\begin{aligned}
\text{parallel:} \quad & \ln \left[\frac{(1 + C_c/C_h)(T_{c_{\text{in}}} - T_{c_{\text{out}}}) + \Delta T_b}{\Delta T_b} \right] = -UA\left(\frac{1}{C_c} + \frac{1}{C_h}\right) \\
\text{counter:} \quad & \ln \frac{\Delta T_a}{(-1 + C_c/C_h)(T_{c_{\text{in}}} - T_{c_{\text{out}}}) + \Delta T_a} = -UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)
\end{aligned} \tag{3.9}$$

Conservation of energy ($Q_c = Q_h$) requires that

$$\boxed{\frac{C_c}{C_h} = -\frac{T_{h_{\text{out}}} - T_{h_{\text{in}}}}{T_{c_{\text{out}}} - T_{c_{\text{in}}}}} \tag{3.10}$$

Then eqn. (3.9) and eqn. (3.10) give

$$\begin{aligned}
\text{parallel:}\quad & \ln\left[\frac{\overbrace{(T_{c_{\text{in}}} - T_{c_{\text{out}}}) + (T_{h_{\text{out}}} - T_{h_{\text{in}}})}^{\Delta T_a - \Delta T_b} + \Delta T_b}{\Delta T_b}\right] \\
& \qquad = \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = -UA\left(\frac{1}{C_c} + \frac{1}{C_h}\right) \\
\text{counter:}\quad & \ln\left(\frac{\Delta T_a}{\Delta T_b - \Delta T_a + \Delta T_a}\right) = \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = -UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right)
\end{aligned} \tag{3.11}$$

Finally, we write $1/C_c = (T_{c_{\text{out}}} - T_{c_{\text{in}}})/Q$ and $1/C_h = (T_{h_{\text{in}}} - T_{h_{\text{out}}})/Q$ on the right-hand side of either of eqns. (3.11) and get for either parallel or counterflow,

$$\boxed{Q = UA\left(\frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a/\Delta T_b)}\right)} \tag{3.12}$$

The appropriate ΔT_{mean} for use in eqn. (3.11) is thus the group on the right, which we call the *logarithmic mean temperature difference* (LMTD):

$$\boxed{\Delta T_{\text{mean}} = \text{LMTD} \equiv \frac{\Delta T_a - \Delta T_b}{\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)}} \tag{3.13}$$

Example 3.1

The idea of a logarithmic mean difference is not new to us. We have already encountered it in Chapter 2. Suppose that we had asked, "What mean radius of pipe would have allowed us to compute the conduction through the wall of a pipe as though it were a slab of thickness $L = r_o - r_i$?" (see Fig. 3.10). To answer this, we write

$$Q = kA\frac{\Delta T}{L} = k(2\pi r_{\text{mean}} l)\left(\frac{\Delta T}{r_o - r_i}\right)$$

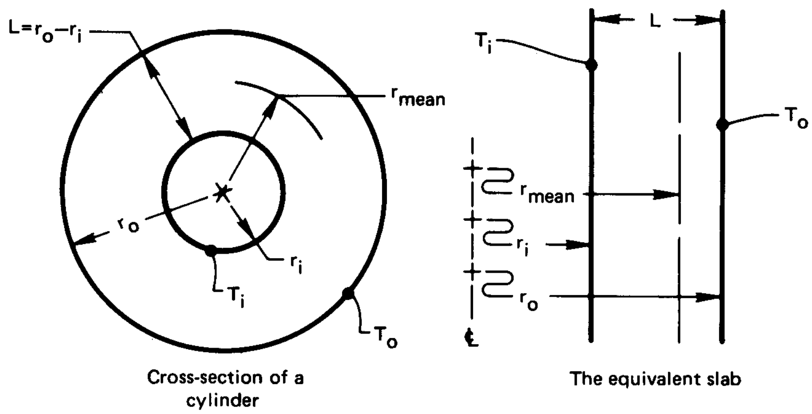


Figure 3.10 Calculation of the mean radius for heat conduction through a pipe.

and then compare it to eqn. (2.21):

$$Q = 2\pi k l \Delta T \, \frac{1}{\ln(r_o/r_i)}$$

It follows that

$$r_{\text{mean}} = \frac{r_o - r_i}{\ln(r_o/r_i)} = \text{logarithmic mean radius} \qquad \blacksquare$$

Example 3.2 Balanced Counterflow Heat Exchanger

Suppose that the heat capacity rates of a counterflow heat exchanger are equal, $C_h = C_c$. Such an exchanger is said to be *balanced.* From eqn. (3.5b), it follows the local temperature different in the exchanger is constant throughout, $\Delta T_{\text{counter}} = T_{h_{\text{in}}} - T_{h_{\text{out}}} = \Delta T_a = \Delta T_b$. Does the LMTD reduce to this value?

Solution. If we substitute $\Delta T_a = \Delta T_b$ in eqn. (3.13), we get

$$\text{LMTD} = \frac{\Delta T_b - \Delta T_b}{\ln(\Delta T_b/\Delta T_b)} = \frac{0}{0} = \text{indeterminate}$$

Therefore it is necessary to use L'Hospital's rule:

$$\lim_{\Delta T_a \to \Delta T_b} \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a / \Delta T_b)} = \frac{\left.\dfrac{\partial}{\partial \Delta T_a}(\Delta T_a - \Delta T_b)\right|_{\Delta T_a = \Delta T_b}}{\left.\dfrac{\partial}{\partial \Delta T_a}\ln\left(\dfrac{\Delta T_a}{\Delta T_b}\right)\right|_{\Delta T_a = \Delta T_b}}$$

$$= \left.\left(\frac{1}{1/\Delta T_a}\right)\right|_{\Delta T_a = \Delta T_b} = \Delta T_a = \Delta T_b$$

So LMTD does indeed reduce to the intuitively obvious result when the capacity rates are balanced. ■

Example 3.3

Water enters the tubes of a small single-pass heat exchanger at 20°C and leaves at 40°C. On the shell side, 25 kg/min of steam condenses at 60°C. Calculate the overall heat transfer coefficient and the required flow rate of water if the area of the exchanger is 12 m^2. (The latent heat, h_{fg}, is 2358.7 kJ/kg at 60°C.)

Solution.

$$Q = \dot{m}_{\text{condensate}} \cdot h_{fg}\Big|_{60^\circ\text{C}} = \frac{25(2358.7)}{60} = 983 \text{ kJ/s}$$

and with reference to Fig. 3.9, we can calculate the LMTD without naming the exchanger "parallel" or "counterflow", since the condensate temperature is constant.

$$\text{LMTD} = \frac{(60 - 20) - (60 - 40)}{\ln\left(\dfrac{60 - 20}{60 - 40}\right)} = 28.85 \text{ K}$$

Then

$$U = \frac{Q}{A(\text{LMTD})} = \frac{983(1000)}{12(28.85)} = 2839 \text{ W/m}^2\text{K}$$

and

$$\dot{m}_{H_2O} = \frac{Q}{c_p \Delta T} = \frac{983{,}000}{4174(20)} = 11.78 \text{ kg/s}$$ ■

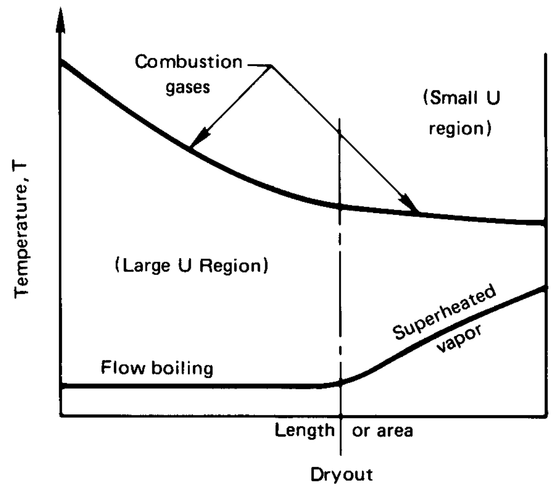


Figure 3.11 A typical case of a heat exchanger in which U varies dramatically.

Extended use of the LMTD

Limitations. The use of an LMTD is limited in two basic ways. The first is that it is restricted to the single-pass parallel and counterflow configurations. This restriction can be overcome by adjusting the LMTD for other configurations—a matter that we take up in the following subsection.

The second limitation—our use of a constant value of U—is harder to deal with. The value of U must be negligibly dependent on T to complete the integration of eqn. (3.7). Even if $U \neq \text{fn}(T)$, the changing flow configuration and the variation of temperature can still give rise to serious variations of U within a given heat exchanger. Figure 3.11 shows a typical situation in which the variation of U within a heat exchanger might be great. In this case, the mechanism of heat exchange on the water side is completely altered when the liquid is finally boiled away. If U were uniform in each portion of the heat exchanger, then we could treat it as two different exchangers in series.

However, the more common difficulty is that of designing heat exchangers in which U varies continuously with position within it. This problem is most severe in large industrial shell-and-tube configurations[1]

[1] Actual heat exchangers can have areas in excess of 10,000 m^2. Large power plant condensers and other large exchangers are often remarkably big pieces of equipment.

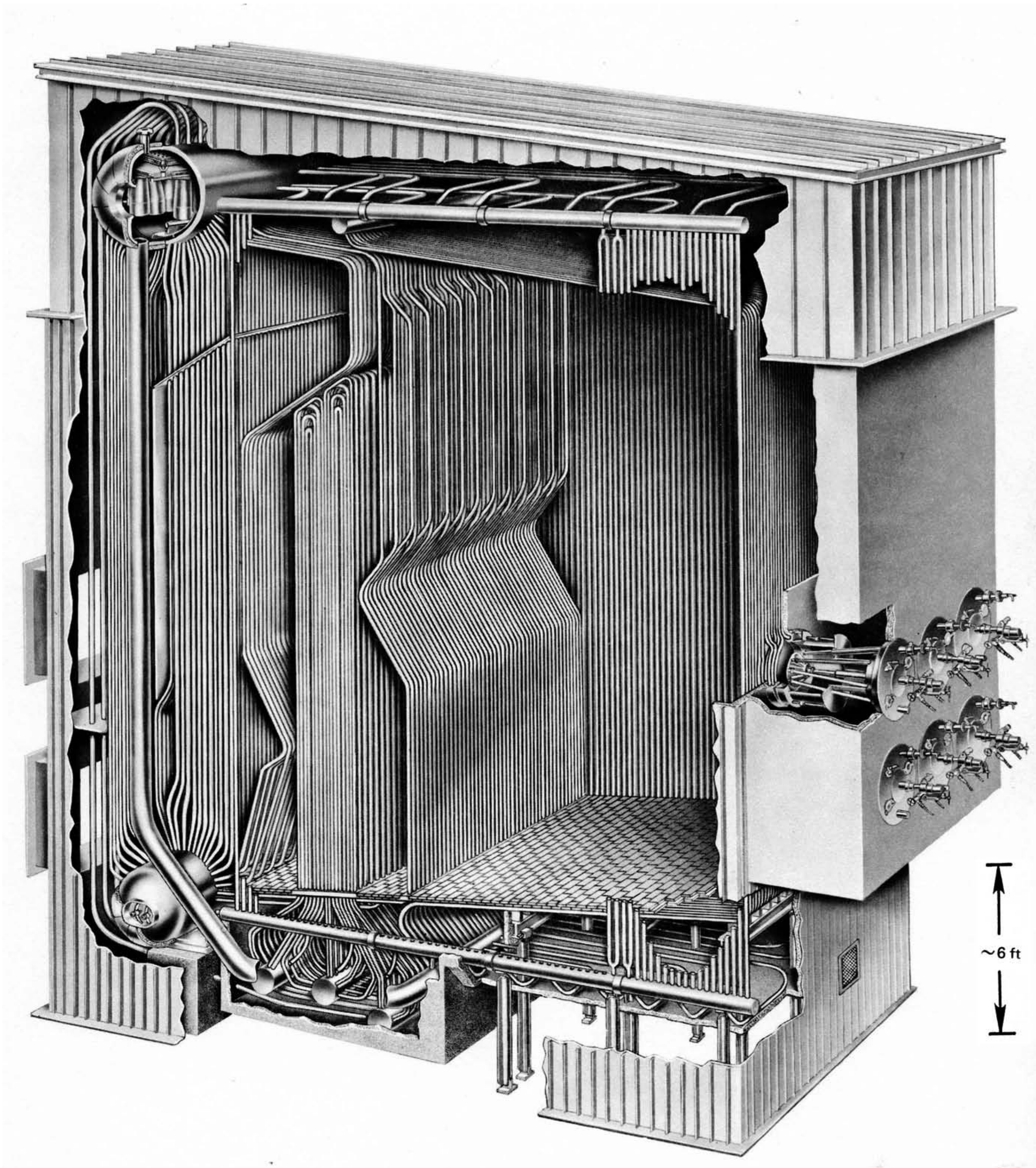


Figure 3.12 The heat exchange surface for a steam generator. This PFT-type integral-furnace boiler, with a surface area of 4560 m^2, is not particularly large. About 88% of the area is in the furnace tubing and 12% is in the boiler (Photograph courtesy of Babcock and Wilcox Co.)

(see, e.g., Fig. 3.5 or Fig. 3.12) and less serious in compact heat exchangers with less surface area. If U depends on the location, analyses such as we have just completed [eqn. (3.1) to eqn. (3.13)] must be done using an average U defined as $\int_0^A U\,dA/A$.

LMTD correction factor, *F*. Suppose we have a heat exchanger in which U can reasonably be taken constant, but one that involves such configurational complications as multiple passes and/or cross-flow. In such cases we must rederive the appropriate mean temperature difference in the same way as we derived the LMTD. Each configuration must be analyzed separately and the results are generally more complicated than eqn. (3.13).

This task was undertaken on an *ad hoc* basis during the early twentieth century. In 1940, Bowman, Mueller and Nagle [3.2] organized such calculations for the common range of heat exchanger configurations. In each case they wrote

$$Q = UA(\text{LMTD}) \cdot F\left(\underbrace{\frac{T_{t_{\text{out}}} - T_{t_{\text{in}}}}{T_{s_{\text{in}}} - T_{t_{\text{in}}}}}_{P}, \underbrace{\frac{T_{s_{\text{in}}} - T_{s_{\text{out}}}}{T_{t_{\text{out}}} - T_{t_{\text{in}}}}}_{R}\right) \tag{3.14}$$

where T_t and T_s are temperatures of tube and shell flows, respectively. The factor F is an LMTD correction that varies from one to zero, depending on conditions. The dimensionless groups P and R have the following physical significance:

- P is the relative influence of the overall temperature difference $(T_{s_{\text{in}}} - T_{t_{\text{in}}})$ on the tube flow temperature. It must obviously be less than one.

- R, according to eqn. (3.10), equals the heat capacity ratio C_t/C_s.

- If one flow remains at constant temperature (as, for example, in Fig. 3.9), then either P or R will equal zero. In this case the simple LMTD will be the correct ΔT_{mean} and F must go to one.

The factor F is defined in such a way that *the LMTD should always be calculated for the equivalent counterflow single-pass exchanger with the same hot and cold temperatures.* This is explained in Fig. 3.13.

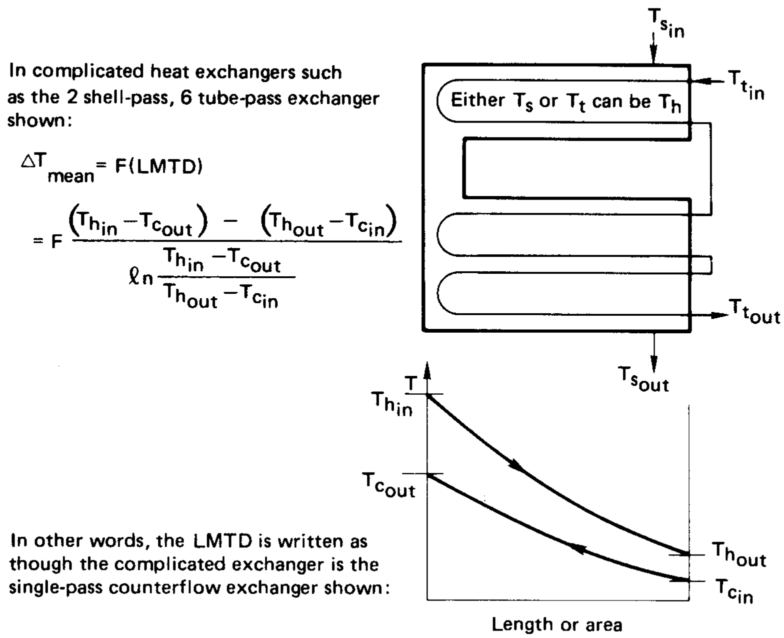
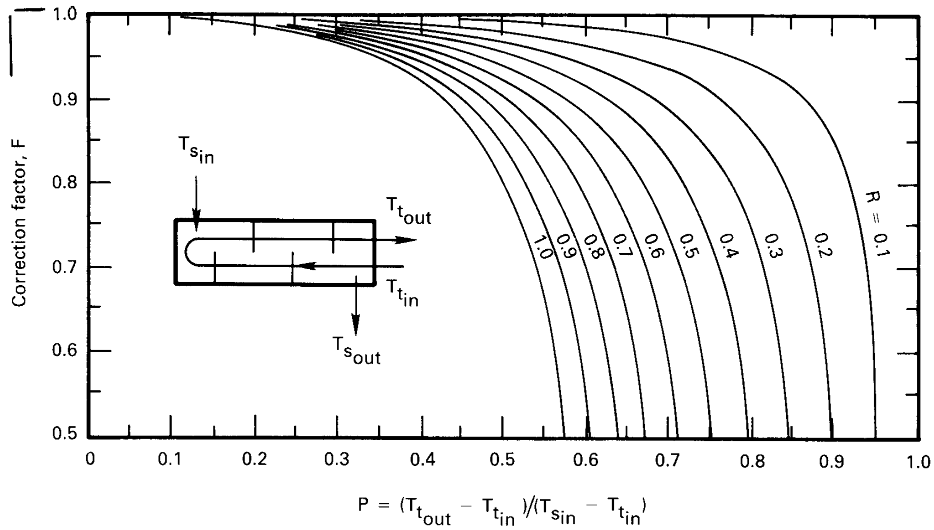


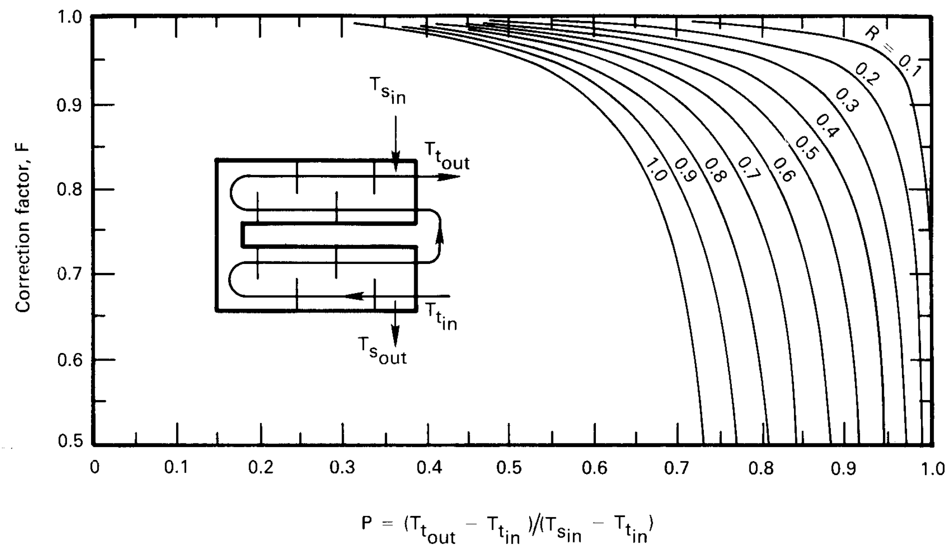
Figure 3.13 The basis of the LMTD in a multipass exchanger, prior to correction.

Bowman *et al.* [3.2] summarized all the equations for F, in various configurations, that had been dervied by 1940. They presented them graphically in not-very-accurate figures that have been widely copied. The TEMA [3.1] version of these curves has been recalculated for shell-and-tube heat exchangers, and it is more accurate. We include two of these curves in Fig. 3.14(a) and Fig. 3.14(b). TEMA presents many additional curves for more complex shell-and-tube configurations. Figures 3.14(c) and 3.14(d) are the Bowman *et al.* curves for the simplest cross-flow configurations. Gardner and Taborek [3.3] redeveloped Fig. 3.14(c) over a different range of parameters. They also showed how Fig. 3.14(a) and Fig. 3.14(b) must be modified if the number of baffles in a tube-in-shell heat exchanger is large enough to make it behave like a series of cross-flow exchangers.

We have simplified Figs. 3.14(a) through 3.14(d) by including curves only for $R \leqslant 1$. Shamsundar [3.4] noted that for $R > 1$, one may obtain F using a simple reciprocal rule. He showed that so long as a heat exchanger has a uniform heat transfer coefficient and the fluid properties are

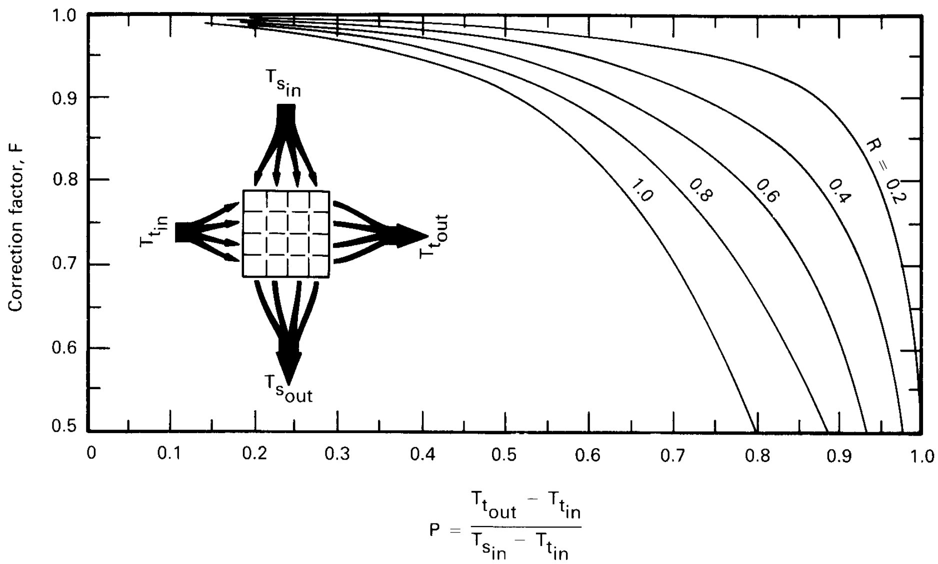


a. *F* for a one-shell-pass, four, six-,... tube-pass exchanger.

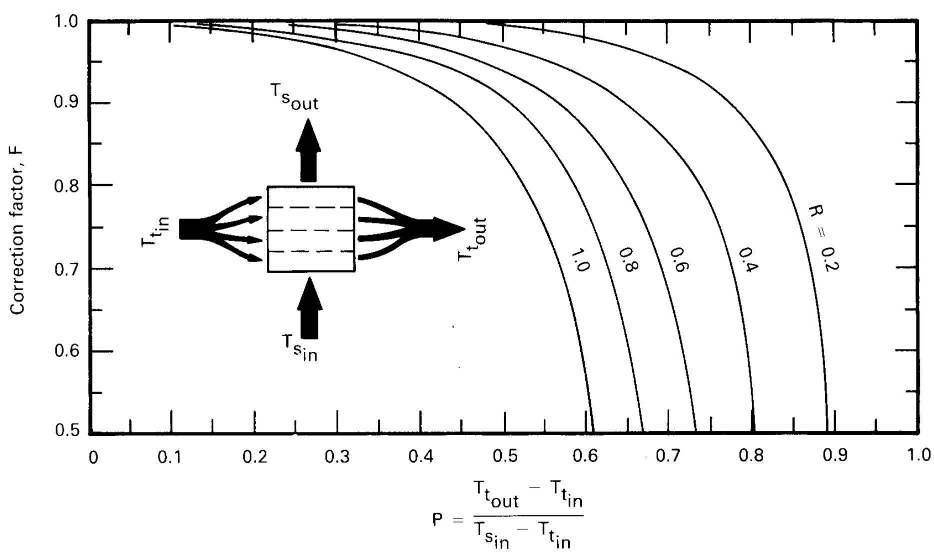


b. *F* for a two-shell-pass, four or more tube-pass exchanger.

Figure 3.14 LMTD correction factors, *F*, for multipass shell-and-tube heat exchangers and one-pass cross-flow exchangers.



c. *F* for a one-pass cross-flow exchanger with both passes unmixed.



d. *F* for a one-pass cross-flow exchanger with one pass mixed.

Figure 3.14 LMTD correction factors, *F*, for multipass shell-and-tube heat exchangers and one-pass cross-flow exchangers.

constant,

$$F(P,R) = F(PR, 1/R) \tag{3.15}$$

Thus, if R is greater than one, we need only evaluate F using PR in place of P and $1/R$ in place of R.

Example 3.4

5.795 kg/s of oil flows through the shell side of a two-shell pass, four-tube-pass oil cooler. The oil enters at 181°C and leaves at 38°C. Water flows in the tubes, entering at 32°C and leaving at 49°C. In addition, $c_{p_{\text{oil}}} = 2282$ J/kg·K and $U = 416$ W/m^{2}K. Find how much area the heat exchanger must have.

Solution.

$$\text{LMTD} = \frac{(T_{h_{\text{in}}} - T_{c_{\text{out}}}) - (T_{h_{\text{out}}} - T_{c_{\text{in}}})}{\ln\left(\dfrac{T_{h_{\text{in}}} - T_{c_{\text{out}}}}{T_{h_{\text{out}}} - T_{c_{\text{in}}}}\right)}$$

$$= \frac{(181-49)-(38-32)}{\ln\left(\dfrac{181-49}{38-32}\right)} = 40.76 \text{ K}$$

$$R = \frac{181-38}{49-32} = 8.412 \qquad P = \frac{49-32}{181-32} = 0.114$$

Since $R > 1$, we enter Fig. 3.14(b) using $P = 8.412(0.114) = 0.959$ and $R = 1/8.412 = 0.119$ and obtain $F = 0.92$.[2] It follows that:

$$Q = UAF(\text{LMTD})$$

$$5.795(2282)(181-38) = 416(A)(0.92)(40.76)$$

$$A = 121.2 \text{ m}^2$$

■

3.3 Heat exchanger effectiveness

We are now in a position to predict the performance of an exchanger once we know its configuration *and* the imposed temperature differences. Unfortunately, we do not often know that much about a system before the design is complete.

[2]Notice that, for a 1 shell-pass exchanger, these R and P lines do not quite intersect [see Fig. 3.14(a)]. Therefore, no single-shell exchanger would give these values.

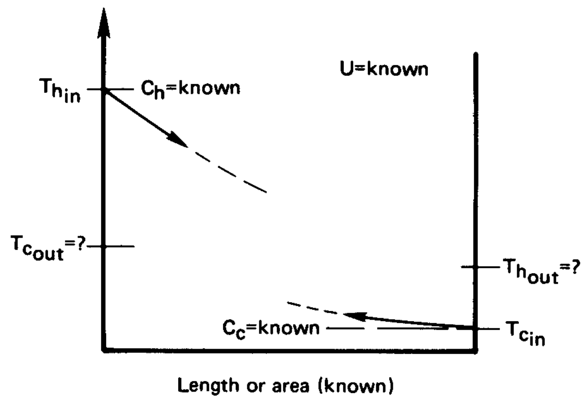


Figure 3.15 A design problem in which the LMTD cannot be calculated a priori.

Often we begin with information such as is shown in Fig. 3.15. If we sought to calculate Q in such a case, we would have to do so by guessing an exit temperature such as to make $Q_h = Q_c = C_h \Delta T_h = C_c \Delta T_c$. Then we could calculate Q from UA(LMTD) or UAF(LMTD) and check it against Q_h. The answers would differ, so we would have to guess new exit temperatures and try again.

Such problems can be greatly simplified with the help of the so-called *effectiveness-NTU method*. This method was first developed in full detail by Kays and London [3.5] in 1955, in a book titled *Compact Heat Exchangers*. We should take particular note of the title. It is with compact heat exchangers that the present method can reasonably be used, since the overall heat transfer coefficient is far more likely to remain fairly uniform.

The heat exchanger effectiveness is defined as

$$\varepsilon \equiv \frac{\text{actual heat transferred}}{\begin{array}{c}\text{maximum heat that could possibly be}\\ \text{transferred from one stream to the other}\end{array}}$$

In mathematical terms, this is

$$\varepsilon = \frac{C_h(T_{h_\text{in}} - T_{h_\text{out}})}{C_\text{min}(T_{h_\text{in}} - T_{c_\text{in}})} = \frac{C_c(T_{c_\text{out}} - T_{c_\text{in}})}{C_\text{min}(T_{h_\text{in}} - T_{c_\text{in}})} \tag{3.16}$$

where C_min is the smaller of C_c and C_h.

It follows that

$$Q = \varepsilon C_\text{min}(T_{h_\text{in}} - T_{c_\text{in}}) \tag{3.17}$$

A second definition that we will need was originally made by E.K.W. Nusselt, whom we meet again in Part III. This is the *number of transfer units* (NTU):

$$\text{NTU} \equiv \frac{UA}{C_{\min}} \tag{3.18}$$

This dimensionless group can be viewed as a comparison of the heat rate capacity of the heat exchanger, expressed in W/K, with the heat capacity rate of the flow.

We can immediately reduce the parallel-flow result from eqn. (3.9) to the following equation, based on these definitions:

$$-\left(\frac{C_{\min}}{C_c} + \frac{C_{\min}}{C_h}\right)\text{NTU} = \ln\left[-\left(1 + \frac{C_c}{C_h}\right)\varepsilon\frac{C_{\min}}{C_c} + 1\right] \tag{3.19}$$

We solve this for ε and, regardless of whether $C_{\min}$ is associated with the hot or cold flow, obtain for the parallel single-pass heat exchanger:

$$\varepsilon \equiv \frac{1 - \exp\left[-(1 + C_{\min}/C_{\max})\text{NTU}\right]}{1 + C_{\min}/C_{\max}} = \text{fn}\left(\frac{C_{\min}}{C_{\max}}, \text{NTU only}\right) \tag{3.20}$$

The corresponding expression for the counterflow case is

$$\varepsilon = \frac{1 - \exp\left[-(1 - C_{\min}/C_{\max})\text{NTU}\right]}{1 - (C_{\min}/C_{\max})\exp\left[-(1 - C_{\min}/C_{\max})\text{NTU}\right]} \tag{3.21}$$

Equations (3.20) and (3.21) are given in graphical form in Fig. 3.16. Similar calculations give the effectiveness for the other heat exchanger configurations (see [3.5] and Problem 3.38), and we include some of the resulting effectiveness plots in Fig. 3.17. The use of effectiveness to rate the performance of an existing heat exchanger and to fix the size of a new one are illustrated in the following two examples.

Example 3.5

Consider the following parallel-flow heat exchanger specification:

$$\begin{aligned} \text{cold flow enters at } 40^\circ\text{C:} \quad & C_c = 20{,}000 \text{ W/K} \\ \text{hot flow enters at } 150^\circ\text{C:} \quad & C_h = 10{,}000 \text{ W/K} \\ A = 30 \text{ m}^2 \qquad & U = 500 \text{ W/m}^2\text{K.} \end{aligned}$$

Determine the heat transfer and the exit temperatures.

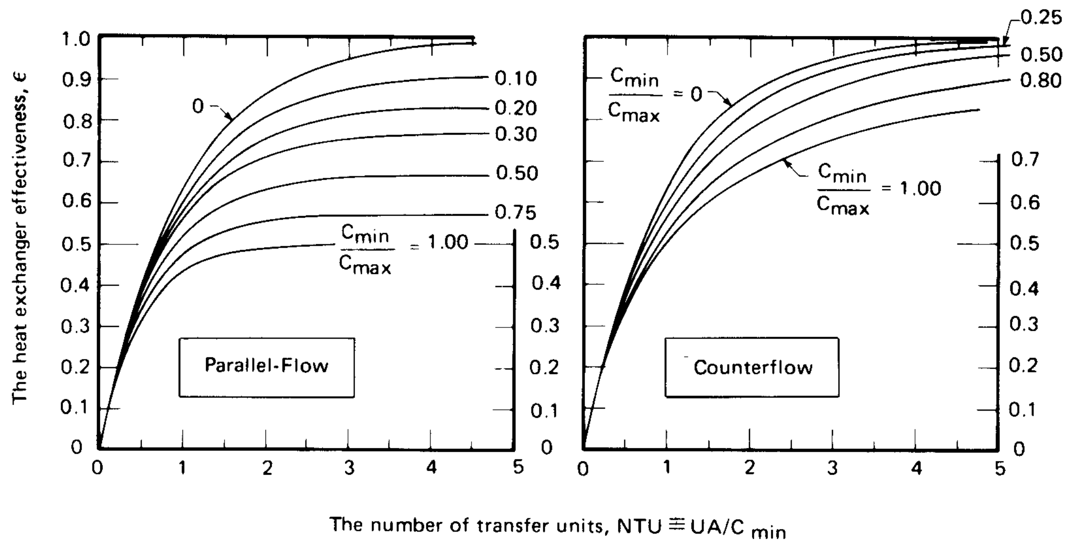


Figure 3.16 The effectiveness of parallel and counterflow heat exchangers. (Data provided by A.D. Kraus.)

Solution. In this case we do not know the exit temperatures, so it is not possible to calculate the LMTD. Instead, we can go either to the parallel-flow effectiveness chart in Fig. 3.16 or to eqn. (3.20), using

$$\text{NTU} = \frac{UA}{C_\text{min}} = \frac{500(30)}{10,000} = 1.5$$

$$\frac{C_\text{min}}{C_\text{max}} = 0.5$$

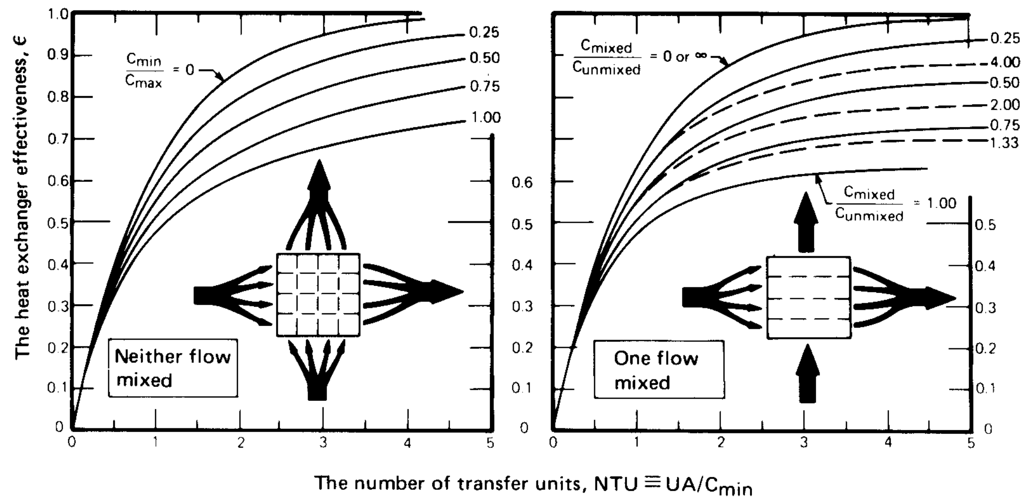
and we obtain $\varepsilon = 0.596$. Now from eqn. (3.17), we find that

$$\begin{aligned} Q = \varepsilon\, C_\text{min}(T_{h_\text{in}} - T_{c_\text{in}}) &= 0.596(10,000)(110) \\ &= 655,600 \text{ W} = 655.6 \text{ kW} \end{aligned}$$

Finally, from energy balances, such as are expressed in eqn. (3.4), we get

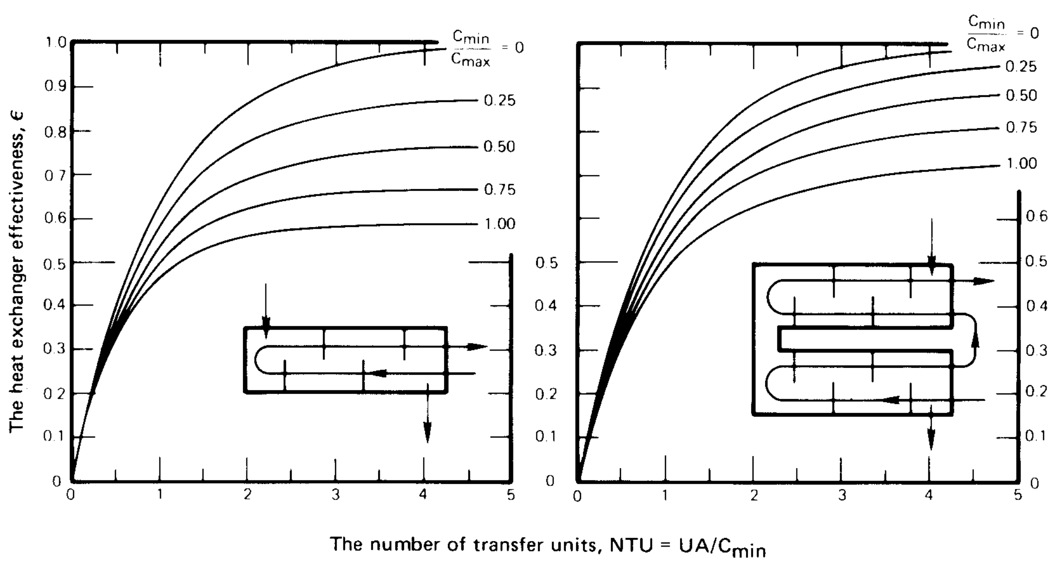
$$T_{h_\text{out}} = T_{h_\text{in}} - \frac{Q}{C_h} = 150 - \frac{655,600}{10,000} = 84.44°\text{C}$$

$$T_{c_\text{out}} = T_{c_\text{in}} + \frac{Q}{C_c} = 40 + \frac{655,600}{20,000} = 72.78°\text{C} \qquad \blacksquare$$



a.) Cross-flow exchanger, neither fluid mixed

b.) Cross-flow exchanger, one fluid mixed



c.) One shell pass, two tube pass exchanger. (Can also be used for 4, 6, 8, 10, 12 tube passes with a maximum error in ε, of 0.040 at $C_{min}/C_{max} = 1$ and large NTU.)

d.) Two shell pass 4 tube pass exchanger. (Can also be used for 4, 6, 8, ... tube passes with reasonable accuracy if there are equal numbers of tube passes in each shell pass.)

Figure 3.17 The effectiveness of some other heat exchanger configurations. (Data provided by A.D. Kraus.)

Example 3.6

Suppose that we had the same kind of exchanger as we considered in Example 3.5, but that the area remained an unspecified design variable. Calculate the area that would bring the hot flow out at 90°C.

Solution. Once the exit cold fluid temperature is known, the problem can be solved with equal ease by either the LMTD or the effectiveness approach. An energy balance [eqn. (3.4a)] gives

$$T_{c_{\text{out}}} = T_{c_{\text{in}}} + \frac{C_h}{C_c}(T_{h_{\text{in}}} - T_{h_{\text{out}}}) = 40 + \frac{1}{2}(150 - 90) = 70°\text{C}$$

Then, using the effectiveness method,

$$\varepsilon = \frac{C_h(T_{h_{\text{in}}} - T_{h_{\text{out}}})}{C_{\min}(T_{h_{\text{in}}} - T_{c_{\text{in}}})} = \frac{10,000(150 - 90)}{10,000(150 - 40)} = 0.5455$$

so from Fig. 3.16 we read NTU $\simeq$1.15 $= UA/C_{\min}$. Thus

$$A = \frac{10,000(1.15)}{500} = 23.00 \text{ m}^2$$

We could also have calculated the LMTD:

$$\text{LMTD} = \frac{(150 - 40) - (90 - 70)}{\ln(110/20)} = 52.79 \text{ K}$$

so from $Q = UA(\text{LMTD})$, we obtain

$$A = \frac{10,000(150 - 90)}{500(52.79)} = 22.73 \text{ m}^2$$

The answers differ by 1%, which reflects graph reading inaccuracy. ■

Single stream heat exchangers. When the temperature of either fluid in a heat exchanger is uniform, the problem of analyzing heat transfer is greatly simplified. We have already noted that no F-correction is needed to adjust the LMTD in this case. The reason is that when only one fluid changes in temperature, the configuration of the exchanger becomes irrelevant. Any such exchanger is equivalent to a single fluid stream flowing through an isothermal pipe.[3]

The single stream limit, in which one stream's temperature is constant, occurs when heat capacity rate ratio $C_{\min}/C_{\max}$ goes to zero. The

[3] We make use of this notion in Section 7.4, when we analyze heat convection in pipes and tubes.

heat capacity rate ratio might *approach* zero because the flow rate or specific heat of one stream is very large compared to the other, as when a high flow mass rate of water cools a very low mass flow rate of air. Alternatively, it might *be* infinite because the flow is absorbing or giving up latent heat (as in Fig. 3.9). Since all heat exchangers are equivalent in this case, it follows that the equation for the effectiveness in any configuration must reduce to the same common expression. This limiting expression can be derived directly from energy-balance considerations (see Problem 3.11), but we obtain it here by letting $C_{\text{min}}/C_{\text{max}} \to 0$ in either eqn. (3.20) or eqn. (3.21). The result is

$$\varepsilon_{\text{single stream}} = 1 - e^{-\text{NTU}} \tag{3.22}$$

Eqn. (3.22) defines the curve for $C_{\text{min}}/C_{\text{max}} = 0$ in all six of the effectiveness graphs in Fig. 3.16 and Fig. 3.17.

Balanced counterflow heat exchangers. In Example 3.2, we saw that when the heat capacity rates are balanced in a counterflow heat exchanger, so that $C_h = C_c$ (or $C_{\text{max}} = C_{\text{min}}$), the temperature difference between the hot and cold streams is constant. In this case, the effectiveness equation [eqn. 3.21] limits to

$$\varepsilon = \frac{\text{NTU}}{1 + \text{NTU}} \tag{3.23}$$

(see Problem 3.37). The balanced counterflow arrangement is used for heat recovery in power cycles and ventilation systems; for example, a warm exhaust air stream may be used to preheat an incoming cold air stream.

3.4 Heat exchanger design

The preceding sections provided means for designing heat exchangers that generally work well in the design of smaller exchangers—typically, the kind of compact cross-flow exchanger used in transportation equipment. Larger shell-and-tube exchangers pose two kinds of difficulty in relation to U. The first is the variation of U through the exchanger, which we have already discussed. The second difficulty is that convective heat transfer coefficients are very hard to predict for the complicated flows that move through a baffled shell.

We shall achieve considerable success in using analysis to predict $\overline{h}$'s for various convective flows in Part III. The determination of $\overline{h}$ in a baffled shell remains a problem that cannot be solved analytically. Instead, it is normally computed with the help of empirical correlations or with the aid of large commercial computer programs that include relevant experimental correlations. The problem of predicting $\overline{h}$ when the flow is boiling or condensing is even more complicated and generally requires the use of empirical correlations of $\overline{h}$ data.

Apart from predicting heat transfer, a host of additional considerations must be addressed in designing heat exchangers. The primary ones are minimizing pumping power and fixed costs.

The pumping power calculation, which we do not treat here in any detail, is based on the principles discussed in a first course on fluid mechanics. It generally takes the following form for each stream of fluid through the heat exchanger:

$$\begin{aligned} \text{pumping power} &= \left(\dot{m}\,\frac{\text{kg}}{\text{s}}\right)\left(\frac{\Delta p}{\rho}\,\frac{\text{N/m}^2}{\text{kg/m}^3}\right) = \frac{\dot{m}\Delta p}{\rho}\left(\frac{\text{N}\cdot\text{m}}{\text{s}}\right) \\ &= \frac{\dot{m}\Delta p}{\rho}\ (\text{W}) \end{aligned} \tag{3.24}$$

where $\dot{m}$ is the mass flow rate of the stream, Δp the pressure drop of the stream as it passes through the exchanger, and ρ the fluid density.

Determining the pressure drop can be relatively straightforward in a single-pass pipe-in-tube heat exchanger or extremely difficult in, say, a shell-and-tube exchanger. The pressure drop in a straight run of pipe, for example, is given by

$$\Delta p = f\left(\frac{L}{D_h}\right)\frac{\rho u_{\text{av}}^2}{2} \tag{3.25}$$

where L is the length of pipe, D_h is the hydraulic diameter, u_{av} is the mean velocity of the flow in the pipe, and f is the Darcy-Weisbach friction factor (see Fig. 7.6).

Optimizing the design of an exchanger is not just a matter of making Δp as small as possible. Often, heat exchange can be augmented by employing fins or roughening elements in an exchanger. (We discuss such elements in Chapter 4; see, e.g., Fig. 4.6). Such augmentation will invariably increase the pressure drop, but it can also reduce the fixed cost of an exchanger by increasing U and reducing the required area. Furthermore, it can reduce the required flow rate of, say, coolant, by increasing the effectiveness and thus balance the increase of Δp in eqn. (3.24).

To better understand the course of the design process, faced with such an array of trade-offs of advantages and penalties, we follow Taborek's [3.6] list of design considerations for a large shell-and-tube exchanger:

- Decide which fluid should flow on the shell side and which should flow in the tubes. Normally, this decision will be made to minimize the pumping cost. If, for example, water is being used to cool oil, the more viscous oil would flow in the shell. Corrosion behavior, fouling, and the problems of cleaning fouled tubes also weigh heavily in this decision.

- Early in the process, the designer should assess the cost of the calculation in comparison with:

  (a) The converging accuracy of computation.

  (b) The investment in the exchanger.

  (c) The cost of miscalculation.

- Make a rough estimate of the size of the heat exchanger using, for example, U values from Table 2.2 and/or anything else that might be known from experience. This serves to circumscribe the subsequent trial-and-error calculations; it will help to size flow rates and to anticipate temperature variations; and it will help to avoid subsequent errors.

- Evaluate the heat transfer, pressure drop, and cost of various exchanger configurations that appear reasonable for the application. This is usually done with specialized computer programs that have been developed and are constantly being improved as new research is included in them.

These are the sort of steps incorporated into the many available software packages for heat exchanger design. However, few students of heat transfer will be called upon to use these routines. Instead, most *will* be called upon at one time or another to design smaller exchangers in the range 0.1 to 10 m^2. The heat transfer calculation can usually be done effectively with the methods described in this chapter. Some useful sources of guidance in the pressure drop calculation are the *Heat Exchanger Design Handbook* [3.7], the data in Idelchik's collection [3.8], the TEMA design book [3.1], and some of the other references at the end of this chapter.

In such a calculation, we start off with one fluid to heat and one to cool. Perhaps we know the flow heat capacity rates (C_c and C_h), certain temperatures, and/or the amount of heat that is to be transferred. The problem can be annoyingly wide open, and nothing can be done until it is somehow delimited. The normal starting point is the specification of an exchanger configuration, and to make this choice one needs experience. The descriptions in this chapter provide a kind of first level of experience. References [3.5, 3.7, 3.9, 3.10, 3.11, 3.12, 3.13] provide a second level. Manufacturer's catalogues are an excellent source of more advanced information.

Once the exchanger configuration is set, U will be approximately set and the area becomes the basic design variable. The design can then proceed along the lines of Section 3.2 or 3.3. If it is possible to begin with a complete specification of inlet and outlet temperatures,

$$\underbrace{Q}_{C\Delta T} = \underbrace{U}_{\text{known}} \underbrace{AF(\text{LMTD})}_{\text{calculable}}$$

Then A can be calculated and the design completed. Usually, a reevaluation of U and some iteration of the calculation is needed.

More often, we begin without full knowledge of the outlet temperatures. In such cases, we normally have to invent an appropriate trial-and-error method to get the area and a more complicated sequence of trials if we seek to optimize pressure drop and cost by varying the configuration as well. If the C's are design variables, the U will change significantly, because $\overline{h}$'s are generally velocity-dependent and more iteration will be needed.

We conclude Part I of this book facing a variety of incomplete issues. Most notably, we face a serious need to be able to determine convective heat transfer coefficients. The prediction of $\overline{h}$ depends on a knowledge of heat conduction. We therefore turn, in Part II, to a much more thorough study of heat conduction analysis than was undertaken in Chapter 2. In addition to setting up the methods ultimately needed to predict $\overline{h}$'s, Part II also deals with many other issues that have great practical importance in their own right.

Problems

3.1 Can you have a cross-flow exchanger in which both flows are mixed? Discuss.