Chapter 12

Areas Related to Circles

In the Chapter

In this chapter, you will be studying the following points:

- Circumference of a circle = $2\pi r$.
- Area of a circle = πr^2.
- Length of an arc of a sector of a circle with radius r and angle with degree measure θ is = $\frac{\theta}{360°} \times \pi r^2$.
- Area of a sector of a circle with radius r and angle with degrees measure θ is = $\frac{\theta}{360°} \times \pi r^2$.
- Area of segment of a circle
  = Area of the corresponding sector – Area of the corresponding triangle.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 12.1

Q.1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Ans. Let the radius of the bigger circle be R cm and radii of two smaller circles are r_1 and r_2, then according to question.

$2\pi R = 2\pi r_1 + 2\pi r_2$

$\Rightarrow \quad 2\pi R = 2\pi (19) + 2\pi (9)$

$\Rightarrow \quad 2\pi R = 2\pi (19 + 9)$

$\Rightarrow \quad R = 28$

Hence, the radius of the circle is 28 cm

Q.2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Ans. Let the radius of the required circle be R and radii of two given circle be r_1 and r_2, then according to question,

$2\pi R^2 = 2\pi r_1^2 + 2\pi r_2^2$

$\Rightarrow \quad 2\pi R^2 = 2\pi (r_1^2 + r_2^2)$

$\Rightarrow \quad R^2 = 64 + 36$

$\Rightarrow \quad R^2 = 100$

$\Rightarrow \quad R = 10$ cm

Q.3. In Fig. depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Ans. We have,

r = Radius of the region representing Gold score = 10.5 cm

r_1 = Radius of the region representing Gold and Red scoring areas
= (10.5 + 10.5) cm = 21cm = $2r$ cm

r_2 = Radius of the region representing Gold, Red and Blue scoring areas
= (21+10.5) cm = 31.5 cm = $3r$ cm

r_3 = Radius of the region representing Gold, Red, Blue and Black scoring areas
= (31.5 + 10.5) cm = 42 cm = $4r$ cm
r_4 = Radius of the region representing Gold, Red, Blue, Black and white scoring areas
= (42 + 10.5) cm = 52.5 cm = $5r$ cm
Now, A_1 = Area of the region representing Gold scoring area

$$= \pi r^2 = \frac{22}{7} \times (10.5)^2 = \frac{22}{7} \times 10.5 \times 10.5$$

$= 22 \times 1.5 \times 10.5 = 346.5 \text{ cm}^2$
A_2 = Area of the region representing Red scoring area
$= \pi (2r)^2 - \pi r^2 = 3\pi r^2 = 3A_1$
$= 3 \times 346.5 \text{ cm}^2 = 1039.5 \text{ cm}^2$
A_3 = Area of the region representing Blue scoring area
$= \pi(3r)^2 - \pi(2r)^2 = 9\pi r^2 - 4\pi r^2$
$= 5\pi r^2 = 5A_1 = 5 \times 346.5 \text{ cm}^2$
$= 1732.5 \text{ cm}^2$
A_4 = Area of the region representing Black scoring area
$= \pi(4\pi r)^2 - \pi(3r)^2 = 7\pi r^2 = 7A_1$
$= 7 \times 346.5 \text{ cm}^2 = 2425.5 \text{ cm}^2$
A_5 = Area of the region representing White scoring area
$= \pi(5r)^2 - \pi(4r)^2 = 9\pi r^2 = 9A_1$
$= 9 \times 346.5 \text{ cm}^2 = 3118.5 \text{ cm}^2$

Q.4. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Ans. Given, diameter of wheel $d_1 = 80$ cm

$\therefore$ Radius, $r = \frac{d_1}{2} = \frac{80}{2} = 40$ cm

$\therefore$ Circumference of wheel of a car =

$$2\pi r = 2 \times \frac{22}{7} \times 40 = \frac{1760}{7} \text{ cm}$$

Circumference of circle = $2\pi r$

Since speed of car = 66 $\text{kmh}^{-1} = \frac{66 \times 1000}{60}$ m min^{-1}

$= 1100\text{m min}^{-1} = 110000 \text{ cm min}^{-1}$

$\therefore$ Wheel of car moves in 1 min = 110000 cm
$\therefore$ Wheel of car moves in 10 min = 1100000 cm
$\therefore$ Number of complete revolution

$$= \frac{1100000}{1760/7} = \frac{7700000}{1760} = 4375$$

Hence, the wheel make complete revolution in 10 min is 4375.

Q.5. Tick the correct answer in the following and justify your choice : If the perimeter and the area of a circle are numeri cally equal, then the radius of the circle is

(A) 2 units (B) p units (C) 4 units (D) 7 units

Ans. (a) Area of circle = πr^2
and area of perimeter = $2\pi r$
According to the given condition,
Area of circle = Area of perimeter
$\therefore$ $\pi r^2 = 2\pi r$
$\Rightarrow$ r = 2 units

EXERCISE 12.2

Q.1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Ans. Here, we have
$r = 6$ cm and $\theta = 60$.
We know that,

$$\text{Area of sector} = \pi r^2 \times \frac{\theta}{360°}$$

$$= \left(\frac{22}{7} \times 6 \times 6 \times \frac{60°}{360°} \right) \text{cm}^2$$

$$= \frac{132}{7} \text{ cm}^2 = 18.86 \text{ cm}^2$$

Q.2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Ans. Since, circumference of circle = 22 cm

$\Rightarrow$ $2\pi r = 22$

$\Rightarrow$ $2 \times \frac{22}{7} r = 22$

$\therefore$ Radius of circle, $r = \frac{7}{2}$ cm

$\therefore$ Area of a quadrant of a circle =

$$= \frac{\pi r^2}{4} = \frac{22}{7} \times \frac{\left(\frac{7}{2}\right)^2}{4} = \frac{11}{14} \times \frac{49}{4} = \frac{77}{8} \text{ cm}^2$$

Q.3. The length of the minute hand of a clock is 14 cm. F ind the area swept by the minute hand in 5 minutes.

Ans. The angle described by the minute hand in 1 miunute = 66°.

∴ The angle described by 1 minute hand in 5 minutes = 30°.

i.e., $\theta = 30°$ and $r = 14$ cm

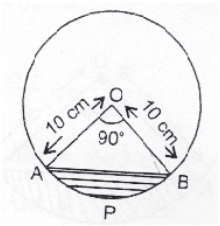
Now, area swept by the minute hand

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{30°}{360°}\right) \text{cm}^2$$

$$= 51.3 \text{ cm}^2$$

Q.4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use $\pi = 3.14$)



Ans. Here, we have

$r = 10$ cm, and $\theta = 90°$

Area of sector (OAPBO)

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= 3.14 \times 10 \times 10 \times \frac{90°}{360°}$$

$$= 3.14 \times 5 \times 5 = 78.5 \text{ cm}^2$$

Area of ΔAOB

$$= \frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times 10 \times 10 \times \sin 90°$$

$$= \frac{1}{2} \times 10 \times 10 \times 1 = 50 \text{ cm}^2$$

Now,

(i) Let APBA is the given minor segment, then

Area of minor segment (APBA)

= Area of sector (OAPBO) – area of Δ(AOB)

$= 78.5 \text{ cm}^2 - 50 \text{ cm}^2$

$= 28.5 \text{ cm}^2$

(ii) Area of major sector

= Area of circle – Area of sector OAPBO

$= (\pi r^2 - 78.5) \text{ cm}^2$

$= (3.14 \times 10 \times 10 - 78.5) \text{ cm}^2$

$= (314 - 78.5) \text{ cm}^2 = 235.5 \text{ cm}^2$

Q.5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Ans. Here, radius (r) = 21 cm

angle (θ) = 60°

(i) ∴ The length of arc (l) = $\frac{\pi r \theta}{180°}$

$$= \frac{\frac{22}{7} \times 21 \times 60°}{180°} \text{ cm}$$

$$= 22 \text{ cm}$$

(ii) Area of sector = $\frac{1}{2} \times l \times r$

$$= \frac{1}{2} \times 22 \times 21 \text{ cm}^2$$

$$= 231 \text{ cm}^2$$

(iii) Area of required segment

$$= r^2 \left(\frac{\pi\theta}{360°} - \frac{1}{2}\sin\theta\right)$$

$$= (21)^2 \left(\frac{22}{7} \times \frac{60}{360°} - \frac{1}{2}\sin 60°\right) \text{cm}^2$$

$$= 441 \left(\frac{11}{21} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right) \text{cm}^2$$

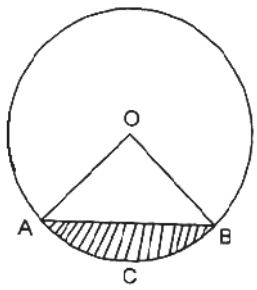
$$= \left(21 \times 11 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$$

$$= \left(231 - \frac{441\sqrt{3}}{4}\right) \text{cm}^2$$

Q.6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Fi nd the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Ans. Here, we have

$r = 15$ cm, $\theta = 60°$



Let OACBO be the given sector and OAB is a triangle. Then

(i) Area of the sector (OACBO)

$$= \pi r^2 \times \frac{\theta}{360^\circ}$$

$$= \left(3.14 \times 15 \times 15 \times \frac{60}{360^\circ}\right) \text{cm}^2$$

$$= 117.75 \text{ cm}^2$$

(ii) Area of the triangle (AOB)

$$= \frac{1}{2} r^2 \sin\theta$$

$$= \frac{1}{2} \times 15 \times 15 \times \frac{\sqrt{3}}{2}$$

$$= \frac{15 \times 15 \times 1.73}{4} = 97.313 \text{ cm}^2$$

Now,

Area of minor segment (ACBA)

= Area of sector (OACBO) – Area of triangle (AOB)

$= (11.75 - 97.313) \text{ cm}^2$

$= 20.437 \text{ cm}^2$

and Area of major segment (ABDA)

= Area of circle – Area of minor segment

$= (\pi r^2 - 20.437) \text{ cm}^2$

$= (3.14 \times 15 \times 15 - 20.437) \text{cm}^2$

$= 686.063 \text{ cm}^2$

Q.7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

Ans. Here, radius (r) = 12 cm

angle (θ) = 120°

∴ Required area of minor segement

$$= r^2 \left(\frac{\pi\theta}{360^\circ} - \frac{1}{2}\sin\theta\right)$$

$$= (12)^2 \left(\frac{3.14 \times 120}{360^\circ} - \frac{1}{2}\sin 120^\circ\right) \text{cm}^2$$

$$= 144 \left(\frac{3.14}{3} - \frac{1}{2} \times \frac{\sqrt{3}}{2}\right) \text{cm}^2$$

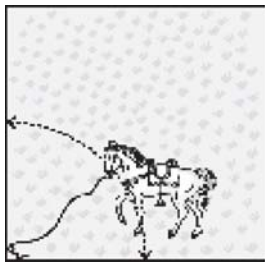
$$= 144 \times \frac{3.14 \times 4 - 3\sqrt{3}}{12} \text{cm}^2$$

$= 12 \times [12.56 - 3 \times 1.73] \text{ cm}^2$

$= 12 \times [12.56 - 5.19] \text{ cm}^2$

$= 12 \times 7.37 \text{ cm}^2$

$= 88.44 \text{ cm}^2$



Q.8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig.). Find (i) the area of that part of the field in which the horse can graze.

(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)

Ans. Here, $\theta = 90^\circ$

r = length of rope = 5.

(i) The are aof part of field horse can graze

= Area of sector

$$= \frac{\pi r^2 \theta}{360^\circ} = 3.14 \times \frac{5 \times 5 \times 90}{360^\circ}$$

$= 19.625 \text{m}^2$

(ii) Now radius (r) = 10 m

$$\text{grazed area} = \frac{3.14 \times 10 \times 10 \times 90}{360^\circ}$$

$= 78.5 \text{ m}^2$

Increase in grazing area = $78.5 \text{m}^2 - 19.625 \text{ m}^2$

$= 58.875 \text{ m}^2$

Q.9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. Find :

(i) the total length of the silver wi re required.

(ii) the area of each sector of the brooch.

Ans. (i) Total length of the silver wire required = circumference of the circle + length of 5 diameters which divide the circle into 10 equal sectors.

$= 2\pi r + 5 \times 2r$

$$= \left(2 \times \frac{22}{7} \times \frac{35}{2} + 5 \times 2 \times \frac{35}{2}\right) \text{mm}$$

$= (110 + 175) \text{ mm}$

$= 285 \text{ mm}$

(ii) The area of each sector of the brooch

$$= \frac{1}{10} \text{ (area of circle)}$$

$$= \frac{1}{10} \times \pi r^2$$

$$= \left(\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}\right) \text{mm}^2$$

$$= \frac{11 \times 35}{4} = \frac{385}{4} \text{ mm}^2$$

$$= 96.25 \text{ mm}^2$$

Q.10. An umbrella has 8 ribs which are equally spaced (see Fig.). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Ans. Total nuimber of ribs = 8

Therefore, angle (θ) made by two consecutive ribs of the centre $= \frac{360°}{8} = 45°$

i.e., $\theta = 45°$

We have, $r = 45$ cm and $\theta = 45°$

Therefore, the area between the two consecutive ribs of the umbrella

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \left(\frac{22}{7} \times 45 \times 45 \times \frac{45}{360°}\right) \text{cm}^2$$

$$= \left(\frac{1}{8} \times \frac{22}{7} \times 45 \times 45\right) \text{cm}^2$$

$$= 795.53 \text{ cm}^2$$

Q.11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Ans. Clearly, the length of the wiper will be radius and angle sweeping by the blades is θ.

i.e., $r = 25$ cm, and $\theta = 115°$.

Therefore, the total area cleaned at each sweep of the blades

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \left(\frac{22}{7} \times 25 \times 25 \times \frac{115°}{360°}\right) \text{cm}^2$$

$$= \left(\frac{625 \times 253}{7 \times 36}\right) \text{cm}^2 = \frac{158125}{252} \text{ cm}^2$$

$$= 627.45 \text{ cm}^2$$

Area swept by two vipers

$$= 2 \times 627.45$$

$$= 1254.90 \text{ cm}^2$$

Q.12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

Ans. Clearly, distance of red coloured light which is thrown by lighthouse is radius (r) and area over light house is thrown by sector angle (θ) i.e., $r = 16.5$ km and $\theta = 80°$.

Now, area of the sea over which the ships are warned.

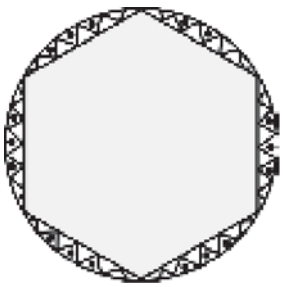
$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \left(3.14 \times \frac{165}{10} \times \frac{165}{10} \times \frac{80°}{360°}\right) \text{km}^2$$

$$= \frac{683892}{360} \text{ km}^2$$

$$= 189.97 \text{ km}^2$$

Q.13. A round table cover has six equal designs as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm². (Use √3 = 1.7)



Ans.

Area of 1 design

= Area of sector AOBC

– Area of equilatral ΔAOB

$$= \left(\frac{60°}{360°} \times \frac{22}{7} \times 28 \times 28 - \frac{\sqrt{3}}{4} \times (28)^2\right) \text{cm}^2$$

$= [410.6 - 33.2]\ cm^2$

$= \frac{232.4}{3}\ cm^2$

Total area covered by design $= \left(6 \times \frac{232.4}{3}\right) cm^2$

$= 464.8\ cm^2$

Cost of making the design = Rs. (0.35 × 464.8)

= Rs. 162.68

Q. 14. Tick the correct answer in the following:

Area of a sector of angle *p* (in degrees) of a circle with radius R is

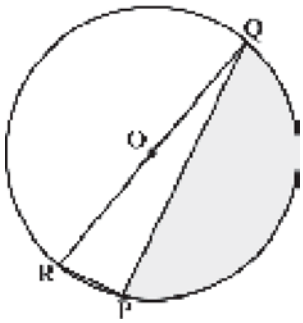
(a) $\frac{p}{180} \times 2\pi R$ (b) $\frac{p}{180} \times \pi R^2$

(c) $\frac{p}{360} \times 2\pi R$ (d) $\frac{p}{720} \times 2\pi R^2$

Ans. (d) $\frac{p}{720} \times 2\pi R^2$

EXERCISE 12.3

Q.1. Find the area of the shaded region in Fig. 12.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



Ans. Here, PQ = 24 cm, PR = 7 cm

$\angle QPR = 90°$ (QR is diameter)

$RQ^2 = PQ^2 + PR^2 = (24)^2 + (7)^2 = 576 + 49 = 625$

$\Rightarrow$ $RQ = \sqrt{625} = 25$ cm

Area of $\Delta QPR = \frac{1}{2} \times PQ \times PR = \frac{1}{2} \times 24 \times 7 = 84\ cm^2$

Radius of circle = $OQ = \frac{RQ}{2} = \frac{25}{2}$ cm

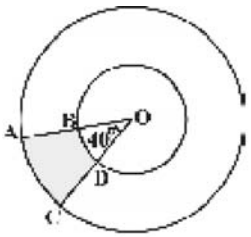
Area of semi-circle $= \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

$= \frac{6875}{28}\ cm^2$

$\therefore$ Area of shaded region $= \left(\frac{6875}{28} - 84\right) cm^2$

$= \frac{4523}{28}\ cm^2$

Q.2. Find the area of the shaded region in Fig., if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and $\angle AOC = 40°$.



Ans. Here, $r_1 = (OA) = 14$ cm

$r_2 = (OB) = 7$ cm

$\theta = 40°$

Area of sector AOC $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{40}{360°} \times \frac{22}{7} \times 14 \times 14\ cm^2 = \frac{616}{9}\ cm^2$

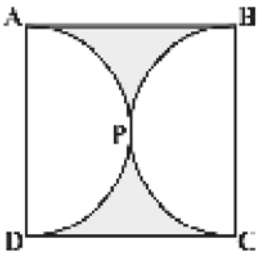
Area of sector BOD $= \frac{40}{360°} \times \frac{22}{7} \times 7 \times 7 = \frac{154}{9}\ cm^2$

Area of shaded region BDCA $= \left(\frac{616}{9} - \frac{154}{9}\right) cm^2$

$= \frac{462}{9}\ cm^2$

$= \frac{154}{3}\ cm^2$

Q.3. Find the area of the shaded region in Fig. , if ABCD is a square of side 14 cm and APD and BPC are semicircles.



Ans. Area of square = $(\text{side})^2 = (14)^2\text{cm}^2 = 196\text{cm}^2$

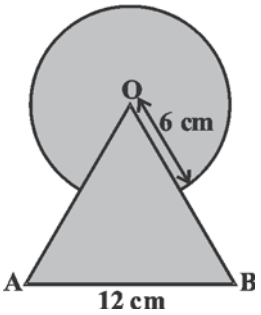
Area of 2 semi-circle $= 2 \times \frac{1}{2}\pi r^2$

$= 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2}\text{ cm}^2$

$= 154\text{ cm}^2$

Area of shaded region $= (196 - 154)\text{ cm}^2 = 42\text{cm}^2$

Q.4. Find the area of the shaded region in Fig. , where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Ans. It is given that side of equilateral triangle $= 12\text{ cm}$

Therefore, area $= \frac{\sqrt{3}}{4}\text{side}^2$

$= \left(\frac{\sqrt{3} \times 12 \times 12}{4}\right)\text{cm}^2$

$= 36\sqrt{3}\text{cm}^2$

Let radius of the circle be r, then

$r = 6\text{ cm}$

Therefore, area $= \pi r^2$

$= \frac{22}{7} \times 6 \times 6 = \frac{792}{7}\text{ cm}^2$

Now, Area of a sector of a circle of radius 6 cm and of angle 60°.

$= \pi r^2 \times \frac{\theta}{360°}$

[Triangle is equilateral, so $\theta = 60°$]

$= \left(\frac{22}{7} \times 6 \times 6 = \frac{60°}{360°}\right)$

$= \frac{132}{7}\text{ cm}^2$

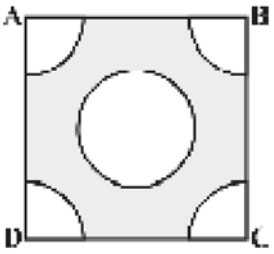
Now, required area

= Area of ΔOAB + Area of circle – Area of sector

$= \left(36\sqrt{3} + \frac{792}{7} - \frac{132}{7}\right)\text{cm}^2$

$= \left(36\sqrt{3} + \frac{660}{7}\right)\text{cm}^2$

Q.5. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. Find the area of the remaining portion of the square.



Ans. Area of remaining portion of square

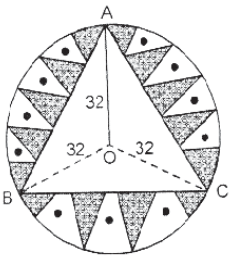
= Area of square – (Area of 4 quadrants of circle + Area of circle)

$= (4)^2 - \left(4 \times \frac{1}{4} \times \frac{22}{7} \times (1)^2 + \frac{22}{7} \times (1)^2\right)$

$= 16 - \left(\frac{22}{7} + \frac{22}{7}\right)$

$= 16 - \frac{44}{7} = \frac{68}{7}\text{ cm}^2$

Q.6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. Find the area of the design.



Ans. Now, area of shaded part

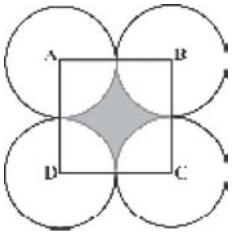
= Area of circle – Area of ΔABC

$= \pi r^2 - 3 \times \frac{1}{2}\text{OB} \times \text{OC} \times \sin 120°$

$= \frac{22}{7} \times 32 \times 32 - \frac{1}{2} \times 32 \times 32 \times \frac{\sqrt{3}}{2}$

$= \left(\frac{22528}{7} - 768\sqrt{3}\right)\text{cm}^2$

Q.7. In Fig., ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles . Find the area of the shaded region.



Ans. Area of the shaded region = Area of the sqaure of side 14 cm– 4[Area of a sector of centre angle 90°]

$$= 14 \times 14 - 4 \times \frac{90°}{360°} \pi \left(\frac{14}{2}\right)^2$$

$$= 196 - \frac{22}{7}(7)^2$$

$$= 196 - 154 = 42 \text{ cm}^2$$

Q.8. Fig. depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find : (i) the distance around the track along its inner edge (ii) the area of the track.

Ans. We have,

The distance between the two inner parallel line segments = 60 m.

Length of each parallel lines = 106 m

Width of track = 10 m

(i) The distance around the track along its inner edge = sum of length of inner parallel lines + 2(semi perimeter of inner circular ends)

$= (106 + 106) + 2(\pi \times 30)$

$$= 212 + 2 \times \frac{22}{7} \times 30 = 212 + 188.57 = 405.57 \text{ cm}$$

(ii) OD = 30 m and Ad = 10 m,

OA = OD + DA = 30 + 10 = 40 m

Area of the track = Area of rectangle ABCD + Area of rectangle EFGH + 2 (Area of semicircle with radius 40 m – Area of semi-circle with radius 30m)

$$= 106 \times 10 + 106 \times 10 + 2\left(\frac{1}{2}\pi(40)^2 - \frac{1}{2}\pi(30)^2\right)$$

$= 1060 + 1060 + \pi[(40)^2 - (30)^2]$

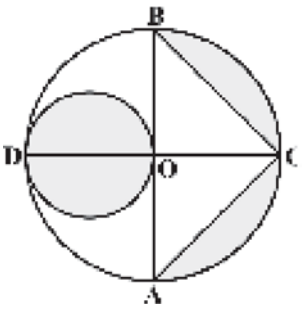
$$= 2120 + \frac{22}{7}(40 + 30)(40 - 30)$$

$$= 2120 + \frac{22}{7} \times 70 \times 10$$

$= 2120 + 2200 = 4320 \text{ m}^2$

Hence, the area of track is 4320 m^2

Q.9. In Fig., AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



Ans. Area of shaded region

= Area of small circle + [Area of semi-circle (with centre O) – Area of ΔABC]

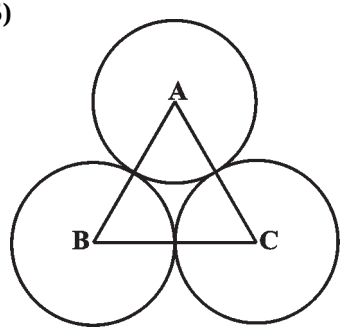
$$= \pi \times \left(\frac{7}{2}\right)^2 + \left(\frac{1}{2} \times \frac{22}{7} \times 7 \times 7 - \frac{1}{2} \times 14 \times 7\right)$$

$$= \frac{22}{7} \times \frac{49}{4} + [77 - 49]$$

$= 38.5 + 28$

$= 66.5 \text{ cm}^2$

Q.10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig.). Find the area of the shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73205$)



Ans. Let the side of the equilateral triangle be 'a' cm, then,

Area = 17320.5 cm^2

$$\Rightarrow \quad \frac{\sqrt{3}}{4} a^2 = 17320.5$$

$$[\text{Area} = \frac{\sqrt{3}}{4}(\text{side})^2]$$

$$\Rightarrow \quad a^2 = \frac{17320.5 \times 4}{\sqrt{3}}$$

$$\Rightarrow \quad a^2 = \frac{17320.5 \times 4}{1.73205} = 40000$$

$$\Rightarrow \quad a = 200$$

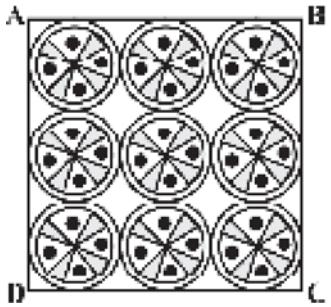
Thus radius of each sctor is 100 cm

Now required Area = Area of $\Delta ABC - 3 \times$ Area of sector with central angle 60° and r = 100 cm

$$= \left[17320.5 - 3\left(\frac{60}{360} \times 3.14 \times 100 \times 100\right)\right] \text{cm}^2$$

$$= (17320.5 - 15700)\ \text{cm}^2 = 1620.5\ \text{cm}^3$$

Q.11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig.). Find the area of the remaining portion of the handkerchief.



Ans. Radius of a circle = 7 cm

Length of side of the sqaure = 3(diameter)

$= 3(14) = 42$ cm

Area of the square = side$^2 = 42^2 = 1764$ cm^2

Area of one circle = $\pi r^2 = \frac{22}{7} \times (7)^2 = 154$

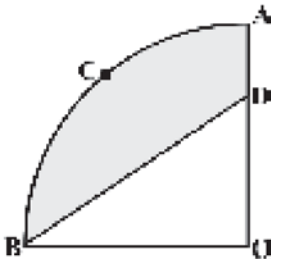
Area of a circle $= 9 \times 154 = 1386$ cm^2

Remaining portion (without design)= Area of the handkerchief – Area of design

$= 1764 - 1386 = 378$ cm^2

Hence, the area of the remaining portion is 378 cm^2.

Q.12. In Fig. , OA≅CB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.



Ans. (i) Area of quadrant OACB

$$= \frac{\pi r^2}{4} = \frac{1}{4} \times \frac{22}{7} \times (3.5)^2\ \text{cm}^2$$

$$= 9.625\ \text{cm}^2$$

(ii) And,

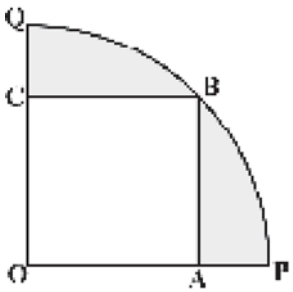
Area of $\Delta BOD = \frac{1}{2} \times 3.5 \times 2\ \text{cm}^2 = 3.5\ \text{cm}^2$

$\therefore$ Area of the shaded portion

$= 9.625\ \text{cm}^2 - 3.5\ \text{cm}^2$

$= 6.125\ \text{cm}^2$

Q.13. In Fig., a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of t he shaded region. (Use $\pi = 3.14$)



Ans. Since OABC is a square

$\therefore$ Diagonal of a square = $\sqrt{2} \times$ side = $\sqrt{2} \times 20$ cm

[OA (side) = 20 cm]

$\therefore$ Radius of circle, r = $20\sqrt{2}$

$\therefore$ Area of quadrant $= \frac{\pi r^2}{4} = \frac{3.14 \times (20\sqrt{2})^2}{4}$

$$= \frac{3.14 \times 800}{4} = 628\ \text{cm}^2$$

Now,Area of square $= (OA)^2 = (20)^2$

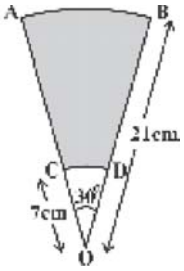
$= 400\ \text{cm}^2$

$\therefore$ Area of shaded region = Area of quadrant – Area of square

$= 628 - 400$

$= 228\ \text{cm}^2$

Q.14. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig.). If ∠AOB = 30°, find the area of the shaded region.



Ans. Given, ∠COD = 30°

r_1 = OC = 7 cm, and

r_2 = OB = 21 cm

Area of sector COD = $\frac{\theta \pi r_1^2}{360} = \frac{30}{360} \times \frac{22}{7} \times (7)^2$

$= \frac{22 \times 7}{12} = \frac{77}{6} = 12.83 \text{ cm}^2$

Area of sector OAB = $\frac{\theta \pi r_1^2}{360} = \frac{30}{360} \times \frac{22}{7} \times (21)^2$

$= \frac{22}{12} \times 3 \times 21$

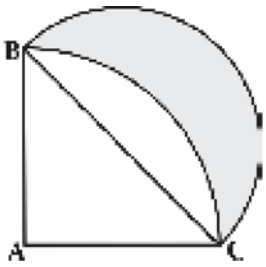
$= \frac{11 \times 63}{6} = \frac{693}{6} = 115.5 \text{ cm}^2$

Area of shaded region = Area of sector OAB – Area of sector COD

$= 115.5 - 12.83 = 102.67 \text{ cm}^2$

$= \frac{308}{3} \text{ cm}^2$

Q.15. In Fig. , ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Ans. Let points P and Q on arcs BPC and BQC respectively with BC as diameter

Now in ΔABC, we have

$BC^2 = AB^2 + AC^2$

(Using pythagoras theorem)

$\Rightarrow \quad BC^2 = 14^2 + 14^2$

$\Rightarrow \quad BC = \sqrt{196 + 196}$

$\Rightarrow \quad BC = 14\sqrt{2}$ cm

Since, BC is a diameter for semicircle BCQB

$\therefore$ Therfore, radius (r) = $7\sqrt{2}$ cm

Now, area of semicircle (BCQB)

$= \frac{\pi r^2}{2}$

$= \frac{22}{7} \times \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2}$

$= 152 \text{ cm}^2$

Area of sector (ACPBA)

$= \frac{1}{4} \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154 \text{ cm}^2$

And, ar (ΔACB) $= \frac{1}{2} \times AC \times AB$

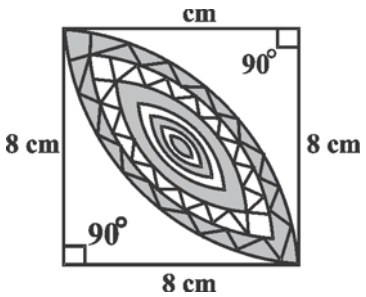
$= \frac{1}{2} \times 14 \times 14 = 98 \text{ cm}^2$

Hence, required area

= [ar (ΔABC) +ar (BCQB)] – ar [(ACPBA)]cm^2

= [(98 + 154) – (154)] cm^2 = 98 cm^2

Q.16. Calculate the area of the designed region in Fig. common between the two quadrants of circles of radius 8 cm each.



Ans. Area of designed region

= 2 [Area of quadrant – Area of triangle]

$= 2\left(\frac{1}{4} \times \frac{22}{7} \times 8 \times 8 - \frac{1}{2} \times 8 \times 8\right)$

$= 2\left(\frac{352}{7} - 32\right)$

$= \frac{256}{7} \text{ cm}^2$

Additional Questions

Q.1. How many times will the wheel of a car rotate in journey of 88 km, if it is given that the diameter of the wheel is 56 cm? (Take $\pi = 22/7$].

Ans. Diameter of wheel = 56 cm

$\therefore \quad r = \frac{56}{2} = 28$ cm

$\therefore$ Circumference of wheel $= 2\pi r$

$$= \left(2 \times \frac{22}{7} \times 28\right) \text{cm} = 176 \text{ cm}$$

In 1 revolution, wheel will cover distance = 176 cm

Total distance = 88 km = 88 × 1000 × 100 cm

$\therefore$ Number of revolutions $= \frac{88 \times 1000 \times 10}{176} = 50{,}000$

Hence, the number of times the wheel will rotate is 50,000.

Q.2. A wire is in the form of a circle of radius 42 cm. It is bent into a square. Determine the side of the square.

Ans. Radius of the circle = 42 cm

Length of the wire = circumference of the circle

$$= 2\pi r = 2 \times \frac{22}{7} \times 42 \text{ cm} = 264 \text{ cm}$$

Let the side of the square be x cm.

Now, the wire is bent into a square.

$\therefore$ Perimeter of the square = Length of the wire

$\Rightarrow 4x = 264 \Rightarrow x = 66$

Hence, the length of the side of the square is 66 cm.

Q.3. If the perimeter of a semi-circular protractor is 36 cm, find its diameter. (Take $\pi = 22/7$]

Ans. Let r be the radius of the protractor, then

Perimeter $= \pi r + 2r$

$\Rightarrow \quad 36 = r(\pi + 2)$

$\Rightarrow \quad 36 = r\left(\frac{22}{7} + 2\right)$

$\Rightarrow \quad 36 = r\left(\frac{22+14}{7}\right) = \frac{36r}{7}$

$\Rightarrow \quad r = \left(\frac{36 \times 7}{36}\right) \text{cm} = 7 \text{ cm}$

$\therefore$ Diameter $(d) = 2r = (2 \times 7)$ cm = 14 cm

Hence, the diameter is 14 cm.

Q.4. Find the radius of a circle where circumference is equal to the sum of the circumferences of two circles of radii 15 cm and 18 cm.

Ans. For first circle, r = 15 cm.

$\therefore$ Its circumference $= 2\pi(15) = 30\pi$

For second circle r = 18 cm

$\therefore$ Its circumference $= 2\pi(18) = 36\pi$

Let R be the radius of the required circle,

$\therefore \quad 2\pi R = 30\pi + 36\pi$

$\Rightarrow \quad 2R = 66$

$\Rightarrow \quad R = 33$

Hence radius of the required circle = 33 cm.

Q.5. Find the area of a sector of a circle of radius 28 cm and central angle 45°.

Ans. $r = 28$ cm, $\theta = 45°$

$\therefore$ Area of sector $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{45}{360} \times \frac{22}{7} \times 28 \times 28$$

$$= \frac{1}{8} \times \frac{22}{7} \times 28 \times 28$$

$= 308 \text{ cm}^2$

Q.6. The wheel of motor cycle is of radius 35 cm. How many revolutions for minute must the wheel make so as to keep a speed of 66 km/h?

Ans. Speed $= \frac{66 \times 1000 \times 100}{60}$ cm/minute

For wheel, $r = 35$ cm

$\therefore$ Circumference $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 35$$

$= 220 \text{ cm} = 2.2 \text{ cm}.$

Number of revolutions

$$= \frac{66 \times 1000 \times 100}{60} \times \frac{1}{220}$$

= 500 revolutions/minute

Q.7. A piece of 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at the centre. Find the radius of the circle.

Ans. Length of arc = 20 cm

$\theta = 60°$

Let radius = r

Length opf arc = $\frac{\theta}{360°} \times 2\pi r$

$$20 = \frac{60°}{360°} \times 2 \times \pi \times r$$

$$\Rightarrow \quad 20 = \frac{1}{3} \times \pi \times r$$

$$\Rightarrow \quad r = \frac{60}{\pi} \text{ cm}$$

Q.8. What is the area of the circle, the circumference of which is equal to the perimeter of a square of side 11 cm.

Ans. It is given that

Side of square = 11 cm

$\therefore$ Perimeter = 4 × side

= 4 × 11 = 44 cm

Now, Circumference of circle

= Perimter of square

$$\Rightarrow \quad 2\pi r = 44$$

$$\Rightarrow \quad r = \frac{44}{2\pi} = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

Now, area of the circle

$= \pi r^2$

$$= \left(\frac{22}{7} \times 7 \times 7\right) \text{cm}^2$$

$= 154 \text{ cm}^2$

Q.9. The area of a circular playgound is 22176 m^2. Find the cost of fencing this ground at the rate of Rs. 50 per meter.

Ans. Let radius of the playground = r m

$\therefore \quad \pi r^2 = 22176$

$$\Rightarrow \quad \frac{22}{7} \times r^2 = 22176$$

$$\Rightarrow \quad r^2 = \frac{22176 \times 7}{22}$$

$$\Rightarrow \quad r^2 = 1008 \times 7$$

$$\Rightarrow \quad r^2 = 7056$$

$$\Rightarrow \quad r^2 = \sqrt{7056} = 844\text{m}$$

Circumference of the circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 84$$

= 528 m

Cost of fencing = 528 × 50

= Rs. 26400

Q.10. Find the area of the sector of a circle of radius 5 cm, if the corresponding arc length is 3.5 cm.

Ans. r = 5 cm

Length of the arc = 3.5 cm

$$\therefore \quad \frac{\theta}{360°} \times 2\pi r = 3.5$$

$$\Rightarrow \quad \frac{\theta}{360°} \times 2 \times \frac{22}{7} \times 6 = 3.5$$

$$\Rightarrow \quad \frac{\theta}{360°} = \frac{3.5}{10 \times 22} \times 7$$

$$\text{Area of sector} = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{3.5 \times 7}{10 \times 22} \times \frac{22}{7} \times (5)^2$$

$$= \frac{3.5}{10} \times 25$$

$$= \frac{87.5}{10} = 8.75 \text{ cm}^2$$

Multiple Choice Questions

Q.1. The area of the sector of a circle of radius r and central angle θ, is :

(a) 1/2 $l.r$ (b) $2\pi r^2 \theta/720$

(c) $2\pi r^2 \theta/360$ (d) $\pi r \theta/360$

Ans. (b)

Q.2. If the perimeter of a circle is equal to that of a square, then the ratio of their area is :

(a) 2 : 7 (b) 14 :11

(c) 7 : 20 (d) 11 : 14

Ans. (b)

Q.3. It is proposed to build a single circular park equal is area of the sum of areas of two circular parks of diameters 165 m and 12 m in a locality. The radius of the new park would be :

(a) 10 m (b) 15 m
(c) 20 m (d) 24 m

Ans. (a)

Q.4. The area of the circle that can be inscribed in a square of side 6 cm is

(a) 36π cm^2 (b) 18π cm^2
(c) 12π cm^2 (d) 9π cm^2

Ans. (d)

Q.5. The area of the sqaure that can be inscribed in a circle of radius 8 cm is :

(a) 256 cm^2 (b) 128 cm^2
(c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

Ans. (b)

Q.6. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is :

(a) 31 cm (b) 25 cm
(c) 62 cm (d) 50 cm

Ans. (d)

Q.7. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameter 36 cm and 20 cm is :

(a) 56 cm (b) 42 cm
(c) 28 cm (d) 16 cm

Ans. (c)

Q.8. If the circumference of a circle and the perimeter of a square are equal, then :

(a) area of circle = Area of the square
(b) Area of circle > Area of the square
(c) Area of circle < Area of the square
(d) Nothing definite can be said about the relation between the areas of the circle and square.

Ans. (b)

Q.9. The diameter of a wheel is 2.8 m. How far it will travel is 1000 revolutions.

(a) 880 m (b) 8800 m
(c) 88m (d) 8700

Ans. (b)

Q.10. The perimeter of a square circumscribing a circle of a radius a unit is :

(a) 2 a unit (b) 4 a unit
(c) 8 a unit (d) 16 a unit

Ans. (c)