

Arithmetic Progressions

In the Chapter

In this chapter, you will be stuyding the following points:

- An **arithmetic progression** (AP) is a list of numbers in which each term is obtained by adding a fixed number *d* to the preceeding term, except the first term. The fixed number *d* is called the **common difference**. The general form of an AP is *a*, *a* + *d*, *a* + 2*d*, *a* + 3*d*, . . .
- A given list of numbers a_1, a_2, a_3, \ldots is an AP, if the differences $a_2 a_1, a_3 a_2, a_4 a_3, \ldots$, give the same value, i.e., if $a_{k+1} a_k$ is the same for different values of k.
- In an AP with first term a and common difference d, the *n*th term (or the general term) is given by $a_n = a + (n-1) d$.
- The sum of the first *n* terms of an AP is given by :

$$S = \frac{n}{2} \left[2a + (n-1)d \right]$$

• If *l* is the last term of the finite AP, say the *n*th term, then the sum of all terms of the AP is given by :

$$S = \frac{n}{2}(a+l)$$

EXERCISE 5.1

Q.1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/4 of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8 % per annum.

Ans. (i) Taxi fare for $1 \text{ km} = \text{Rs. } 15 = a_1$

Taxi fare for 2 kms = Rs. 15 + Rs. 8 = Rs. 23 = a_2 Taxi fare for 3 kms = Rs. 23 + Rs. 8 = Rs. 31 = a_3 Taxi fare for 4 kms = Rs. 31 + Rs. 8 = Rs. 39 = a_4

and so on.

- $a_2 a_1 = Rs. 23 Rs. 15 = Rs. 8$
- $a_3 a_2 = Rs. 31 Rs. 23 = Rs. 8$

$$a_4 - a_3 = Rs.39 - Rs.31 = Rs.8$$

i.e., $a_{k+1} - a_k$ is the same every time.

So, this list of numbers form an arithmetic progression with the first term a = Rs. 15 and the common difference d = Rs. 8.

(ii) Suppose, the amount of air present in the cylinder be 'a' units

 \therefore According to the question, the list giving the air present in the cylinder is given by

$$a, a - \frac{a}{4} = \frac{3a}{4}, \frac{3a}{4} - \frac{1}{4} \times \frac{3a}{4} = \frac{12a - 3a}{16} = \frac{9a}{16}, \dots$$

Here,
$$a_1 = a, a_2 = \frac{3a}{4}, a_3 = \frac{9a}{16}...$$

Now, $a_2 - a_1 = \frac{3a}{4} - a = -\frac{a}{4}$

$$a_3 - a_2 = \frac{9a}{16} - \frac{3a}{4} = \frac{9a - 12a}{16} = -\frac{3a}{16}$$

$$\Rightarrow a_2 - a_1 \neq a_3 - a_2$$

- $\therefore \quad a_1, a_2, a_3, \dots, \text{ do not form an A.P.}$ (iii) Initial cost = Rs. 150. Cost rises by Rs.50 each subsequent metre. $\therefore \quad \text{Series is, 150, 200, 250, \dots}$ $\therefore \quad 200 - 150 = 250 - 200 = 50$
 - \therefore d is constant
 - \therefore Series is an A.P.
 - (iv) Here, P = Rs. 10000, r = 8% p.a.

Amount after 1 year =
$$10000 \left(1 + \frac{8}{100}\right)^1$$

$$=10000 \times \frac{108}{100} = \text{Rs.}10800$$

Amount after 2 years =
$$10000 \left(1 + \frac{8}{100}\right)^2$$

$$=10000 \times \left(\frac{108}{100}\right)^2 = \text{Rs.11664}$$

... The succeeding terms are 10000, 10800, 11668 ...

Here 'd' is not a constant as $10800 - 10000 \neq 11664 - 10800$

 \therefore It is not an A.P.

Q.2. Write first four terms of the AP, when the first term *a* and the common difference *d* are given as follows:

(i) a = 10, d = 10 (ii) a = -2, d = 0(iii) a = 4, d = -3 (iv) $a = -1, d = \frac{1}{2}$ (v) a = -1.25, d = -0.25Ans. (i) Here, a = 10 and d = 10 \therefore A.P. is 10, 10 + 10, 10 + 2 × 10, 10 + 3 × 10, ... i.e., 10, 20, 30, 40,

(ii) Here a = -2 and d = 0 \therefore A.P. is $-2, -2+0, -2+2 \times 0, ...$ i.e., -2, -2, -2, -2 (iii) Here, a = 4 and d = -3 \therefore A.P. is 4, 4 + (-3), 4 + 2 × (-3), 4 + 3 × (-3), ... i.e. 4, 1, -2, -5, (iv) Here, a = -1 and $d = \frac{1}{2}$: A.P. is $-1, -1 + \frac{1}{2}, -1 + 2 \times \frac{1}{2}, -1 + 3 \times \frac{1}{2}, ...$ $=-1, -\frac{1}{2}, 0, \frac{1}{2}, \dots$ (v) Here, a = -1.25 and d = -0.25 \therefore A.P. is $-1.25, -1.25 - 0.25, -1.25 - 2 \times 0.25, ...$ $=-1.25, -1.50, -1.75, \ldots$ Q.3. For the following APs, write the first term and the common difference: (i) $3, 1, -1, -3, \ldots$ (ii) $-5, -1, 3, 7, \ldots$ (iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ (iv) 0.6, 1.7, 2.8, 3.9, ... **Ans.** (i) Here, given, A.P. is 3, 1, -1, -3, ... a = 3, d = 1 - 3 = -2.*.*.. (ii) Here, given A.P. is – 5, –1, 3, 7, Hence, a = -5, d = -1 - (-5)= -1 + 5=4.(iii) Here, given A.P. is $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ Hence, $a = \frac{1}{3}$, $d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$.

(iv) Here, given A.P. is 0.6, 1.7, 2.8, Hence, a = 0.6, d = 1.7 - 0.6 = 1.1.

Q.4. Which of the following are APs ? If they form an AP, find the common difference *d* and write three more terms.

(i) 2, 4, 8, 16, ... (ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, (iii) 12, $\frac{5}{2}$, 5, 2, 5, 7

(iii) -1.2, -3.2, -5.2, -7.2, ...(iv) -10, -6, -2, 2, ...(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$ (vi) 0.2, 0.22, 0.222, 0.2222, ...(vii) 0, -4, -8, -12, ...

142 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

(viii)
$$-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \cdots$$

(ix) 1, 3, 9, 27, ...
(x) $a, 2a, 3a, 4a, ...$
(xi) $a, a^2, a^3, a^4, ...$
(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$
(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$
(xiv) 1², 3², 5², 7², 7³, ...
Ans. (i) Given series is
 $2, 4, 8, 16, ...$
Here, $a_2 - a_1 = 4 - 2 = 2$
 $a_3 - a_2 = 8 - 4 = 4$
 $a_4 - a_3 = 16 - 8 = 8, ...$
 $\therefore a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$
 \therefore The given series is not A.P.
(ii) Given series is 2, $\frac{5}{2}, 3, \frac{7}{2}, ...$
Here, $a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2},$
 $a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2},$
 $a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2},$
 $\therefore d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... = \frac{1}{2}.$
 \therefore It is an A.P.
the next three terms are $\frac{7}{2} + \frac{1}{2}, \frac{7}{2} + 2 \times \frac{1}{2}, \frac{7}{2} + 3 \times \frac{1}{2}$
i.e., $4, \frac{9}{2}, 5$.
(iii) Given series is $-1.2, -3.2, -5.2, -7.2, ...$
 $a_2 - a_1 = -3.2 - (-1.2)$
 $= -3.2 + 1.2$
 $= -2.0$

 $\begin{array}{rl} a_3 - a_2 & = -5.2 - (-3.2) \\ & = -5.2 + 3.2 \\ & = -2.0 \end{array}$

 $a_4 - a_3 = -7.2 - (-5.2)$ = -7.2 + 5.2 = -2.0

It is anA.P.

 $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -2.$

The next three terms are $-7.2 + (-2), -7.2 + 2 \times (-2), -7.2 + 3 \times (-2)$ -9.2, -11.2, -13.2i.e., (iv) Here, given series is -10, -6, -2, 2, ... $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4,$ $a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$ $a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$ $\therefore \quad d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 4.$ It is an A.P. The next three terms are $2+3, 2+2 \times 4, 2+3 \times 4$ *i.e.*, 6, 10, 14. (v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$ $\begin{array}{ll} a_2 - a_1 &= 3 + \sqrt{2} - 3 = \sqrt{2} \\ a_3 - a_2 &= (3 + 2\sqrt{2}) - (3 + \sqrt{2}) \\ &= 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2} \end{array}$:. $d = a_2 - a_1 = a_3 - a_2 = \sqrt{2}$. It is an A.P. The next three terms are $3 + 3\sqrt{2} + \sqrt{2}, 3 + 3\sqrt{2} + \sqrt{2} \times \sqrt{2}, 3 + 3\sqrt{2} + 3 \times \sqrt{2}$ i.e. $3 + 4\sqrt{2}, 3 + 5\sqrt{2}, 3 + 6\sqrt{2}$ (vi) Here, the given series is 0.2, 0.22, 0.222, ... $a_2 - a_1 = 0.22 - 0.2 = 0.02,$ $a_3 - a_2 = 0.222 - 0.22 = 0.002, \dots$ $a_2 - a_1 \neq a_3 - a_2$ It is not an A.P. (vii) Here, the given series is $0, -4, -8, -12, \ldots$ $a_2 - a_1 = -4 - 0 = -4$ $a_3 - a_2 = -8 - (-4) = -8 = 4 = -4$ $a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$ $\therefore \quad d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = -4.$ \therefore It is an A.P. The next three terms are $-12 + (-4), -12 + 2 \times (-4), -12 + 3(-4)$ i.e., -16, -20, 25(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$ $a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$ $a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

$$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$$

 $\therefore \quad d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 0.$ \therefore It is an A.P. The next three terms are $-\frac{1}{2}+0, -\frac{1}{2}+2\times 0, -\frac{1}{2}+3\times 0$ i.e. $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$. (ix) Here, the given series is 1, 3, 9, 27, $a_2 - a_1 = 3 - 1 = 2,$ $a_3^2 - a_2^1 = 9 - 3 = 6,$ $a_4 - a_3^2 = 27 - 9 = 18$ $\therefore \quad a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$ \therefore It is not A.P. (x) Here, the given series is *a*, 2*a*, 3*a*, 4*a*, $a_2 - a_1 = 2a - a = a$, $a_3^2 - a_2^1 = 3a - 2a = a,$ $a_4 - a_3^2 = 4a - 3a = a, \dots$ Clearly, it is an A.P. whose common difference is a. The next three terms are $4a+a, 4a+2 \times a, 4a+3 \times a$ 5a, 6a, 7a,i.e. (xi) Here, the given series is a, a^2, a^3, a^4, \dots $a_2 - a_1 = a^2 - a = a(a - 1)$ $a_3^2 - a_2^1 = a^3 - a^2 = a^2 (a - 1)$ $a_4^2 - a_3^2 = a^4 - a^3 = a^3 (a - 1)$ $\therefore \quad a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$ \therefore It is not A.P.

(xii) Here, the given series is

$$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$$

 $a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$
 $a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$
 $\therefore d = a_2 - a_1 = a_3 - a_2 = \sqrt{2}$
 $\therefore d = a_2 - a_1 = a_3 - a_2 = \sqrt{2}$
 $\therefore \text{ It is an A.P.}$
The next three terms are
 $\sqrt{32} + \sqrt{2}, \sqrt{32} + 2\sqrt{2}, \sqrt{32} + 3\sqrt{2}$
i.e., $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$
or $\sqrt{50}, \sqrt{72}, \sqrt{98}$.
(xiii) Here, the given series is
 $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$
 $a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3} - (\sqrt{2} - 1)$
 $a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3} (\sqrt{3} - \sqrt{2})$
 $a_2 - a_1 \neq a_3 - a_2$
It is not A.P.
(xiv) Here, the given series is
 $1^2, 3^2, 5^2, 7^2, \dots$
 $a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$
 $a_3 - a_2 = 5^2 - 3^2 = 25 - 16 = 9$
 $a_4 - a_3 = 7^2 = 49 - 25 = 24$
 $\therefore a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$
 \therefore It is not A.P.
(xv) Here, the given series is
 $1^2, 5^2, 7^2, 7^3, \dots$
 $a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$
 $a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$
 $\therefore a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 24$.
It is an A.P.
The next three terms are
 $73 + 24, 73 + 2 \times 24, 73 + 3 \times 24$
i.e., $97, 73 + 48, 73 + 72$
or. $97, 121, 145$.

EXERCISE 5.2

Q.1. Fill in the blanks in the following table, given that *a* is the first term, *d* the common difference and a_n the *n*th term of the AP:

	a	d	n	a _n
(i)	7	3	8	-
(ii)	-18	-	10	0
(iii)	-	-3	18	-5
(iv)	-18.9	2.5	_	3.6
(v)	3.5	0	105	-

Ans. (i) 28.
Hint:

$$a_n = a + (n-1) d$$

 $= 7 + (8-1) 3$
 $= 7 + 21$
 \Rightarrow
 $a_n = 28.$
(ii) 2.
Hint: Using the formula
 $a_n = a + (n-1) d$
 $0 = -18 + (10-1) d$
 $18 = 9d$
 $2 = d$

144 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

(iii) 46 Hint: Using the formula $a + (n-1) d = a_n$ a + (18 - 1)(-3) = -5 $a + 17 \times (-3) = -5$ a - 51 = -5a = -5 + 51=46.a (iv) 10. Hint : Using the formula $a + (n-1) d = a_n$ -18.9 + (n-1)2.5 = 3.6(n-1)2.5 = 3.6 + 18.9(n-1)2.5 = 22.5 $n-1 = \frac{22.5}{2.5}$ n - 1 = 9n = 9 + 1 = 10.(v) 3.5 Hint: On using the formula, $a_n = a + (n-1) d$ = 3.5 + (105 - 1) = 0= 3.5 + 0= 3.5Q.2. Choose the correct choice in the following and justify : (i) 30th term of the AP: 10, 7, 4, ..., is (a) 97 (b)77 (d) - 87(c) - 77(ii) 11th term of the AP: $-3, -\frac{1}{2}, 2, ...,$ is (a) 28 (b) 22 $(d) - 48 \frac{1}{2}$ (c) - 38**Ans.** (i) Here, a = 10, n = 30, d = 7 - 10 = -3Using the formula, $a_n = a + (n-1)d$ $a_{30} = 10 + (30 - 1)(-3)$ $=10+29 \times (-3)$ =10-87 $a_{30} = -77$

 \Rightarrow

30th term of A.P. = -77. (c)

(ii) Here, a = -4, n = 11, $d = -\frac{1}{2}$ - (-3)

 $=-\frac{1}{2}+3$

 $= -3 + 10 \times \frac{5}{2}$ $= -3 + 25 \Longrightarrow a_{11} = 22$ (b) Q.3. In the following APs, find the missing terms in the boxes : (i) 2, , 26 (ii) , 13, , 3 (iii) 5, $[], [], 9\frac{1}{2}$ \neg , \Box , \Box , \Box , δ (iv) - 4.,38, , , , , , , , -22 (v) **Ans.** (i) Here, n = 3, a = 2, and $a_n = l = 26$ $a_n = a + (n-1) d$ *.*.. \Rightarrow $2\ddot{6} = 2 + (3-1)d$ 2d = 26 - 2 \Rightarrow \Rightarrow 2d = 24 $d = \frac{24}{2}$ \Rightarrow d = 12. \Rightarrow Hence, missing term = a + d = 2 + 12 = 14. (ii) Here, 2nd term = 13a + d = 13...(i) \Rightarrow 4th term = 3and a + 3d = 3 \Rightarrow ...(ii) On solving equation (i) and equation (ii), we get a = 18, d = -5 \therefore 1st term = 18 3rd term = a + 2d $= 18 + 2 \times (-5)$ = 18 - 10= 8Hence required terms are 18 and 8. (iii) Here, a = 5 and $a_4 = 9\frac{1}{2} = \frac{19}{2}$ $a + 3d = \frac{19}{2}$ \Rightarrow

 $=\frac{5}{2}$

 $a_{11} = -3 + (11 - 1)\left(\frac{5}{2}\right)$

 $a_n = a + (n-1)d$

Using the formula

⇒	$5 + 3d = \frac{19}{2}$
⇒	$3d = \frac{19}{2} - 5$
⇒	$3d = \frac{9}{2}$
⇒	$d = \frac{3}{2}$
∴	2nd term = $a + d = 5 + \frac{3}{2} = \frac{10+3}{2} = \frac{13}{2}$
	3rd term = $a + 2d = 5 + 2 \times \frac{3}{2} = 8$
	\therefore Missing terms are $\frac{13}{2}$ and 8.
	(iv) Here. $a = -3$ (i)
and	$a_c = 6$
\Rightarrow	$a+5d^{\circ}=6$
\Rightarrow	-4 + 5d = 6 [using (i)]
\Rightarrow	5d = 6+4
\Rightarrow	5d = 10
\Rightarrow	d = 2
	Missing terms are $a + d$, $a + 2d$, $a + 3d$, $a + 4d$
	$=-4+2, -4+2 \times 2, -4+3 \times 2, -4+4 \times 2$
	=-2, -4+4, -4+6, -4+8
	=-2,0,2,4.
	(v) $a_2 = 38$
\Rightarrow	$a + d^2 = 38$ (i)
and	$a_{c} = -22$
	$a + 5d^{\circ} = -22$ (ii)
	On solving equation (i) and equation (ii), we get
	a = 53, d = -15
	Hence missing terms are
	$= a, a_{x}, a_{z}, a_{z}$
	= a, a + 2d, a + 3d, a + 4d
	$=53, 53+2 \times (-15), 53+3 \times (-15),$
	$53 + 4 \times (-15)$
	=53, 23, 8, -7
	Q.4. Which term of the AP : 3, 8, 13, 18,, is
78?	
	Ans. Given series is
	3, 8, 13, 18,
	Here, $a = 3, d = 8 - 3 = 5$
	Let 78 be the <i>n</i> th term fo the given A.P.
	Using the formula,

 $a + (n-1) d = a_n$ 3 + (n-1)(5) = 78 \Rightarrow (n-1)5 = 78 - 3 \Rightarrow (n-1)5 = 75 \Rightarrow $n-1 = \frac{75}{5}$ \Rightarrow n - 1 = 15 \Rightarrow \Rightarrow n = 15 + 1 \Rightarrow п =16 Hence 16th term of A.P. is 78. Q.5. Find the number of terms in each of the following APs : (i) 7, 13, 19, ..., 205 (ii) 18, 15 $\frac{1}{2}$, 13, ..., -47 Ans. (i) Given series is 7, 13, 19,, 205 Here, a = 7, d = 13 - 7 = 6, $a_n = 205$, n = ?Using the formula, $a + (n-1)d = a_n$ 7 + (n-1)6 = 205 \Rightarrow (n-1) 6 = 205 - 7 \Rightarrow \Rightarrow (n-1) 6 = 198n - 1 = 33 \Rightarrow n = 33 + 1 \Rightarrow \Rightarrow п =34 Hence, the given A.P. contains 34 terms.

(ii) Given series is

$$8, 15 \frac{1}{2}, 13, \ldots, -47$$

Here, a = 18, $d = \frac{31}{2} - 18 = \frac{-5}{2}$, $a_n = -47$, n = ?Using the formula,

$$a + (n-1) d = a_n$$

$$\Rightarrow \qquad 18 + (n-1)\left(\frac{-5}{2}\right) = -47$$
$$\Rightarrow \qquad (n-1)\left(\frac{-5}{2}\right) = -47 - 18$$
$$\Rightarrow \qquad (n-1)\left(\frac{-5}{2}\right) = -65$$

$$\Rightarrow \qquad (n-1)\left(\frac{1}{2}\right) = -65$$
$$\Rightarrow \qquad n-1 = 26+1$$
$$\Rightarrow \qquad n = 27.$$

Hence, the given A.P. contains 27 terms.

146	Lifeskills'	Complete	NCERT	Solutions	Class-2	X Mathematics
-----	-------------	----------	-------	-----------	---------	---------------

Q.6. Check whether - 150 is a term of the AP : 11, 8, 5, 2 . . . Ans. Let -150 is a *n*th term of A.P. *.*... -150 = a + (n-1)d \Rightarrow -150 = 11 + (n-1)(-3)-3(n-1) = -150-11 \Rightarrow -3(n-1) = -161 \Rightarrow $(n-1) = \frac{161}{3}$ \rightarrow $n = \frac{161}{3} + 1 = \frac{164}{3} = 54\frac{1}{3}$ \Rightarrow Since *n* cannot be a fraction. \therefore -150 is not a term of 11, 8, 5, 2, Q.7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73. **Ans.** We have, $a_{11} = a + 10d = 38$...(i) $a_{16} = a + 15d = 73$...(ii) On subtracting equation (ii) from equation (i), we get $-5a = -35 \Longrightarrow d = 7$ From (i) $a + 10 \times 7 = 38 \implies a = 38 - 70 = -32$ $a_{31} = a + 30d$ Now, $= -32 + 30 \times 7$ = -32 + 210 $a_{31} = 178$ Hence, 31st term of an A.P. is 178.

Q.8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Ans. We have, $a_3 = a + 2d = 12$...(i) last term $a_{50} = a + 49d = 106$...(ii) On solving equation (i) and equation (ii), we get a = 8, d = 2. $a_{29} = a + 28d$ $= 8 + 28 \times 2$ = 8 + 56 $a_{29} = 64$ Hence, 29th term of A.P. is 64.

Q9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero? Ans. We have

 $a_{3} = a + 2d = 4 \qquad \dots(i)$ $a_{9} = a + 8d = -8 \qquad \dots(ii)$ On solving equation (i) and equation (ii), we get a = 8, d = -2Let $a_{n} = 0$, then $a + (n-1) d = a_{n}$ $\Rightarrow 8 + (n-1) (-2) = 0$

(n-1)(-2)=4 \Rightarrow n - 1=4 \Rightarrow \Rightarrow =5п Hence, 5th term of this A.P. is zero. Q.10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference. Ans. We have, $a_{17} - a_{10} = 7$ (a+16d) - (a+9d) = 7.... 7d = 7 \Rightarrow \Rightarrow d =7Hence, common difference =1 Q.11. Which term of the AP: 3, 15, 27, 39, ... will be 132 more than its 54th term? Ans. We have $a_{n} - a_{54}$ =132[a + (n-1)d] - [a + 53d]=132 \Rightarrow (n-1-53)d = 132 \Rightarrow \Rightarrow (n-54)d = 132(n-52) 12 = 132 \Rightarrow (d = 15 - 3 = 12)n - 54= 11 \Rightarrow =65 \Rightarrow п

Hence, 65th term is 132 more than its 54th term.

Q.12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Ans. Let
$$d_1 = d_2 = d$$

∴ We have, $a_{100} - b_{100} = 100$
 $= (a+99d) - (b+99d) = 100$
 $\Rightarrow \qquad a-b = 100$
Now, $a_{1000} - b_{1000}$
 $= (a+999d) - (b+999d)$
 $= a - b$

$$= 100.$$
 [using (i)]

Q.13. How many three-digit numbers are divisible by 7?

Ans. The three digit numbers divisible by 7 are 105, 112, 119,, 994.

Here,
$$a = 105$$
, $d = 112 - 105 = 7$, $a_n = 994$, $n = ?$

Now,

$$a + (n-1) d = a_n$$

 $\Rightarrow 105 + (n-1) 7 = 994$
 $\Rightarrow (n-1) 7 = 994 - 105$

 \Rightarrow

$$(n-1)7 = 889$$

$$n-1 = \frac{889}{7}$$

$$\begin{array}{c} \Rightarrow \\ \Rightarrow \\ \Rightarrow \\ \end{array} \qquad \qquad n = 127 + 1 \\ n = 128. \end{array}$$

Hence, there are 128 three-digit numbers which are divisible by 7.

Q.14. How many multiples of 4 lie between 10 and 250?

	Ans. Multiples of 4 between 10 and 250 are 12,
16,.	, 248
Here	e, $a = 12, d = 16 - 12 = 4, a_{\rm p} = 248$
<i>:</i>	$a+(n-1)d = a_n$
\Rightarrow	$12 + (n-1)4 = 2\ddot{4}8$
\Rightarrow	(n-1)4 = 248 - 12
\Rightarrow	(n-1)4 = 236
\Rightarrow	n - 1 = 59
\Rightarrow	n = 60
	Hence, 60 multiples of 4 lie between 10 and 250.
	Q.15. For what value of n , are the n th terms of
two	APs: 63, 65, 67, and 3, 10, 17, equal?
	Ans. We have,
	63 + (n-1)2 = 3 + (n-1)7
\Rightarrow	(7-2)(n-1) = 63-3
\Rightarrow	5(n-1) = 60
	60
\Rightarrow	$n-1 = \frac{60}{5}$
	5
\Rightarrow	n - 1 = 12
\Rightarrow	<i>n</i> =13
	Q.16. Determine the AP whose third term is 16
and	the 7th term exceeds the 5th term by 12.
	Ans. Here, $a+2d = 16$
and	(a+6d) - (a+4d) = 12
	or $6d - 4d = 12$
	or $2d = 12$
	or $d = 6$
	Using this value in equation (i), we have
	$a + 2 \times 6 = 16$
\Rightarrow	a + 12 = 16
\Rightarrow	a=4
	Hence, A.P. is $4, 4 + 6, 4 + 2 \times 6, 4 + 3 \times 6 i.e., 4, 10$,
16,2	22,
	Q.17. Find the 20th term from the last term of
the	AP: 3, 8, 13,, 253.
	Ans. <i>n</i> th term from the last term of the A.P. $= l - l$
(<i>n</i> –	1) <i>d</i>

The 20th term from the last term of the A.P. 3, 8, 13, 253 is

$$\begin{array}{l} a_{20} &= 253 - (20 - 1) \, 5 \\ &= 253 - 95 \\ &= 158. \end{array}$$

Q.18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Ans. We have (a + 3d) + (a + 7d) = 24

2a + 10d = 24or a + 5d = 12or (a+5d)+(a+9d)=44and 2a + 14d = 44or a + 7d = 22or On solving equations (i) and (ii), we get a = -13, d = 5.Hence, first three terms of this A.P. are $-13, 13+5, -13+2 \times 5$ -13, -8, -3.i.e.,

Q.19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Ans. Here, we have,

	$a = 5000, d = 200, a_{n} = 7000, n = ?$
Now	$a + (n-1) d = a_n$
\Rightarrow	5000 + (n-1)200 = 7000
\Rightarrow	(n-1) 200 = 7000 - 5000
\Rightarrow	$(n-1)\ 200 = 2000$
\Rightarrow	$(n-1) = \frac{2000}{200}$

$$\Rightarrow \qquad n-1 = 10$$

$$\Rightarrow \qquad n = 11 \text{ years.}$$

Hence, in 11th year (i.e., in 2005) of his services he drew an annual salary of Rs. 7000.

Q.20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the *n*th week, her weekly savings become Rs 20.75, find *n*.

Ans. Here, we have

$$a = 5, d = 1.75, a_n = 20.75, n = ?$$

Now,

$$a + (n-1)d = a_n$$

$$5 + (n-1)1.75 = 20.75$$

$$(n-1)1.75 = 20.75 - 5$$

$$(n-1)1.75 = 15.75$$

$$n-1 = \frac{15.75}{1.75}$$

$$n-1 = 9$$

$$n = 9 + 1$$

$$n = 10$$

In 10th week Ramkali's saving were Rs. 20.75.

EXERCISE 5.3

37 = 4,

 \Rightarrow

- Q.1. Find the sum of the following APs: (i) 2, 7, 12, . . ., to 10 terms. (ii) -37, -33, -29, . . ., to 12 terms. (iii) 0.6, 1.7, 2.8, . . ., to 100 terms.
- (iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$, to 11 terms.

Ans. (i) Here a = 2, d = 7 - 2 = 5, n = 10Using the formula

S_n =
$$\frac{n}{2}$$
 [2a + (n − 1) d]
S₁₀ = $\frac{10}{2}$ [2×2+(10−1)5]
= 5 [4+9×5]
= 5[4+45]
= 5×49
∴ S₁₀ = 245.
(ii) Here, a = -37, d = -33 - (-37) = -33 +

$$n = 12.$$

Now, $S_n = \frac{n}{2} [2a + (n-1)d]$

$$\Rightarrow S_{12} = \frac{12}{2} [2 \times (-37) + (12 - 1) 4]$$

= 6 [-74 + 44]
= 6 \times (-30)
$$\therefore S_{12} = -180$$

(iii) Here, a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100

Now, $S_n = \frac{n}{2} [2a + (n-1)d]$

$$\Rightarrow \qquad S_{100} = \frac{100}{2} [2 \times (0.6) + (100 - 1) 1.1] \\= 50 [1.2 + 108.9] \\= 50 [110.1] \\\therefore \qquad S_{100} = 5505.$$

(iv) Here, we have

$$a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}, n = 11$$

Now, $S_n = \frac{n}{2} [2a + (n-1)d]$ $\Rightarrow \qquad S_{11} = \frac{11}{2} \left[2 \times \frac{1}{15} + (11-1)\frac{1}{60} \right]$

$$= \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right]$$
$$= \frac{11}{2} \times \frac{9}{30}$$
$$S_{11} = \frac{33}{20}.$$

Q.2. Find the sums given below :

(i)
$$7 + 10 \frac{1}{2} + 14 + ... + 84$$

(ii) $34 + 32 + 30 + ... + 10$
(iii) $-5 + (-8) + (-11) + ... + (-230)$
Ans. (i) Here, $a = 7$
 $d = 10 \frac{1}{2} - 7$

$$= \frac{21}{2} - 7 = \frac{7}{2}$$

$$= \frac{21}{2} - 7 = \frac{7}{2}$$

$$a + (n-1)d = a_n$$

$$\Rightarrow \qquad 7 + (n-1)\frac{7}{2} = 84$$

$$\Rightarrow \qquad (n-1)\frac{7}{2} = 77$$

$$\Rightarrow \qquad n-1 = 22$$

 \Rightarrow п Now using the formula

$$S_n = \frac{n}{2} (a + a_n)$$

$$\Rightarrow \qquad S_{23} = \frac{23}{2} (7 + 84)$$

$$= \frac{23 \times 91}{2}$$

=23

$$\begin{array}{rcl} & \ddots & \mathrm{S}_{23} &= \frac{2093}{2} = 1046 \, \frac{1}{2} \\ (\text{ii}) & \mathrm{Here}, \, a = 34, \, d = 32 - 34 = -2, \, 1, \, l = a_n = 10 \\ & a + (n-1)d &= a_n \\ \Rightarrow & 34 + (n-1)(-2) &= 10 \\ \Rightarrow & (n-1)(-2) &= -24 \\ \Rightarrow & n-1 &= 12 \\ \Rightarrow & n &= 13. \end{array}$$

Now using the formula,

$$S_n = \frac{n}{2} (a+l)$$

 $-\frac{13}{4}$

We have,
$$S_{13} = \frac{13}{2}(34+10)$$

	_ /	2 ^ '				
<i>.</i>	$S_{13} = 28$	86.				
(iii)	Here, $a = -5, d = -5$	-8 -	(-5) =	= -8 + 3	5 = -3, l	=
$a_{\rm n} = -23$	30.					
	a (<i>n</i> −1) <i>d</i>	= a	l_n			
\Rightarrow	-5 + (n-1)(-3)	=-	230			
\Rightarrow	(n-1)(-3)	=-	225			
\Rightarrow	n-1	=	75			
\Rightarrow	n	=	76			

Now using the formula, $S_n = \frac{n}{2}(a+l)$

the sum =
$$\frac{76}{2} [-5 + (-230)]$$

= $38 \times (-5 - 235)$
= $38 \times (-235)$
S₇₆ = -8930.

Q.3. In an AP:

...

(i) given $a = 5, d = 3, a_n = 50$, find *n* and S_n. (ii) given a = 7, $a_{13} = 35$, find d and S_{13} . (iii) given $a_{12} = 37$, d = 3, find a and S_{12} . (iv) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10} . (v) given d = 5, $S_9 = 75$, find a and a_9 . (vi) given a = 2, d = 8, $S_n = 90$, find n and a_n . (vii) given a = 8, $a_n = 62$, $S_n = 210$, find *n* and *d*. (viii) given $a_n = 4$, d = 2, $S_n^n = -14$, find *n* and *a*. (ix) given a = 3, n = 8, S = 192, find d. (x) given l = 28, S = 144, and there are total 9 terms. Find a.

 $S_n = \frac{n}{2}(a+l)$

 $=\frac{16}{2}(5+50)$

Ans. (i) Here, $a = 5, d = 3, a_n = 50$ Using the formula

	a + (n - 1)d	$=a_n$
\Rightarrow	5 + (n - 1) 3	=50
\Rightarrow	(<i>n</i> -1)3	=45
\Rightarrow	n-1	=15
\Rightarrow	n	=16

Now,

$$=8 \times 55$$

⇒ $S_{n} = 440.$
(ii) Given, $a=7, a_{13} = 35, d=?, S_{13} = ?$
 $a_{13} = a + 12d$

∴ $35 = 7 + 12d$

⇒ $12d = 28$

⇒ $d = \frac{7}{3}$

and $S_{13} = \frac{13}{2}(7 + 35)$
 $= \frac{13}{2} \times 42$
 $= 13 \times 21$

∴ $S_{13} = 273.$
(iii) Given $a_{13} = 37, d=3, a=? S_{13} = ?$

? $a_{12} = a + 11d$ We have, $37^{2} = a + 11 \times 3$ \Rightarrow a = 37 – 33 \Rightarrow a =4 \Rightarrow $S_{12} = \frac{12}{2}(4+37)$

and

 $= 6 \times 41$ $S_{12} = 246.$ (iv) given $a_3 = 15$, $S_{10} = 125$, d = ?, $a_{10} = ?$ *.*.. $a_3^{10} = a + 2d$ 13 = a + 2dWe have, \Rightarrow

and
$$S_{10} = \frac{10}{2} (a + a_{10})$$

 $\Rightarrow \qquad 125 = 5 (a + a + 9d)$

$$\Rightarrow 2a+9d = 25$$

Solving equation (i) and equation (ii), we get
$$d = -1, a = 17$$
$$\therefore a_{10} = a + (10-1) d$$
$$= a + 9d$$
$$= 17 + 9(-1)$$
$$= 17 - 9$$
$$= 8$$
$$\therefore d = -1, a_{10} = 8.$$

 $d = -1, a_{10} = 8.$ (v) given $d = 5, S_9 = 75, a = ?, a_9 = ?$

We have,

 \Rightarrow

 $S_9 = \frac{9}{2}(a+a_9)$

75 =
$$\frac{9}{2}(a+a+8d)$$

150	Lifeskills'	Complete	NCERT	Solutions	Class-X	Mathematics
-----	-------------	----------	-------	-----------	---------	-------------

75 $=\frac{9}{2}(2a+8d)$ \Rightarrow 75 = 9 (a + 4 × 5) \Rightarrow 75 = 9a + 180 \Rightarrow 9*a* = -105 \Rightarrow $a = -\frac{35}{3}$ \Rightarrow $a_{0} = a + 8d$ *.*..

$$=-\frac{35}{3} + 8 \times 5$$
$$=-\frac{35}{3} + 40$$
$$=\frac{85}{3}$$

$$\therefore \quad a = \frac{-35}{3} \text{ and } a_9 = \frac{85}{3}$$
(vi) given $a = 2, d = 8, S_n = 90, n = ?, a_n = ?$
We have
$$a_n = a + (n - 1) d$$

$$= 2 + (n - 1) 8$$

$$= 2 + 8n - 8$$

$$= 8n - 6$$

and

 $S_n = \frac{n}{2} [a + a_n]$ 90 = $\frac{n}{2}[a+8n-6]$ \Rightarrow 90 = $\frac{n}{2}[2+8n-6]$ \Rightarrow 90 = $\frac{n}{2}[8n-4]$ \Rightarrow 90 = $4n^2 - 2n$ \Rightarrow $4n^2 - 2n - 90 = 0$ \Rightarrow $2n^2 - n - 45 = 0$ \Rightarrow (which is quadratic equation) $\Rightarrow 2n^2 - 10n + 9n - 45$ =0 (using factorization method) \Rightarrow 2n(n-5) + 9(n-5) = 0 \Rightarrow (n-5)(2n+9) = 0 $n = 5, n = \frac{-9}{2}$ \Rightarrow

The number of terms can not be negative *n* = 5

and

$$a_n = a + (n - 1) d$$

$$\Rightarrow a_5 = 2 + (5 - 1) 8$$

$$= 2 + 32$$

$$= 34.$$
(vii) given $a = 8, a_n = 62, S_n = 210$, find n and d .
We have

$$a_n = a + (n - 1)d$$

$$\Rightarrow 62 = 8 + (n - 1) d$$

$$\Rightarrow (n - 1)d = 54$$

and

$$210 = \frac{n}{2} [8+62]$$

35n = 210
n = 6.

 $S_n = \frac{n}{2} [a + a_n]$

Substituting the value of n in equation (i), we get

 \Rightarrow \Rightarrow \Rightarrow

or

or

or

or

or

...

$$(6-1) d = 54$$

$$d = \frac{54}{5}$$
(viii) given $a_n = 4, d = 2, S_n = -14$, find n and a .
$$a_n = a + (n-1) d$$

$$\Rightarrow \qquad 4 = a + (n-1) 2$$

$$\Rightarrow \qquad 4 = a + 2n - 2$$

$$\Rightarrow \qquad 6 = a + 2n$$
and
$$S_n = \frac{n}{2} (a + a_n)$$

$$-14 = \frac{n}{2} (a + 4)$$

$$n = -\frac{-28}{a + 4}$$

Substituting this value of n in equation (i), we get

$$6 = a + \frac{2 \cdot (-28)}{a+4}$$

or $6 (a+4) = a^2 + 4a - 56$
or $a^2 - 2a - 80 = 0$
or $a^2 - 10a + 8a - 80 = 0$
or $a(a-10) + 8 (a-10) = 0$
or $(a+8) (a-10) = 0$
or $a = -8, a = 10$
 $\therefore n = -\frac{28}{a+4}$

 $=\frac{-28}{-8+4}$, (when a=-8) or $\frac{-28}{10+4}$, (when a = 10) or $\frac{-28}{14}$ =7 or - 7(it is not possible)

: a = -8, n = 7.(ix) given a = 3, n = 8, S = 192, find d.

We have,

$$S = \frac{\pi}{2} [a + a_n]$$

$$192 = \frac{8}{2} [3 + a_8]$$

$$3 + a_8 = 48$$

$$a_8 = 45$$
Now,

$$a_n = a + (n - 1) d$$

$$\Rightarrow \qquad 45 = 3 + (8 - 1)d$$

$$\Rightarrow \qquad 7d = 42$$

$$\Rightarrow \qquad d = \frac{42}{7}$$

$$\Rightarrow \qquad d = -6$$

(x) given l = 28, S = 144, and there are total 9 terms. Find a.

S = $\frac{n}{2}[a+l]$

 $144 = \frac{9}{2}[a+28]$ or *a*+28 =32 \Rightarrow

We have,

a =4. \Rightarrow Q.4. How many terms of the AP : 9, 17, 25, ... must be taken to give a sum of 636?

Ans. Here, a = 9, d = 17 - 9 = 8, $S_n = 636 = 8$ Now, using the formula,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

636 = $\frac{n}{2} [2 \times 9 + (n-1) 8]$

 \Rightarrow

$$\Rightarrow \qquad 636 = \frac{n}{2} \left[18 + 8n - 8 \right]$$

$$\Rightarrow \qquad 636 = \frac{n}{2} [10 + 8n]$$

$$636 = 5n + 4n^{2}$$

$$4n^{2} + 5n - 636 = 0$$

$$n = \frac{-5 \pm \sqrt{(5)^{2} - 4 \times 4 \times 9 - 636)}}{2 \times 4}$$
(using quadratic formula)

$$n = \frac{-5 \pm \sqrt{25 + 10176}}{8}$$
$$= \frac{-5 \pm \sqrt{10201}}{8}$$
$$= \frac{-5 \pm 101}{8}$$
$$= \frac{-5 \pm 101}{8} \text{ or } \frac{-5 - 101}{8}$$

 \therefore $n = \frac{96}{8}$; since negative value of *n* is not

possible. \Rightarrow n=12.

 \Rightarrow

 \Rightarrow

...

Q.5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. Here, a = 5, $a_n = l = 45$, $S_n = 400$ $S_n = \frac{n}{2}[a+l]$ We have $400 = \frac{n}{2} [5+45)$ n =16. $\mathbf{a} + (\mathbf{n} - 1) d = \mathbf{a}_{\mathbf{n}}$ Now, $5 + (16 - 1) d = 45^{"}$ \Rightarrow 5 + 15d = 45 \Rightarrow 15d = 40 \Rightarrow $d = \frac{8}{3}$ \Rightarrow $d = 2\frac{2}{3}$ \Rightarrow

Q.6. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Ans. Here a = 17, $l = a_n = 350$, d = 9Now using $a + (n-1)^n d = a_n$ 17 + (n-1)9 = 350 \Rightarrow

152 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

\Rightarrow	(n-1)	9 =333
\Rightarrow	n-1	=37
\Rightarrow	n	=38

using

...

. .

$$S_{n} = \frac{n}{2} [2a + (n-1)d], \text{ we get}$$

$$S_{38} = \frac{38}{2} [2 \times 17 + (38-1)9]$$

$$= 19 [34 + 333]$$

$$= 19 \times [367]$$

$$S_{38} = 6973$$
Find the sum of first 22 terms of an AP

Q.7. Pin which d = 7 and 22nd term is 149.

Ans. Here,
$$n = 22$$
, $d = 7$, $a_{22} = 149$, $S_{22} = ?$
We have, $a + (n-1)d = a_n$
 $\Rightarrow a + (22-1)7 = 149$
 $\Rightarrow a + 21 \times 7 = 149$
 $\Rightarrow a + 147 = 149$
 $\Rightarrow a = 2$
using $S_n = \frac{n}{2} [a + a_n]$, we get

$$S_{22} = \frac{22}{2} [a + a_{22}]$$

= $\frac{22}{2} [2 + 149]$

$$S_{22} = 1661.$$

Q.8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans. Here, $n = 51, a_2 = 14, a_3 = 18$ $d^2 = a_3 - a_2$ *.*... =18-14=4 $a_{3} = a + 2d$ Now, $18 = a + 2 \times 4$ \Rightarrow a = 10 \Rightarrow $S_{12} = \frac{51}{2} [2 \times 10 + (51 - 1) 4]$ *.*... $=\frac{51}{2}[20+200]$ $=\frac{51}{2}\times 220$ $= 51 \times 110$

 $S_{51} = 5610.$

Q.9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first *n* terms.

Ans. Here, we have

 \Rightarrow \Rightarrow

 \Rightarrow

...

$$S_{7} = \frac{7}{2} [2a + (7-1)d]$$

$$49 = \frac{7}{2} [2a + 6d]$$

$$49 = 7a + 21d$$

$$a + 3d = 7$$
...(i)

and
$$S_{17} = \frac{17}{2} [2a + (17 - 1)d]$$

$$\Rightarrow \frac{17}{2} [2a+16d] = 289$$
$$\Rightarrow a+8d = 17$$

$$\Rightarrow a+8d = 17 \dots$$
(ii)
On solving equations (i) and (ii), we get
 $a=1, d=2$

(::)

Sn =
$$\frac{n}{2} [2a + (n-1)d]$$

= $\frac{n}{2} [2 \times 1 + (n-1)2]$
= $\frac{n}{2} [2 + 2n - 2] = n^2$

Thus, $S_n = n^2$.

Q.10. Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below :

(i)
$$a_{1} = 3 + 4n$$
 (ii) $a_{2} = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Ans. (i) Here, $a_n = 3 + 4n$ Put $n = 1, 2, 3, 4, \dots$ successively, we get $a_1 = 3 + 4 \times 1 = 3 + 4 = 7$ $a_2^{1} = 3 + 4 \times 2 = 3 + 8 = 11$ $a_3 = 3 + 4 \times 3 = 3 + 12 = 15, \dots$ $\therefore a = 7, d = a_2 - a_1 = 11 - 7 = 4.$ Clearly, a_1 , a_2 , a_3 , ... is an A.P. with common

difference 4.

Now,
$$S_{15} = \frac{15}{2} [2 \times 7 + (15 - 1)4]$$

 $= \frac{15}{2} [14 + 56]$
 $= \frac{15}{2} \times 70 = 525$

 \Rightarrow

(ii) Here, $a_n = 9 - 5n$ Put $n = 1, 2, 3, \dots$ successively, we get $a_1 = 9 - 5 \times 1 = 9 - 5 = 4$ *.*.. $a_2^{'} = 9 - 5 \times 2 = -1,$ $a_3 = 9 - 5 \times 3 = -6, \dots$ a = 4, d = -1 - 4 = -5.. .

Clearly, a_1 , a_2 , a_3 , ... is an A.P. with common difference -5.

Now,
$$S_{15} = \frac{15}{2} [2 \times 4 + (15 - 1)(-5)] = \frac{15}{2} [8 - 70]$$
$$= \frac{15}{2} [-62] = -465.$$

Q.11. If the sum of the first *n* terms of an AP is $4n - n^2$, what is the first term (that is S₁)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the *n*th terms.

Ans. Here, $S_n = 4n - n^2$, $\therefore S_1 = 4 \times 1 - (1)^2 = 4 - 1$ =3. First term is 3. Now, $S_2 = 4 \times 2 - (2)^2 = 8 - 4 = 4$ Sum of two terms is 4. 2nd term = $S_2 - S_1 = 4 - 3 = 1$ Now, $S_3 = 4 \times 3 - (3)^2 = 12 - 9 = 3$ \therefore 3rd term = $S_3 - S_2 = 3 - 4 = -1$ $S_9 = 4 \times 9 - (9)^2 = 36 - 81 = -45$ Now $\mathbf{S}_{10}^{'} = 4 \times 10 - (10^2 = 40 - 100 = -60)$ and 10th term $= S_{10} - S_9 = -60 - (-45)$ *.*.. = -60 + 45 = -15Similarly, nth term = $S_n - S_{n-1}$ = $4n - n^2 - 4(n-1) + (n-1)^2$ $=4n-n^2-4n+4+n^2+1-2n$ = 5 - 2n.

Q.12. Find the sum of the first 40 positive integers divisible by 6.

Ans. The positive integers divisible by 6 are. Here, a = 6, d = 12 - 6 = 6, n = 40

$$S_{40} = \frac{40}{2} [2 \times 6 + (40 - 1) 6]$$

= 20 [12 + 39 × 6]
= 20 [12 + 234]
= 20 × 246

Thus, $S_{40} = 4920.$

Q.13. Find the sum of the first 15 multiples of 8.

Ans. The multiples of 8 are, 8, 16, 24, 32, Here, a = 8, d = 16 - 8 = 8, n = 15

$$\therefore \qquad S_{15} = \frac{15}{2} [2 \times 8 + (15 - 1) 8]$$
$$= \frac{15}{2} [16 + 112]$$
$$= \frac{15}{2} \times 128$$
$$= 15 \times 64$$

Thus, $S_{15} = 960$.

Q.14. Find the sum of the odd numbers between 0 and 50.

Ans. The odd numbers between 0 and 50 are 1, 3, 5, 7, 9, 49 Here, a = 1, d = 3 - 1 = 2, $a_n = l = 49$ Using the formula $a + (n-1)d = a_n$ 1 + (n-1)2 = 49(n-1)2 = 48n - 1 = 24

n =25

Thus,

$$S_{n} = \frac{25}{2} [2 \times 1 + (25 - 1)2]$$
$$= \frac{25}{2} [2 + 48]$$
$$= \frac{25}{2} \times 50$$
$$= 25 \times 25$$
$$S_{n} = 625$$

 $S = \frac{n}{2} [2a + (n-1)d]$

Q.15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by **30 days?**

Ans. Here the series is :
200, 250, 300,
So,
$$a = 200, d = 50, n = 30$$
, (given)
∴ $S_{30} = \frac{30}{2} [2 \times 200 + (30 - 1) 50]$
 $= 15 [400 + 29 \times 50]$

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

= 15 [400 + 1450] $= 15 \times 1850$ = 27.750

The penalty for the cost of construction is Rs. 27,750.

Q.16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Ans. Here, n = 7, $S_n = 700$, d = -20 $S_n = \frac{n}{2} [2a + (n-1)d]$ *.*... $700 = \frac{7}{2} \left[2a + (7-1)(-20) \right]$ \Rightarrow $200 = 2a + 6 \times (-20)$ \Rightarrow 200 = 2a - 120 \Rightarrow 2a = 200 + 1202a = 320 \Rightarrow $=\frac{320}{2}$ а \Rightarrow a = 160Ist Prize = Rs.160. 2nd Prize = 160 - 20 = Rs. 1403rd Prize = 140 - 20 = Rs. 1204th Prize = 120 - 20 =Rs. 100 5th Prize = 100 - 20 =Rs. 80 6th Prize = 80 - 20 =Rs. 60

7th Prize = 60 - 20 =Rs. 40.

Q.17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. No. of trees that planted by all 3 sections of classes, I, II, III, XII are 3, 6, 9, ... 36 respectively.

Here, d = 6 - 3 = 9 - 6 = = 3. Hence, it is an A.P. Here, $a = 3, a_n = 36, n = 12$.

Total no. of trees,
$$S_n = \frac{n}{2} [a + a_n]$$
$$= \frac{12}{2} [3 + 36]$$

 $= 6 \times 39$ = 234.

Q.18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in Fig.. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = 22/7$)



[Hint : Length of successive semicircles is *l*₁, *l*₂, *l*₃, *l*₄,..., with centres at A, B, A, B, ..., respectively.] Ans. Here, series is

or $\pi r_1, \pi r_2, \pi r_3, \dots, \pi \times 0.5, \pi \times 1.0, \pi \times 1.5, \pi \times 2.0, \dots$

or
$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

Here,
$$a = , d = \frac{3\pi}{2} - p = \frac{\pi}{2}, n = 13$$

$$S_{n} = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{13}{2} \left[2 \times \frac{\pi}{2}, +(13-1), \frac{\pi}{2} \right]$$

$$= \frac{13}{2} [\pi + 6\pi] \qquad [\pi = \frac{22}{7}]$$

$$= \frac{13}{2} \times 7\pi = \frac{13}{2} \times 7 \times \frac{22}{7} = 143$$

Total length = 143 cm.

Q.19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how many logs are in the top row?



Ans. Clearly, logs stacked in each row form a sequence

20, 19, 18, 17, 16 15, It is an A.P. with a = 20, d = 19 - 20 = -1Let Sⁿ = 200, then

$$\frac{n}{2} [2 \times 20 + (n-1)(-1)] = 20$$

$$\Rightarrow \qquad n(40 - n + 1) = 400$$

$$\Rightarrow \qquad 41n - n^2 = 400$$

$$\Rightarrow \qquad n^2 - 41n + 400 = 0$$

$$\Rightarrow \qquad (n - 16)(n - 25) = 0$$

$$\Rightarrow \qquad n = 16 \text{ or } 25$$

For n = 25, we have

 $a_{25} = a + 24d$

$$=20+24(-1)$$

 \therefore Number of logs in 25th row is -4, which is not possible.

So, n = 25 is not possible.

n = 16

...

Thus, 200 logs are placed in 16 rows.

- $\therefore \quad \text{Number of logs in the 16th row (top row)} = a_{16}$ = a + 15d
 - = 20 + 15(-1) = 20 15 = 5.

Q.20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig.).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[**Hint :** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Ans. Distance travelled to pick up 1 potato = $2 \times 5 = 10 \text{ m}$

Distance travelled to pick up II potato

$$= 2 \times (5+3) = 16 \,\mathrm{m}$$

Distance travelled to pick up III potato

 $= 2 \times (5 + 3 + 3) = 22 \text{ m etc.}$

The series is 10, 16, 22, Here, n = 10, a = 10, d = 16 - 10 = 6

:
$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow S_{10} = \frac{10}{2} [2 \times 10 + (10 - 1) 6]$$
$$= 5 [20 + 54]$$
$$= 5 \times 74 = 370$$
Total distance travelled = 370 m

EXERCISE 5.4 (Optional)

Q.1. Which term of the AP : 121, 117, 113, ..., is its first negative term?

	[Hint : Find <i>n</i> for	$a_{n} < 0$]
	Ans. Here, <i>a</i> = 12	1, d = 117 - 121 = -4.
:: U	Using a_n	< 0
\Rightarrow	a + (n - 1) d	< 0
\Rightarrow	121 + (n-1)(-4)	< 0
\Rightarrow	(n-1)(-4)	<-121
\Rightarrow	-4n + 4	<-121
\Rightarrow	-4n	<-125
\Rightarrow	4n	>125
\Rightarrow	n	$> \frac{125}{4}$
\Rightarrow	п	>311/4
	T (1)	

Q.2. The sum of the third and the seventh term of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Ans. Given,
$$a_3 + a_7 = 6$$

 $\Rightarrow a + (3-1)d + a + (7-1)d = 6$
 $[a_n = a + (n + 1)d]$
 $\Rightarrow a + 2d + a + 6d = 6$
 $\Rightarrow 2a + 8d = 6$
 $\Rightarrow a + 4d = 3$
and $a_3a_7 = 8$
 $(a + 2d)(a + 6d) = 8$
 $\Rightarrow (3 - 4d + 2d)(3 - 4d + 6d) = 8$
 $\Rightarrow (3 - 2d)(3 + 2d) = 8$
 $\Rightarrow 9 - 4d^2 = 8$
 $\Rightarrow -4d^2 = 8 - 9$
 $\Rightarrow -4d^2 = -1$

∴ Its first negative term is 32nd.

\Rightarrow	d^2	$=rac{1}{4}$
\Rightarrow	d	$=+\frac{1}{2}$

When
$$d = \frac{1}{2}$$

 $\therefore \qquad a + 4 \times \frac{1}{2} = 3$

$$\begin{array}{ccc} \Rightarrow & a+2 & =3 \\ \Rightarrow & a & =3-2 \\ \Rightarrow & a & =1 \end{array}$$

When
$$d = -\frac{1}{2}$$

 $\therefore \qquad a + 4\left(-\frac{1}{2}\right) = 3$
 $\Rightarrow \qquad a - 2 = 3$
 $\Rightarrow \qquad a = 3 + 2$

$$\Rightarrow$$
 $a = 5$

when
$$a = , d = \frac{1}{2}, \left[S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

= $8\left[2 + \frac{15}{2}\right]$
= 4×19
= 76
 $S_{16} = \frac{16}{2}\left[2 \times 5 + (16-1)\left(\frac{1}{2}\right)\right]$

 $S_{16} = \frac{16}{2} \left[2 \times 1 + (16 - 1) \left(\frac{1}{2} \right) \right]$

Also

when
$$a = 5, d = \frac{-1}{2}$$

$$= 8 \left[10 - \frac{15}{2} \right]$$

= 4 [20 - 15]
= 4 × 5
= 20

 \therefore The sum of first 16 terms of A.P. is 76 or 20.

Q.3. A ladder has rungs 25 cm apart. (see Fig.). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the

bottom rungs are 2 $\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[Hint : Number of rungs =
$$\frac{250}{25}$$
 +1]



Ans. Distance between top and bottom rungs

$$=2\frac{1}{2}$$
 m $= 2.5 \times 100$ cm
= 250 cm

Number of rungs =
$$\frac{250}{25} = 10$$

:. $a = 25, l = a_n = 45, n = 10.$ The length of the wood required for the rungs = sum of 10 rungs

$$= \frac{10}{2} [25 + 45]$$

= 5 × 70 = 350 cm

Q.4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

[Hint: $S_{x-1} = S_{49} - S_x$] Ans. According to question $S_{x-1} = S_{49} - S_x$

$$\Rightarrow \frac{x-1}{2} [2 \times 1 + (x-1-1).1]$$
$$= \frac{49}{2} [2 \times 1 + (49-1)(1)] - \frac{x}{2} [2 \times 1 + (x-1)1]$$

$$\Rightarrow \frac{x-1}{2} [2x+x-2] = \frac{49}{2} [2+48] - \frac{x}{2} [2+x-1]$$

$$\Rightarrow \frac{x(x-1)}{2} = 25 \times 49 - \frac{x(x+1)}{2}$$

$$\Rightarrow x^2 - x = 2450 - x2 - x$$

$$\Rightarrow 2x^2 = 2450$$

$$\Rightarrow x^{2|} = 1225$$

$$\Rightarrow x = +35.$$

Since x is a counting number, so $x \neq -35.$

 \therefore x = 35.

Q.5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. (see Fig.). Calculate the total volume of concrete required to build the terrace.

[Hint: Volume of concrete required to build the



Ans. Volumes of concrete required to build :

First step =
$$\frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4} \text{ m}^3$$

Second step = $\left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50 = \frac{50}{4} \text{ m}^3$

Third step =
$$\left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50 = \frac{75}{4} \text{ m}^3$$

Here, n = no. of steps = 15, (Given)

$$= \frac{25}{4} + \frac{50}{4} + \frac{75}{4} + \dots$$
$$= \frac{25}{4} [1 + 2 + 3 + \dots + 15] \qquad [n = 15]$$
$$= \frac{25}{4} \times \frac{15}{2} [2 \times 1 + (15 - 1) \times 1]$$
$$= \frac{25}{4} \times \frac{15}{2} \times 16 = 750 \text{ m}^3$$

Additional Questions

Q.1. Is 0 a term of the AP : 31, 28,25 ...? Justify your answer.

Ans. $a_n = 0$ a = 31, d = 28 - 31 = -3 $\therefore \qquad a_n = a + (n-1)d$ 0 = 31 + (n-1)(-3) $\Rightarrow \qquad 3n - 3 = +31$ $\Rightarrow \qquad 3n = 28$ $\Rightarrow \qquad n = \frac{28}{3}$ which is not a natural number.

Hence 0 is not a term of the given A.P.

Q.2. The taxi fare after each kilometer, when the fare is Rs.15 for the first km and Rs. 8 for each additional km, does not form an AP as the total fare (in Rs.) after each km is 15, 8, 8, 8,

Is the statement true ? Give reasons.

Ans. Fare after 1 km = Rs. 15 Fare after 2 km = Rs.23 Fare after 3 km = Rs. 31 Fare after 4 km = Rs.39 Now the series is 13, 23, 31, 39, $a_2 - a_1 = 23 - 15 = 8$ $a_3 - a_2 = 31 - 23 = 8$ $a_4 - a_3 = 39 - 31 = 8$ Now, d = 8 and = 15. ∴ the given series is an A.P. Hence the given statement is true.

Q.3. For the A.P. : -3, -7, -11, ... can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20} ? Given reason for your answers.

Ans.
$$a_{30} - a_{20} = [a + (30 + 1)d] - [a + (20 - 1)d]$$

= 29d - 19d = 10d

158 | Lifeskills' Complete NCERT Solutions Class-X Mathematics

As d = -7(-3) = -7 + 3 = -4 $a_{30} - a_{20} = 10 \times (-4) = -40.$ *.*... Q.4. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c. **Ans.** As the numbers are in AP, \therefore 7 - a = b - 7 = 23 - b = c - 23 $\Rightarrow b-7=23-b \Rightarrow 2b=30 \Rightarrow b=15$ \therefore 7 - a = b - 7 \Rightarrow 7 - a = 15 - 7 \Rightarrow a = -1 Again $b-7 = c-23 \Longrightarrow 15-7 = c-23$ $\Rightarrow c = 31$ Hence a = -1, b = 15, c = 31. Q.5. The 9th term of an AP is zero, prove that 29th term is twice the 19th term. **Ans.** Let for the AP first term = aand common difference = d $a_{n} = a + (n-1)d$ As $a_n = a + 8d = 0 \Longrightarrow a = -8d$ *.*... $a_{29} = a + 28d = -8d + 28d = 20d$ Now and $a_{19} = a + 18d = -8d + 18d = 10d$ $a_{29} = 2a_{19}$ Hence 6, $3k^2 + 4k + 4$ are three consecutive terms of an A.P. Ans. Clearly $2(2k^2+3k+6)$ $=(k^2+4k+8)+3k^2+4k+4)$ $4k^2 + 6k + 12 = 4k^2 + 8k + 12$ \Rightarrow $6k - 8k = 0 \Longrightarrow k = 0$. \Rightarrow Q.7. Find whether 55 is a term of the AP : 7, 10, 13, ... or not. If yes, find which term it is . **Ans.** a = 7, d = 10 - 3 = 3Let $a_{\rm m} = 55$ a + (n-1)d = 55*.*.. =55 \Rightarrow $7 + (n-1) \times 3$ =55 \Rightarrow $(n-1) \times 3$ $(n-1) = 16 \Longrightarrow n = 17$ \Rightarrow Yes, 17th term = 55. Q.8. Find the sum of last 10 terms of the A.P.: 8, 10, 12,, 126. Ans. Writing the AP in reverse order 126, 124, 12, 10, 8 a = 126, d = 124 - 126 = -2

As
$$S_n = \frac{n}{2} [2a + (n-1)d]$$

 $S_{10} = \frac{10}{2} [2a + 9d]$
 $= 5 [2 \times 126 + 9 \times (-2)]$
 $= 5[252 - 18]$
 $= 5 \times 234 = 1170.$

Q.9. Find the sum of first seven numbers which are multiples of 2 as well as of 9.

Ans. Let $S_7 = 18 + 36 + 54 + \dots 7$ terms a = 18, d = 18, n = 7

$$S_{7} = \frac{7}{2} [2a + 6d]$$
$$= \frac{7}{2} [2 \times 18 + 6 \times 18]$$
$$= \frac{7}{2} [36 + 108]$$
$$= \frac{7}{2} \times 144 = 7 \times 72 = 504$$

Q.10.Kanishka was given her pocket money on Jan. 1st, 2012. She puts Re. 1 on Day 1, Rs. 2 on Day 2, Rs. 3 on Day 3 and continued doing so till the end of the month, into her piggy bank. She also spent Rs. 204 of her pocket money, and found that at the end of the month she still had Rs. 100 with her. How much was her pocket money for the month ?

Ans.
$$S_n = 1 + 2 + 3 + \dots + 31$$

 $a = 1, d = 1, a_n = 31, n = 31$
 $S_{31} = \frac{31}{2} [1 + 31]$
 $= 31 \times 16 = 496$
Money in the piggy Bank = Rs. 496.
Money spend = Rs. 204.
Money with her = Rs. 100
Total pocket money = 496 + 204 +

= Rs. 800.

100

Multiple Choice Questions

Q.1.	If the sum of <i>n</i> terms of an A.P. is $2n^2 + 5n$, then its 2nd terms is :			
		(c) 11	(d) 13	

Ans. (c)

Q.3.	The sum of first five	e positive integers divisible	difference is 2, then the sum of the first 6 terms			
	by 6 :			is :		
	(a) 180	(b) 90		(a) 0	(b) 5	
	(c) 45	(d) 30		(c) 6	(d) 15	
Ans. (b)				Ans. (a)		
Q.4.	4. The sum of first 16 terms of the AP : 10, 6, 2, is :			The famous math	nematician associated with	
				finding the sum of the first 100 natural		
	(a) - 320	(b) 320		numbers is :		
	(c) - 352	(d) - 400		(a) Pythagoras	(b) Newton	
Ans.	(a)			(c) Gauss	(d) Euclid	
Q.5.	What is the common difference of an A.P. in		Ans.	. (c)		
	which $a_{24} - a_{17} = -28$?		Q.9. The sum of first five multiples of 3 is :			
	(a) 8	(b) –8		(a) 45	(b) 55	
	(c)-4	(c)4		(c) 65	(d) 75	
Ans. (c)			Ans.	. (a)		
Q.6.	6. The sum of first 10 multiples of 2 is :			Q.10. Which term of the AP: 21, 42, 63, 84, is		
	(a) 100	(b) 110		210?		
	(c) 130	(d) 120		(a) 9th	(b) 10th	
Ans.	(b)			(c) 11th	(d) 12th	
Q.7. If the first term of an AP is -5 and the common			Ans.	. (b)		