Chapter 4

Quadratic Equations

In the Chapter

In this chapter, you will be studying the following points:

- ★ A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.
- ★ A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
- ★ If we can factorise $ax^2 + bx + c$, $a \neq 0$, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- ★ A quadratic equation can also be solved by the method of completing the square.
- ★ Quadratic formula: The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

  $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, provided $b^2 - 4ac > 0$.
- ★ A quadratic equation $ax^2 + bx + c = 0$ has
  (i) two distinct real roots, if $b^2 - 4ac > 0$,
  (ii) two equal roots (i.e., coincident roots), if $b^2 - 4ac = 0$, and
  (iii) no real roots, if $b^2 - 4ac < 0$.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 4.1

Q.1. Check whether the following are quadratic equations :

(i) $(x+1)^2 = 2(x-3)$
(ii) $x^2 - 2x = (-2)(3-x)$
(iii) $(x-2)(x+1) = (x-1)(x+3)$
(iv) $(x-3)(2x+1) = x(x+5)$
(v) $(2x-1)(x-3) = (x+5)(x-1)$
(vi) $x^2 + 3x + 1 = (x-2)^2$
(vii) $(x+2)^3 = 2x(x^2-1)$
(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Ans. (i) L.H.S. $= (x+1)^2 = x^2 + 2x + 1$

R.H.S. $= 2(x-3) = 2x - 6$

$\therefore \quad x^2 + 2x + 1 = 2x - 6$

$\Rightarrow \quad x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow \quad x^2 + 7 = 0$

$\therefore$ It is of the form $ax^2 + bx + c = 0$, where

$a = 1, b = 0, c = 7$

Hence, the given equation is a quadratic equation.

(ii) R.H.S. $= (-2)(3-x) = -6 + 2x$

$\therefore \quad x^2 - 2x = -6 + 2x$

$\Rightarrow \quad x^2 - 2x + 6 - 2x = 0$

$\Rightarrow \quad x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$ where

$a = 1, b = -4, c = 6$

Hence, the given equation is a quadratic equation.

(iii) L.H.S. $= (x-2)(x+1)$

$= x^2 - x - 2$

R.H.S. $= (x-1)(x+3)$
$= x^2+2x-3$

$\therefore \quad x^2-x-2 = x^2+2x-3$

$\Rightarrow -x-2-2x+3=0$

$\Rightarrow \quad -3x+1 = 0$

It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(iv) L.H.S. $= (x-3)(2x+1)$
$= 2x^2-6x+x-3$
$= 2x^2-5x-3$

R.H.S. $= x(x+5) = x^2+5x$

$\therefore \; 2x^2-5x-3 = x^2+5x$

$\Rightarrow \; 2x^2-x^2-5x-3=0$

$\Rightarrow \quad x^2-3x-10 = 0$

It is of the form $ax^2+bc+c=0$ where

$a=1, b=-10, c=-3$

Hence, the given equation is a quadratic equation.

(v) L.H.S. $= (2x-1)(x-3)$
$= 2x^2-6x-x+3$
$= 2x^2-7x+3$

R.H.S. $= (x+5)(x-1)$
$= x^2+5x-x-5$
$= x^2+4x-5$

$\therefore \; 2x^2-7x+3 = x^2\,4x-5$

$\Rightarrow \; 2x^2-x^2-7x-4x+3+5=0$

$\Rightarrow \; x^2-11x+8\,0$

It is of the form $ax^2+bx+c=0$ where

$a=1, b=-11, c=8$.

Hence, the given equation is a quadratic equation.

(vi) $x^2+3x+1 = (x-2)^2$

$\Rightarrow \quad x^2+3x+1 = x^2-4x+4$

$\Rightarrow \quad 7x-3=0$

It is not of the form $ax^2+bc+c=0$

Hence, the given equation is not quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1)$

$\Rightarrow \quad x^3+6x^2+12x+8 = 2x^3-2x$

$\Rightarrow \; x^3-6x^2-14x-8=0$

It is not of the form $ax^2+bx+c=0$

$\therefore$ The given equation is not quadratic equation.

(viii) $x^3+4x^2-x+1 = (x-2)^3$

$\Rightarrow \quad x^3-4x^2-x+1 = x^3-6x^2+12x-8$

$\Rightarrow \quad 2x^2-13x+9=0$

It is of the form $ax^2+bx+c=0$

Hence, the given equation is quadratic equation.

Q.2. Represent the following situations in the form of quadratic equations :

(i) The area of a rectangular plot is 528 m². The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Ans. (i) Let the breadth of rectangular plot $= xm$

$\therefore$ The length of rectangular plot $= (2x+1)m$.

$\therefore$ Area of rectangular plot $= x.\,(2x+1)m^2$

$\therefore$ According to the equation,

$x(2x+1) = 528$

$\Rightarrow \quad 2x^2+x = 528$

$\Rightarrow \quad 2x^2+x-528 = 0$

(ii) Let two consecutive positive integers are x and $x+1$.

Their product $= x(x+1)$.

$\therefore$ According to question

$x(x+1) = 306$

or $\quad x^2+x = 306$

or $\quad x^2+x-306 = 0$

(iii) Let Rohan's present age $= x$ years

$\therefore$ Rohan's mother's present age $= (x+26)$ years

$\therefore$ Rohan's and his mother's age 3 years from now will be $(x+3)$ years and $(x+26)+3$, i.e., $(x+29)$ years respectively

Hence, according to the question.

$(x+3)(x+29) = 360$

$\Rightarrow \quad x^2+29x+3x+87 = 360$

$\Rightarrow \quad x^2+32x-273=0$

(iv) Let original speed of train $= x$ km/h

Time taken to cover 480 km $= \dfrac{480}{x}$ hours.

New speed $= (x-8)$ km/h.

New time taken to cover 480 km $= \dfrac{480}{x-8}$ km/h.

$\therefore$ According to question

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$\Rightarrow \quad 480\left(\frac{1}{x-8}-\frac{1}{x}\right)=3$

$\Rightarrow \quad \frac{x-(x-8}{x(x-8)}=\frac{1}{160}$

$\Rightarrow \quad x^2-8x=1280$

$\Rightarrow \quad x^2-8x-1280=0$

EXERCISE 4.2

Q.1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2-3x-10=0$

(ii) $2x^2+x-6=0$

(iii) $2x^2+7x+5\sqrt{2}=0$

(iv) $2x^2-x+\frac{1}{8}=0$

(v) $100x^2-20x+1=0$

Ans. (i) We have, $x-3x-10=0$

$\Rightarrow \quad x^2-5x+2x-10=0$

$\Rightarrow \quad x(x-5)+2(x-5)=0$

$\Rightarrow \quad (x-5)(x+2)=0$

$\therefore \quad x-5=0 \text{ or } x+2=0$

$\Rightarrow \quad x=5 \text{ or } x=-2$

The roots of quadratic equation are 5 and – 2.

(ii) We have, $2x^2+x-6=0$

$\Rightarrow \quad 2x^2+4x-3x-6=0$

$\Rightarrow \quad 2x(x+2)-3(x+2)=0$

$\Rightarrow \quad (x+2)(2x-3)=0$

$\Rightarrow \quad x+2=0 \text{ or } 2x-3=0$

$\Rightarrow \quad x=-2 \text{ or } x=3/2$

$\therefore$ The roots of quadratic equation are –2 and 3/2.

(iii) We have, $\sqrt{2}x^2+7x+4\sqrt{2}=0$

$\Rightarrow \quad \sqrt{2}x^2+5x+2x+5\sqrt{2}=0$

$\Rightarrow \quad x(\sqrt{2}+5)+\sqrt{2}(\sqrt{2}x+5)=0$

$\Rightarrow \quad (\sqrt{2}x+5)(x+\sqrt{2})=0$

$\Rightarrow \quad \sqrt{2}x+5=0 \text{ or } x+\sqrt{2}=0$

$\Rightarrow \quad x=-5\sqrt{2} \text{ or } x=-\sqrt{2}$

Hence $-5\sqrt{2}$ and $-\sqrt{2}$ are the roots of the given quadratic equation.

(iv) We have $2x^2-x+\frac{1}{8}=0$

$\Rightarrow \quad 16x^2-8x+1=0$

$\Rightarrow \quad 16x^2-4x-4x+1=0$

$\Rightarrow \quad 4x(4x-1)-(4x-1)=0$

$\Rightarrow \quad (4x-1)-(4x-1)=0$

$\Rightarrow \quad 4x-1=0 \text{ or } 4x-1=0$

$\Rightarrow \quad x=\frac{1}{4} \text{ or } \frac{1}{4}.$

Hence given quadratic equation has repeated roots $\frac{1}{4}$.

(v) We have $100x^2-20x+1=0$

$\Rightarrow \quad 100x^2-10x-10x+1=0$

$\Rightarrow \quad 10x(10x-1)-(10x-1)=0$

$\Rightarrow \quad (10x-1)(10x-1)=0$

$\Rightarrow \quad 10x-1=0, 10x-1=0$

$\Rightarrow \quad x=\frac{1}{10},\frac{1}{10}.$

$\therefore$ Given quadratic equation has repeated roots $\frac{1}{10}$.

Q.2. Solve the problems given in Example 1.

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

Ans. (i) Let the number of marbles John had be x.

Then the number of marbles Jivanti had $=45-x$.

According to the given question

$(x-5)(45-x-5)=124$

$\Rightarrow \quad x-5(40-x)-124=0$

$\Rightarrow \quad 40x-x^2-200+5x-124=0$

$\Rightarrow \quad -x^2+45x-324=0$

$\Rightarrow \quad x^2-45x+324=0$

$\Rightarrow \quad x^2-36x-9x+324=0$

$\Rightarrow \quad x(x-36)-9(x-36)=0$

$\Rightarrow \quad (x-36)(x-9)=0$

$\Rightarrow \quad x-36=0 \text{ or } x-9=0$

$x=36 \text{ or } x=9$

Hence, the number of marbles john had = 36 and the number of marbles jivanti had = 45 – 36 = 9.

(ii) Let the number of toys produced on that day be x.

And, cost of production $= (55 - x)$

According to the given question,

$$x(55-x) = 750$$
$$\Rightarrow 55x - x^2 - 750 = 0$$
$$\Rightarrow x^2 - 55x + 750 = 0$$
$$\Rightarrow x^2 - 30x - 25x + 750 = 0$$
$$\Rightarrow x(x-30) - 25(x-30) = 0$$
$$\Rightarrow (x-30)(x-25) = 0$$
$$\Rightarrow x - 30 = 0 \text{ or } x - 25 = 0$$
$$\Rightarrow x = 30 \text{ or } x = 25$$

Hence, the number of toys produced on that day are 30 or 25.

Q.3. Find two numbers whose sum is 27 and product is 182.

Ans. Let first number be x.

$\therefore$ Second number $= 27 - x$.

According to question

$$x(27-x) = 182$$
$$\Rightarrow 27x - x^2 = 182$$
$$\Rightarrow x^2 - 27x + 182 = 0$$
$$\Rightarrow x^2 - 14x - 13x + 182 = 0$$
$$\Rightarrow x(x-14) - 13(x-14) = 0$$
$$\Rightarrow x - 14 = 0 \text{ or } x - 13 = 0$$
$$\Rightarrow x = 14 \text{ or } x = 13$$

Hence, the number are 14 and 13.

Q.4. Find two consecutive positive integers, sum of whose squares is 365.

Ans. Let two consecutive positive integers be x and $x + 1$.

$\therefore$ According to question

$$x^2 + (x+1)^2 = 365$$
$$\Rightarrow x^2 + x^2 + 2x + 1 = 365$$
$$\Rightarrow 2x^2 + 2x - 364 = 0$$
$$\Rightarrow x^2 + x - 182 = 0$$
$$\Rightarrow x^2 + 14x - 13x - 182 = 0$$
$$\Rightarrow x(x+14) - 13(x+14) = 0$$
$$\Rightarrow (x+14)(x-13) = 0$$
$$\Rightarrow x = -14, x = 13$$

Since x is a positive integer, so $x \neq -14$

$$\therefore x = 13$$

Thus, consecutive positive integers are 13 and 14.

Q.5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Ans. Let the length of base of right triangle be x cm.

$\therefore$ Height of right triangle $= (x - 7)$ cm

Length of hypotenuse = 13 cm, (given)

Using pythagoras theorem,

$$x^2 + (x-7)^2 = (13)^2$$
$$\Rightarrow x^2 + x^2 - 14x + 49 = 169$$
$$\Rightarrow 2x^2 - 14x - 120 = 0$$
$$\Rightarrow x^2 - 7x - 60 = 0$$
$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$
$$\Rightarrow x(x-12) + 5(x-12) = 0$$
$$\Rightarrow (x-12)(x+5) = 0$$
$$\Rightarrow x - 12 = 0 \text{ or } x + 5 = 0$$
$$\Rightarrow x = 12 \text{ or } x = -5$$

Since the length cannot be negative.

$\therefore$ The other sides of right triangle are 12 cm and (12 – 7) cm, i.e., 5 cm.

Q.6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Ans. Let the number of pottery articles on a particular day be x.

$\therefore$ The cost of production of each article

$$= \text{Rs. } (2x + 3)$$

So, the total cost of production on that day

$$= \text{Rs. } x(2x + 3)$$

$$\therefore x(2x+3) = 90$$
$$\Rightarrow 2x^2 + 3x = 90$$
$$\Rightarrow 2x^2 + 3x - 90 = 0$$
$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$
$$\Rightarrow x(2x+15) - 6(2x+15) = 0$$
$$\Rightarrow (2x+15)(x-6) = 0$$
$$\Rightarrow 2x + 15 = 0, x - 6 = 0$$
$$\Rightarrow x = \frac{-15}{2}, x = 6$$

The number of articles cannot be negative

$\therefore$ The number of pottery articles are 6.

Cost of each article $= \text{Rs. } (2 \times 6 + 3)$

$= \text{Rs. } 15.$

EXERCISE 4.3

Q.1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x_2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Ans. (i) We have, $2x^2 - 7x + 3 = 0$

Is the same as,

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Now, $$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow \quad x^2 - 2 \cdot \frac{7}{2} \cdot x + \frac{3}{2} = 0$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \frac{49}{16} = 0$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$$

$$\Rightarrow \quad \left(x - \frac{7}{4}\right) = + \frac{5}{4}$$

$$\Rightarrow \quad x = \frac{5}{4} + \frac{7}{4} \text{ and } x = \frac{-5}{4} + \frac{7}{4}$$

$$\Rightarrow \quad x = 3 \text{ and } x = \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

is the same as $$x^2 + \frac{1}{2}x - 2 = 0$$

$$\Rightarrow \quad \left\{x + \frac{1}{2}\left(\frac{1}{2}\right)\right\}^2 - \left\{\frac{1}{2}\left(\frac{2}{2}\right)\right\}^2 - 2 = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 - \frac{33}{16} = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right) = \pm \frac{\sqrt{33}}{4}$$

$$\Rightarrow \quad x = \frac{\sqrt{33}}{4} - \frac{1}{4} \text{ and } x = \frac{-\sqrt{33}}{4} - \frac{1}{4} \textbf{ Ans.}$$

(iii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

is as same as, $$x^2 \sqrt{3}x + \frac{3}{4} = 0$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{4}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{3}{4} = 0$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{4}\right)^2 - \frac{3}{4} + \frac{3}{4} = 0$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{4}\right)^2 = 0$$

$$\Rightarrow \quad x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2} \text{ (repeated roots)}$$

(iv) we have $2x^2 + x + 4 = 0$

$$\Rightarrow \quad x^2 + \frac{1}{2}x + 2 = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 - \frac{1}{16} + 2 = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 + \frac{31}{16} = 0$$

$$\Rightarrow \quad \left(x + \frac{1}{2}\right)^2 = \frac{-31}{16} < 0$$

which is not real.

Hence, the quadratic equation has no real roots.

Q.2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Ans. (i) we have, $2x^2 - 7x + 3 = 0$

It is of the form, $ax^2 + bx + c = 0$

where, $a = 2, b = -7, c = 3$

Now, $D = b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 3$

$= 49 - 24$

$= 25 > 0$

Hence the roots of the quadratic equation exist.

Now quadratic formula gives,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{25}}{2 \cdot 2}$$

$$= \frac{7 \pm 5}{4}$$

$$\Rightarrow \quad x = \frac{7+5}{4} \text{ and } \frac{7-5}{4}$$

$$\therefore \quad x = 3 \text{ and } \frac{1}{2}$$

(ii) We have, $2x^2 + x - 4 = 0$

Here, $a = 2, b = 1, c = -4$.

Now $D = b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times (-4)$

$= 1 + 32$

$= 33 > 0$

Hence, the roots of the quadratic equation exist.

Now quadratic formula gives,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-1 \pm \sqrt{33}}{2 \cdot 2}$$

$$\Rightarrow \quad x = \frac{-1+\sqrt{33}}{4} \text{ and } \frac{-1-\sqrt{33}}{4} \text{ and.}$$

(iii) We have, $4x^2 + 4\sqrt{3}x + 3 = 0$

It is of the form, $ax^2 + bc + c = 0$

where, $a = 4, b = 4\sqrt{3}, c = 3$.

Now $D = b^2 - 4ac$

$= (4\sqrt{3})^2 - 4 \times 4 \times 3$

$= 48 - 48$

$= 0$

Hence, quadratic equation has two equal real roots.

Now, quadratic formula gives,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-4\sqrt{3} + 0}{2 \cdot 4}$$

$$= \frac{-4\sqrt{3}}{8}$$

$$\Rightarrow \quad x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$$

(iv) Given, $2x^2 + x + 4 = 0$

Here, $a = 2, b = 1, c = -4$

Now, $D = b^2 - ac$

$= (1)^2 - 4 \times 2 \times 4$

$= 1 - 32$

$= -31 < 0$

Hence, the quadratic equation has no real roots.

Q.3. Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Ans. (i) We have, $x - \frac{1}{x} = 3$

$$\Rightarrow \quad \frac{x^2 - 1}{x} = 3$$

$$x^2 - 1 = 3x$$

$$\Rightarrow \quad x^2 - 3x - 1 = 0$$

which is a quadratic equation.

Here, $a = 1, b = -3, c = -1$

$D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1)$

$= 9 + 4 = 13 > 0$

$$\therefore \quad x = \frac{3+\sqrt{13}}{2} \text{ and } \frac{3-\sqrt{13}}{2}$$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

or $$\frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

or $\dfrac{-11}{x^2+4x-7x-28}=\dfrac{11}{30}$

or $x^2-3x-28=-30$

or $x^2-3x+2=0$

which is a quadratic equation

Here, $a=1, b=-3, c=2$

Now $D=b^2-4ac=(-3)^2-4\times1\times2$

$=9-8$

$=1>0$

Hence quadratic equation has real roots

$\therefore \quad x=\dfrac{3\pm\sqrt{1}}{2\times1}$

$\Rightarrow \quad x=\dfrac{3\pm1}{2}$

$\therefore \quad x=2, 1.$

Q.4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$ .Find his present age.

Ans. Let the Rehman's present age be x.

$\therefore$ Rehman's age 3 years ago $=x-3$.

and Rehman's age 5 years from now $=x+5$.

$\therefore \quad \dfrac{1}{x-3}+\dfrac{1}{x+5}=\dfrac{1}{3}$

$\Rightarrow \quad \dfrac{(x+5)+(x-3)}{(x-3)(x+5)}=\dfrac{1}{3}$

$\Rightarrow \quad \dfrac{2x+2}{(x-3)(x+5)}=\dfrac{1}{3}$

$\Rightarrow \quad x^2+5x-3x-15=6x+6$

$\Rightarrow \quad x^2-4x-21=0$

which is quadratic equation

Here, $a=1, b=-4, c=-21$.

The roots of quadratic equation are

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot1\cdot(-2)}}{2\cdot1}$$

$$=\frac{4\pm\sqrt{16+84}}{2}$$

$$=\frac{4\pm\sqrt{100}}{2}$$

$$x=\frac{4+10}{2} \text{ and } \frac{4-10}{2}$$

$\Rightarrow \quad x=7$ and -3

Since the age cannot be negative,

$\therefore$ The present ageo f Rehman is 7 years.

Q.5. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Let Shefali's marks in Mathematics be x.

$\therefore$ Shefali's marks in English is $30-x$.

Now Shafali's new marks in Mathematics $=x+2$

Shaflai's new marks in English $=30-x-3$

$=27-x$.

$\therefore \quad (x+2)(27-x)=210$

$\Rightarrow \quad 27x-x^2+54-2x=210$

$\Rightarrow \quad x^2-25x+156=0$

which is quadratic equation

Here, $a=1, b=-25, c=156$.

The root of quadratic equation are

$$x=\frac{-(-25)\pm\sqrt{(-25)^2-4\cdot1\cdot156}}{2\cdot1}$$

$$=\frac{25\pm\sqrt{625-624}}{2}$$

$$=\frac{25\pm1}{2}$$

$\Rightarrow \quad =\dfrac{25+1}{2}$ and $\dfrac{25-1}{2}$

$\therefore \quad x=13$ or 12

The marks in Mathematics and English are 13 and 17 or 12 and 18 respectively.

Q.6. The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Ans. Let the length of shorter side AD $=x$ m.

$\therefore$ The length of longer side AB $=(30+x)$ m

$\therefore$ Length of diagonal BD $=(x+60)$ m.

$\therefore$ Using Pythagoras theorem,

$AB^2+AD^2=BD^2$

$\Rightarrow \quad (30+x)^2+(x)^2=(x+60)^2$

$\Rightarrow \quad 900+x^2+60x+x^2=x^2+3600+120x$

$\Rightarrow \quad x^2-60x-2700=0$

$\Rightarrow \quad x^2-90x+30x-2700=0$

(by factorization method)

$\Rightarrow \quad x(x-90)+30(x-90)=0$

$\Rightarrow \quad (x-90)(x+30)=0$

$\Rightarrow \quad x=90 \text{ or } -30$

Since the length of the sides cannot be negative.

$\therefore$ Shorter side AD = 90 metre

and longer side AB = 90 + 30 = 120 metre

Q.7. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans. Let the larger number = x.

$\therefore$ The square of smaller number = 8x

$\therefore \quad x^2-8x=180$

$\Rightarrow \quad x^2-8x-180=0$

which is quadratic equation

Here, $a=1, b=-8, c=-180$

$\therefore$ The quadratic formula gives

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4\cdot 1\cdot(-180)}}{2\cdot 1}$$

$$=\frac{8\pm\sqrt{64+720}}{2}$$

$$=\frac{8\pm\sqrt{784}}{2}$$

$$\Rightarrow \quad x=\frac{8\pm 28}{2}$$

$$\Rightarrow \quad x=\frac{8+28}{2} \text{ or } \frac{8-28}{2}$$

$\Rightarrow \quad x=18 \text{ or } -10$

Case I. when $x=18$, we have

Square of the smaller number = $8x$

$=8\times 18=144$

$\therefore$ Smaller number = ± 12

Thus, the numbers are 18, 12 or 18, – 12

Case II. When $x=-10$, we have

Square of the smaller number = $8x$

$=8(-10)=-80$

which is not possible as square of a number is always positive.

Hence, the required numbers are 18, 12 or 18, –12.

Q.8. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans. Let the original speed of train be x km/h

$\therefore$ The time taken to travel the journey = $\frac{360}{x}$ h

The new speed of the train = $(x+5)$ km/h

$\therefore$ The time taken to travel the journey = $\frac{360}{x+5}$ h.

Therefore, $\frac{360}{x}-\frac{360}{x+5}=1$

$$\Rightarrow \quad \frac{360(x+5-x)}{x(x+5)}=1$$

$\Rightarrow \quad x^2+5x=1800$

$\Rightarrow \quad x^2+5x-1800=0$

which is quadratic equation.

Here, $a=1, b=5, c=-1800$

Now, the quadratic equation gives,

$$x=\frac{-5\pm\sqrt{(5)^2-4\cdot 1\cdot(-1800)}}{2\cdot 1}$$

$$=\frac{-5\pm\sqrt{25+7200}}{2}$$

$$=\frac{-5\pm\sqrt{7225}}{2}$$

$$\Rightarrow \quad x=\frac{-5\pm 85}{2}$$

$$\Rightarrow \quad x=\frac{-5+85}{2} \text{ or } \frac{-5-85}{2}$$

$=40 \text{ or } -45$

$\therefore$ The speed of the train cannot be negative.

$\therefore$ The original speed of the train = 40 km/h.

Q.9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans. Let the time taken by smaller tap be x hours.

In 1 hour the part of tank filled = $1/x$

$\therefore$ The time taken by larger tap = $(x-10)$ hours

In 1 hour the part of tank filled = $\frac{1}{x-10}$

$\therefore \quad \frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$

$\Rightarrow \quad \frac{x-10+x}{x(x-10)}=\frac{8}{75}$

$\Rightarrow \quad 150x-750=8x^2-80x$

$\Rightarrow \quad 8x^2-230x+750=0$

$\Rightarrow \quad 4x^2-115x+375=0$

Here, $a=4$, $b=-115$, $c=375$

By using quadratic formula, we get

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-115)\pm\sqrt{(-115)^2-4\cdot 4\cdot 375}}{2\cdot 4}$$

$$= \frac{115\pm\sqrt{13225-6000}}{8}$$

$$= \frac{115\pm\sqrt{7225}}{8}$$

$$= \frac{115\pm 85}{8}$$

$$\therefore \quad x = \frac{115+85}{8} \text{ or } \frac{115-85}{8}$$

$$= \frac{200}{8} \text{ or } \frac{30}{8}$$

$$= 25 \text{ or } \frac{15}{4}$$

$$= 25 \text{ hrs or } 3\frac{3}{4} \text{ hrs} < 9\frac{3}{8}$$

(we can neglect it)

Time taken by I tap = 25 hrs.

Time taken by II tap = 15 hrs.

Q.10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

Ans. Let the average speed of passenger train be x km/hr.

$\therefore$ The average speed of express train

$= (x+11)$ km/hr.

Total distance = 132 km.

Time taken by express train to travel 132 km

$$= \frac{132}{x+11} \text{ hr.}$$

Time taken by passenger train to travel 132 km

$$= \frac{132}{x} \text{ hr.}$$

$$\therefore \quad \frac{132}{x}-\frac{132}{x+11}=1$$

$$\Rightarrow \quad \frac{x+11-x}{x(x+11)}=\frac{1}{132}$$

$\Rightarrow \quad x^2+11x = 11\times 132$

$\Rightarrow \quad x^2+11x-1432 = 0$

Using quadratic formula, we get

$$x = \frac{-11\pm\sqrt{(11)^2-4\cdot(-1452)\cdot 1}}{2\cdot 1}$$

$$= \frac{-11\pm\sqrt{121+5808}}{2}$$

$$= \frac{-11\pm\sqrt{5929}}{2} = \frac{-11\pm 77}{2}$$

$$\therefore \quad x = \frac{-11-77}{2} \text{ or } x=\frac{-11+77}{2}$$

$$x = \frac{-88}{2} = -44, x=\frac{66}{2} = 33 \text{ km/hr.}$$

and Since speed cannot be negative

Speed of passenger train = 33 km/hr

Speed of express train = (33 + 11) km/hr = 44 km/hr.

Q.11. Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Ans. Let the sideof I square be x m.

$\therefore$ The side of II square $= \frac{4x-24}{4}$ m

$= (x-6)$ m

$\therefore \quad x^2+(x-6)^2 = 468$

$\Rightarrow \quad x^2+x^2-12x+36 = 468$

$\Rightarrow \quad 2x^2-12x+36 = 468$

$\Rightarrow \quad 2x^2-12x-432 = 0$

$\Rightarrow \quad x^2-6x-216 = 0$

which is a quadratic equation

Here, $a = 1, b = -6, c = -216$.

$\therefore$ The quadratic formula gives

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot (-216)}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{36+864}}{2}$$

$$= \frac{6 \pm \sqrt{900}}{2}$$

$$= \frac{6 \pm 30}{2}$$

$$\Rightarrow x = \frac{6+30}{2} \text{ or } \frac{6-30}{2}$$

$$\Rightarrow x = \frac{36}{2} \text{ or } \frac{-24}{2}$$

$$\Rightarrow x = 18 \text{ or} -12$$

The sides of square cannot be negative, so x = 18

The sides of these two square are 18 m and $(18-6)$ m = 12m.

EXERCISE 4.4

Q.1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Ans. (i) we have, $2x^2 - 3x + 5 = 0$

Here, $a = 2, b = -3, c = 5$

$\therefore$ Discriminant, $b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5$

$= 9 - 40$

$= -31 < 0$

Hence, the roots are not real, i.e., imaginary.

(ii) We have, $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 4, b = -4\sqrt{3}, c = 4$

$\therefore$ Disriminant, $b^2 - 4ac = (-43)^2 - 4 \times 3 \times 4$

$= 48 - 48$

$= 0$

$\therefore$ The given quadratic equation has two equal real roots

$\therefore$ The quadratic formula give,

$$x = \frac{-(-4\sqrt{3}) \pm \sqrt{0}}{2 \cdot 3}$$

$$= \frac{4\sqrt{3} \pm 0}{6}$$

$$\Rightarrow x = \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$$

$\therefore$ The equal roots are $\frac{2\sqrt{3}}{3}$ and $\frac{2\sqrt{3}}{3}$.

(iii) We have, $2x^2 - 6x + 3 = 0$

Here, $a = 2, b = -6, c = 3$

$\therefore$ Discriminant, $b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24$

$= 12 > 0$

$\therefore$ The given quadratic equation has two distinct real roots.

The roots are,

$$x = \frac{-(-6) \pm \sqrt{12}}{2 \cdot 2}$$

$$= \frac{6+2\sqrt{3}}{4}, \frac{6-2\sqrt{3}}{4}$$

$$= \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$$

Two distinct real roots are $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$.

Q.2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x-2) + 6 = 0$

Ans. (i) We have $2x^2 + kx + 3 = 0$

Here, $a = 2, b = k, c = 3$

$\therefore$ The given quadratic equation has two equal real roots.

$\therefore$ Discriminant $b^2 - 4ac = 0$

$\Rightarrow k^2 - 4 \times 2 \times 3 = 0$

$\Rightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = +2\sqrt{6}$.

(ii) We have, $kx(x-2) + 6 = 0$

$\Rightarrow kx^2 \; 2kx + 6 = 0$

Here, $a = k, b = -2k, c = 6$

∴ The given equation has two equal real roots.

∴ Discriminant, $b^2 - 4ac = 0$

⇒ $(-2k)^2 - 4 \times k \times 6 = 0$

⇒ $4k^2 - 24k = 0$

⇒ $4k^2 = 24k$

⇒ $4k^2 - 24k = 0$

⇒ $4k(k - 6) = 0$

⇒ $k = 0, k = 6$

Q.3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m²? If so, find its length and breadth.

Ans. Let the breadth of rectangle be x.

∴ The lengthof rectangle $= 2x$

Area $= 800$ (given)

∴ $x2x = 800$

⇒ $2x^2 = 800$

⇒ $x^2 = \frac{800}{2}$

$= 400$

⇒ $x = +20.$

The sides of rectangle cannot be negative, therefore $x = 20$.

The value of x is real to design of grove is possible.

Length and breadth of rectangle are 40 m and 20 m respectively.

Q.4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans. Let age of one of the friends = x year.

Then, age of the other friend = $(20 - x)$ years

Four years ago their ages were

$x - 4$ and $20 - x - 4$ i.e., $16 - x$.

∴ $(x - 4)(16 - x) = 48$

⇒ $16x - x^2 - 64 + 4x = 48$

⇒ $x^2 - 20x + 112 = 0$

Here $a = 1, b = -20, c = 112$

∴ Discriminant, $b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112$

$= 400 - 448 = -48 < 0$

The given equation has not real roots.

hence, the given situation is not possible.

Q.5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m₂? If so, find its length and breadth.

Ans. Let the length of rectangular park = x m.

∴ The breadth of rectangular park = $\frac{400}{2}$,

Its Perimeter = 80 m, (given)

∴ $2\left(x + \frac{400}{x}\right) = 80$

⇒ $x^2 + 400 = 40x$

⇒ $x^2 - 40x + 400 = 0$

⇒ $x^2 - 20x - 20x + 400 = 0$

(by factorization method)

⇒ $x(x - 20) - 20(x - 20) = 0$

⇒ $(x - 20(x - 20) = 0$

⇒ $x = 20, 20$

Yes, it is possible to design the rectangular park of sides 20m, 20m.

Additional Questions

Q.1. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Ans. Consider equation :

$x^2 - 3x + 1 = 0$ is an equation with integral coefficients but its roots are not integers.

Q.2. Does there exists a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

Ans. Consider the equation $x^2 - 6x + 7 = 0$.

$$x = \frac{-(-6) \cdot \sqrt{(-6)^2 - 4 \cdot 1 \cdot 7}}{2}$$

$$x = \frac{6 \cdot \sqrt{36 - 28}}{2}$$

$$= \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2}$$

$$= 3 \pm \sqrt{2}$$

Thus there exists an equation whose coefficients are rational but both of its roots are irrational.

Q.3. Is 0.2 a root of a the equation $x^2 - 0.4 = 0$? Justify.

Ans. $x^2 - 0.4 = 0$

⇒ $x^2 = 0.4$

⇒ $x = + \sqrt{0.40}$

⇒ $x = +0.6$

Hence 0.2 is not a root of the given equation.

Q.4. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

Ans. Consider the quadratic equation

$$\sqrt{3}x^2 - 7\sqrt{3}x + 12\sqrt{3} = 0$$

Now, $$x = \frac{7\sqrt{3} \pm \sqrt{(7\sqrt{3})^2 - 4\sqrt{3}.12\sqrt{3}}}{2\sqrt{3}}$$

$$x = \frac{7\sqrt{3} \pm \sqrt{147-144}}{2\sqrt{3}}$$

$$x = \frac{7\sqrt{3} \pm \sqrt{3}}{2\sqrt{3}} = \frac{7 \pm 1}{2} = 4, 3$$

Hence a quadratic equation exists whose coefficients are all distinct irrationals but both the roots are rationals.

Q.5. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Ans. Let the natural number $= x$.

$\therefore$ According to question

$$x^2 - 84 = 3(x+8$$

$$\Rightarrow x^2 - 84 = 3x + 24$$

$$\Rightarrow x^2 - 3x - 108 = 0$$

$$\Rightarrow (x-12)(x+9) = 0$$

$$\Rightarrow x - 12 = 0 \text{ or } x + 9 = 0$$

$$\Rightarrow x = 12 \text{ or } x = -9$$

$\therefore x = 12$ as x is a natural number.

Hence required natural number is 12.

Q.6. If Zeba were younger by 5 years than when she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

Ans. Let Zeba's age $= x$ years

5 years ago Zeba's age $= (x-5)$ years

Now, according to the question,

$$(x-5)^2 = 5x + 11$$

$$\Rightarrow x^2 + 25 - 10x = 5x + 11$$

$$\Rightarrow x^2 + 15x + 14 = 0$$

$$\Rightarrow (x-14)(x-1) = 0$$

$$\Rightarrow x = 14, 1$$

Clearly $x = 14$

Hence Zeba's age = 14 years.

Q.7. At t minutes past 2 p.m., the time needed by the minutes hand of a clock to show 3 p.m. was found to be 3 minutes less than $\frac{t^2}{4}$ minutes.

Ans. According to question,

$$60 - t = \frac{t^2}{4} - 3$$

$$\Rightarrow 240 - 4t = t^2 - 12$$

$$\Rightarrow t^2 - 12 = 240 - 4t$$

$$\Rightarrow t^2 + 4t - 252 = 0$$

$$\Rightarrow t^2 + 18t - 14t - 252 = 0$$

$$\Rightarrow t(t+18) - 14(t+19) = 0$$

$$\Rightarrow (t-14)(t+18) = 0$$

$$\Rightarrow t = 14, t = -18$$

As $t > 0 \therefore t = 14$.

Hence, $t = 14$ minutes

Q.8. Find the value of p for which the quadratic equation has real and equal roots $2x^2 + px + \frac{9}{2} = 0$.

Ans. The given quadratic equatuion is :

$$2x^2 + px + \frac{9}{2} = 0$$

Here, $a = 2, b = p, c = \frac{9}{2}$

For real and equal roots :

$$b^2 - 4ac = 0$$

$$\Rightarrow p^2 - 4 \times 2 \times \frac{9}{2} = 0$$

$$\Rightarrow p^2 = 36$$

$$\Rightarrow p = +6$$

Q.9. Find the value of m so that the quadratic equation $mx(x-7) + 49 = 0$ has two equal roots.

Ans. $mx(x-7) + 49 = 0$

$$\Rightarrow mx^2 - 7mx + 49 = 0$$

Comparing with

$$ax^2 + bx + c = 0$$

$$a = m, b = -7m, c = 49$$

Now $$D = b^2 - 4ac$$

$$= (-7m)^2 - 4 \times m \times 49$$

$$= 49m^2 - 4 \times m \times 49$$

For equal roots, $D = 0$

$$49m^2 - 4 \times m \times 49 = 0$$

$$\Rightarrow m^2 - 4m = 0$$

$$\Rightarrow m(m-4) = 0$$

$$\Rightarrow m = 0 \text{ or } m = 4.$$

Q.10. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$.

Ans. Given quadratic equation is :

$$9x^2 - 6ax + (a^2 - b^2)\ 0=$$

$$\Rightarrow (3x)^2 - 2 \times (3a) \times x + a^2 - b^2 = 0$$

$\Rightarrow \quad (3x-a)^2 - b^2 = 0$

$\Rightarrow \quad [3x-a-b][3x-a+b] = 0$

[Using $a^2 - b^2 = (a+b)(a-b)$]

$\Rightarrow \quad [3x-(a+b)][3x-(a-b)] = 0$

$\therefore \quad 3x = a+b$

or $\quad 3x = a-b$

or $\quad x = \frac{a+b}{3}$

abd $\quad x = \frac{a-b}{3}$

Multiple Choice Questions

Q.1. Which of the following questions has the sum of its roots as 3 ?

(a) $x^2+3x-5=0$

(b) $-x^2+3x+3=0$

(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x - 1$

(d) $3x^2-3x-3=0$

Ans. (b)

Q.2. A natural number when increaseby 12 equal 160 times its reciprocal. Find the number.

(a) 12 (b) 20

(c) 8 (d) 15

Ans. (c)

Q.3. A natural number whose square diminished by 84 is equal to thrice of 8 more than the given number is :

(A) 12 (b) 18

(c) 13 (d) 10

Ans. (a)

Q.4. The roots of the quyadratic equation $2x^2-x-6=0$ is :

(a) $-2, \frac{3}{2}$ (b) $2, -\frac{3}{2}$

(c) $-2, \frac{-3}{2}$ (d) $2, \frac{3}{2}$

Ans. (b)

Q.5. The value of k for which roots of the quadratic equation $kx^2+2x+3=0$ are equal is :

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) 3 (d) -3

Ans. (a)

Q.6. If $\frac{1}{2}$ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then value of k is :

(a) 2 (b) -2

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Ans. (a)

Q.7. If the discriminant of the equation $6x^2-bx+2=0$ is 1, then the value of 'b' is :

(a) 7 (b) -7

(c) $+7$ (d) $\pm\sqrt{3}$

Ans. (c)

Q.8. The quadratic equation $2x^2-\sqrt{5}x+1=0$ has :

(a) two distinct real number

(b) two equal real roots

(c) no real roots

(d) more than two real roots.

Ans. (c)

Q.9. The roots of the quadratic equation $x^2-3x+2=0$

(a) $(1, -2)$ (b) $(-1, 2)$

(c) $(-1, -2)$ (d) $1, 2)$

Ans. (d)

Q.10. Which constant should be added and subtracted to siove the quadratic equation $4x^2-\sqrt{3}x-5=0$ by the method of completing the squares ?

(a) $\frac{9}{16}$ (b) $\frac{3}{16}$

(c) $\frac{3}{4}$ (d) $\frac{\sqrt{3}}{4}$

Ans. (b)