Chapter 2

Polynomials

In the Chapter

In this chapter, will be studying studied the following points:

- Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
- A quadratic polynomial in x with real coefficients in of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points, where the graph of $y = p(x)$ intersects the x -axis.
- A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.
- If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$$

- If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

and $\alpha\beta\gamma = \frac{-d}{a}$

- The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that
  $p(x) = g(x)\, q(x) + r(x)$,
  where $r(x) = 0$ or degree $r(x) <$ degree $g(x)$.
- **Terms :** Various parts of an algebraic expression separated by + or – operations are called the terms of the expression.

  Example : (i) $3x^4 - 2x^3 + 8x^2 + 6 - 3x$ is an algebraic expression consisting of 5 terms, which are $3x^4$, $-2x^3$, $8x^2$, 6 and $-3x$.
  (ii) $x^3 + 3x^2y - 4xy^2 + y^3 + 8x + 3$ is an algebraic expression consisting of 6 terms, which are x^3, $3x^2y$, $-4xy^2$, y^2 $8x$ and 3.

  Constants : A symbol having a fixed numerical value is called a constant.

  Example : 3, 2, $\frac{6}{5}$, –9, π etc.

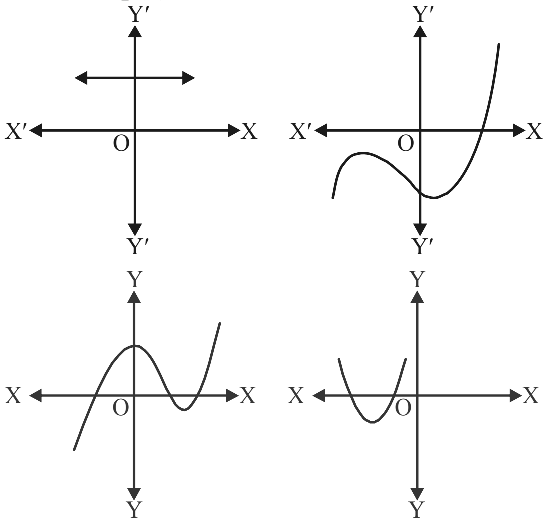
  Coefficients : Any factor in a term of an algebraic expression along with the sign of the term is called the coefficient of the product of the other factors.

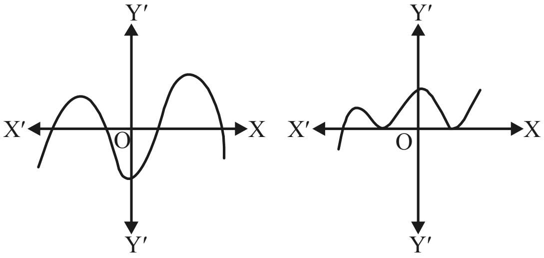
  Example :(i) In an algebraic expression $7x^3 - 8x^2 + 5x - 3$, the coefficients of x^3, x^2 and x are 7, – 8 and 5 respectively. We also say that –3 is the constant term.
  (ii) In $3x^4 - 5x^3 + 8x^2 + \sqrt{3}x + 5$, the coefficents, x^4, x^3, x^2 and x are 3, –5, 8 and $\sqrt{3}$ respectively.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 2.1

Q.1. The graph of $y = p(x)$ are given in Fig. 2.1 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.





Ans. (i) Graph $y = p(x)$ does not cut the x-axis at any point, so the given polynomial has no zero.

(ii) Graph $y = p(x)$ cuts the x-axis at one point, so the given polynomial has one zero.

(iii) Graph $y = p(x)$ cuts the x-axis at three point, so the given polynomial has three zeroes.

(iv) Graph $y = p(x)$ cuts the x-axis at two point, so the given polynomial has two zeroes.

(v) Graph $y = p(x)$ cuts the x-axis at four point, so the given polynomial has four zeroes.

(vi) Graph y = p(x) cuts the x-axis at three points, so the given polynomial has three zeroes.

EXERCISE 2.2

Q.1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(*i*) $x^2 - 2x - 8$ (*ii*) $4s^2 - 4s + 1$

(*iii*) $6x^2 - 3 - 7x$ (*iv*) $4u^2 + 8u$

(*v*) $t^2 - 15$

Ans. (*i*) We have

$$x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = (x - 4)(x + 2)$$

So, the value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$

i.e., when $x = 4$ or $x - 2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and –2.

Now,

$$\text{Sum of zeroes} = (4) + (-2) = \frac{2}{1} = \frac{-(\text{Coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of zeroes} = (4) - (2) = \frac{-8}{1} = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

(*ii*) We have

$$4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1 = (2s - 1)(2s - 1)$$

So, the value of $4s^2 - 4s + 1$ is zero, when $2s - 1 =$ 0 or $2s - 1 = 0$

i.e., when $s =$ or $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Now,

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{1}{2} = \frac{1}{1} = \frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

(iii) We have,

$$\begin{aligned} 6x^2 - 7x - 3 &= 6x^2 - 9x + 2x - 3 \\ &= 3x(2x-3) + (2x-3) \\ &= (3x+1)(2x-3) \end{aligned}$$

So, the value of $6x^2 - 7x - 3$ is zero when $(3x + 1) = 0$ or $(2x - 3) = 0$.

i.e., $x = -$ or $x = \frac{3}{2}$

Now,

Sum of zeroes $= \frac{-1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Prduct of zeroes $= \frac{-1}{3} \times \frac{3}{2} = \frac{-3}{6}$

$= \frac{\text{Constatnt term}}{\text{Coefficient of } x^2}$

(iv) We have,

$$4u^2 + 8u = 4u(u+2)$$

So, the value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$

i.e., when $u = 0$ or $u = -2$

$\frac{\text{Constant term}}{\text{Coefficient of } u^2}$ $(t+\sqrt{15})(t-\sqrt{15})$ $\frac{1}{2}, \sqrt{\frac{1}{3}}$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2

Now,

Sum of zeroes $= 0 + (-2) = \frac{-2}{1}$

$=$

(*v*) We have

$$t^2 - 15 = (t)^2 - \left(\sqrt{15}\right)^2$$

$=$

So, the value of $t^2 - 15$ is zero when $\left(t + \sqrt{15}\right) = 0$ or $\left(t - \sqrt{15}\right) = 0$

i.e, when $t = -\sqrt{15}$ or $t = \sqrt{15}$

Therefore, the zeroes $t^2 - 15$ are $-\sqrt{15}$ are $\sqrt{15}$.

Now,

Sum of the zeroes $= \left(-\sqrt{5}\right) + \left(\sqrt{15}\right)$

$= \frac{0}{1}$

$= \frac{+(\text{Constant term})}{\text{Coefficient of } t^2}$

Product of the zeros $= \left(-\sqrt{15}\right) \times \left(\sqrt{15}\right)$

$= \frac{-15}{1}$

$= \frac{+(\text{Constant term})}{\text{Coefficient of } t^2}$

(*vi*) We have

$$\begin{aligned} 3x^2 - x - 4 &= 3x^2 - 4x + 3x - 4 \\ &= x(3x-4) + 1(3x-4) \\ &= (x+1)(3x-4) \end{aligned}$$

So, the value of $3x^2 - x - 4$ is zero, when $(x + 1) = 0$ or $(3x - 4) = 0$

i.e, $x = -1$ or $= \frac{4}{3}$

Now,

Sum of the zeroes $= -\frac{4}{3} = \frac{-3+4}{3} = \frac{1}{3}$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of the zeros $= -1 \times \frac{4}{3} = -\frac{4}{3}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Q.2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(*i*) $\frac{1}{4}, -1$ (*ii*) (*iii*) $0, \sqrt{5}$

(*iv*) $1, 1$ (*v*) $-\frac{1}{4}, \frac{1}{4}$ (*vi*) $4, 1$

Ans. (i) Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β. Then

$$\alpha + \beta = \frac{1}{4} \Rightarrow \frac{-b}{a} = \frac{1}{4} \Rightarrow a = -4b$$

and $\alpha\beta = -1 \Rightarrow \frac{c}{a} = -1 \Rightarrow c = -a$

If $a = 1$, then $b = -\frac{1}{4}$ and $c = -1$

So, quadratic polynomial which fits the given condition is

$x^2 - \frac{1}{4}(x-1)$ or $4x^2 - x - 4$.

(*ii*) Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β. Then

$$\alpha + \beta = \frac{1}{4} \Rightarrow \frac{-b}{a} = \frac{1}{4} \Rightarrow a = -4b$$

and $$\alpha\beta = -1 \Rightarrow \frac{c}{a} = -1 \Rightarrow c = -a$$

If $a = 1$, then $b = -\frac{1}{4}$ and $c = -1$

So, quadratic polynomial which fits the given condition is

$x^2 - \frac{1}{4}x - 1$ or $4x^2 - x - 4$.

$x^2 - \sqrt{2}x + \frac{1}{3}$ or $3x^2 - 3\sqrt{2}x + 1$

(*iii*) Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β. Then

$$\alpha + \beta = \sqrt{0} \Rightarrow -\frac{b}{a} = 0 \Rightarrow -b = 0$$

and $$\alpha\beta = \sqrt{5} \Rightarrow \frac{c}{a} = \sqrt{5} \Rightarrow c = a\,.\sqrt{5}$$

If $a = 1$, then $b = 0$ and $c = \sqrt{5}$

So, quadratic polynomial which fits the given condition is

$x^2 + 0x + \sqrt{5}$ or $x^2 + \sqrt{5}$

(*iv*) Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β. Then

$$\alpha + \beta = 1 \Rightarrow -\frac{b}{a} = 1 \Rightarrow -b = 0$$

and $$\alpha\beta = 1 \Rightarrow \frac{c}{a} = 1 \Rightarrow c = a$$

If $a = 1$, then $b = -1$ and $c = 1$

So, quadratic polynomial which fits the given condition is $x^2 + x + 1$

(*v*) Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β. Then

$$\alpha + \beta = -\frac{1}{4} \Rightarrow \frac{-b}{a} = -\frac{1}{4} \Rightarrow b = -\frac{1}{4}a$$

and $$\alpha\beta = \frac{1}{4} \Rightarrow \frac{c}{a} = \frac{1}{4} \Rightarrow c = \frac{1}{4}a$$

If $a = 1$, then $b = \frac{1}{4}$ and $c = \frac{1}{4}$.

So, quadratic polynomial which fits the given condition is

$x^2 + \frac{1}{4}x + \frac{1}{4}$ or $4x^2 + x + 1$

(*vi*) Let the quadratic polynomial be $ax^2 + bx + c$, and its zeroes be α and β. Then

$$\alpha + \beta = 4 \Rightarrow \frac{-b}{a} = 4 \Rightarrow -b = 4a$$

and $$\alpha\beta = 1 \Rightarrow \frac{c}{a} = 1 \Rightarrow c = a$$

If $a = 1$, then $b = -4$ and $c = 1$.

So, quadratic polynomial which fits the given condition is $x^2 - 4x + 1$.

EXERCISE 2.3

Q.1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(*i*) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(*ii*) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(*iii*) $p(x) = x^4 - 5x^2 + 6$, $g(x) = 2 - x^2$.

Ans. (i)

$$\begin{array}{r|l} x^2 - 2 & x^3 - 3x^2 + 5x - 3 \quad (\; x - 3 \\ & \underline{\overset{-}{x^3} \qquad\quad \overset{+}{-\,2x}} \\ & -3x^2 + 7x - 3 \\ & \underline{\overset{+}{-\,3x^2} \qquad \overset{-}{+\,6}} \\ & 7x - 9 \end{array}$$

We stop the division process, because the degree of the remainder is less than the degree of the divisor. Hence, here the quotient is $x - 3$ and the remainder is $7x - 9$.

(*ii*)

$$\begin{array}{r|l} x^2 - x + 1 & x^4 - 3x^2 + 4x + 5 \quad (\; x^2 + x - 3 \\ & \underline{\overset{-}{x^4} \overset{-}{+\,x^2} \qquad \overset{+}{-\,x^3}} \\ & x^3 - 4x^2 + 4x + 5 \\ & \underline{\overset{-}{x^3} \overset{+}{-\,x^2} \overset{-}{+\,x}} \\ & -3x^2 + 3x + 5 \\ & \underline{\overset{+}{-\,3x^2} \overset{-}{+\,3x} \overset{+}{-\,3}} \\ & 8 \end{array}$$

We stop the division process, because the degree of the remainder is less than the degree of the divisor. Hence, here the quotient is $x^2 + x - 3$ and the remainder is 8.

(*iii*)

$$
\begin{array}{l}
-x^2-2 \,\big)\, x^4 - 5x + 6 \,\big(\, -x^2 - 2 \\
\qquad\qquad x^4 - 2x^2 \\
\qquad\qquad - \quad + \\
\hline
\qquad\qquad 2x^2 - 5x + 6 \\
\qquad\qquad 2x^2 - 4 \\
\qquad\qquad - \quad + \\
\hline
\qquad\qquad -5x + 10
\end{array}
$$

Hence, here quotient is $-(x^2 + 2)$ and remainder is $-5x + 10$.

Q.2. Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm.

(*i*) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(*ii*) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(*iii*) $p(x) = x^4 - 5x^2 + 6, g(x) = 2 - x^2$.

Ans. (*i*)

$$
\begin{array}{l}
t^2-3 \,\big)\, 2t^4 - 3t^3 - 2t^2 + 9t - 12 \,\big(\, 2t^2 + 3t + 4 \\
\qquad\quad 2t^4 - 6t^2 \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 3t^3 + 4t^2 - 9t - 12 \\
\qquad\quad 3t^3 - 9t \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 4t^2 - 12 \\
\qquad\quad 4t^2 - 12 \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 0
\end{array}
$$

Here, the remainder is zero, hence $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(*ii*)

$$
\begin{array}{l}
x^2 + 3x + 1 \,\big)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2 \,\big(\, 3x^2 - 4x + 2 \\
\qquad\quad 3x^4 + 9x + 3x^2 \\
\qquad\quad - \quad - \quad - \\
\hline
\qquad\quad -4x^3 + 10x^2 - 2x + 2 \\
\qquad\quad -4x^3 - 12x^2 - 4x \\
\qquad\quad + \quad + \quad + \\
\hline
\qquad\quad 2x^2 + 6x + 2 \\
\qquad\quad 2x^2 + 6x + 2 \\
\qquad\quad - \quad - \quad - \\
\hline
\qquad\quad 0
\end{array}
$$

Here, the remainder is zero, hence $x^2 - 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(*iii*)

$$
\begin{array}{l}
x^3 - 3x + 1 \,\big)\, x^5 - 4x^3 + x^2 + 3x + 1 \,\big(\, x^2 - 1 \\
\qquad\quad x^5 - 3x^3 + x^2 \\
\qquad\quad - \quad + \quad - \\
\hline
\qquad\quad -x^3 + 3x + 1 \\
\qquad\quad -x^3 + 3x - 1 \\
\qquad\quad + \quad - \quad + \\
\hline
\qquad\quad 2
\end{array}
$$

Here, the remainder is not zero, hence $x^3 - 3x + 1$ is not a factor of $x^5 + 4x^3 - x^2 + 3x + 1$.

Q.3. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, it two of its zeroes are $\sqrt{\frac{5}{3}}$ and $\sqrt{\frac{5}{3}}$.

[CBSE 2009 (C)

Ans. Let $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x$ – and $\alpha = \sqrt{\frac{5}{3}}$, $\beta = -\sqrt{\frac{5}{3}}$

$\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of $p(x)$ then

$\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)\left(x^2 - \frac{5}{3}\right)$ is a factor of $p(x)$.

Now, by applying division algorithm :

$$
\begin{array}{l}
\qquad\qquad\quad 3x^2 + 6x - 3 \\
x^2 - \frac{5}{3} \,\big)\, 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
\qquad\quad 3x^4 - 5x^2 \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 6x^3 + 3x^2 - 10x - 5 \\
\qquad\quad 6x^3 - 10x \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 3x^2 - 5 \\
\qquad\quad 3x^2 - 5 \\
\qquad\quad - \quad + \\
\hline
\qquad\quad 0
\end{array}
$$

So, $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$$= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) + 0$$

$$= \left(x^2 - \frac{5}{3}\right)\{3x^2 + 3x + 3x + 3\}$$

$$= \left(x^2 - \frac{5}{3}\right)\{3x(x + 1) + 3(x + 1)\}$$

$$= \left(x^2 - \frac{5}{3}\right)(3x + 3)(x + 1)$$

There, the zeroes of the given fourth degree polynomial $p(x)$ are

$$\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1, -1$$

Q.4. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder, are $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Ans. We have

Dividend $p(x) = x^3 - 3x^2 + x + 2$

Divisor $g(x) = ?$

Quotient $q(x) = x - 2$

and Remainder $r(x) = -2x + 4$

According to division algorithm

Divident $p(x)$ = Divisor $g(x)$ × Quotient $q(x)$ + Remainder $r(x)$

$\Rightarrow$ $x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

$\Rightarrow$ $x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$

$\Rightarrow$ $x^3 - 3x^2 + 3x - 2 = g(x) \times (x - 2)$

$$\Rightarrow g(x) = \frac{x3 - 3x^2 + 3x - 2}{(x-2)}$$

i.e.

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x - 2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \end{array}$$

Therefore, $g(x) = x^2 - x + 1$.

Q.5. Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) $deg\ p(x) = deg\ q(x)$

(ii) $deg\ q(x) = 0$

(iii) $deg\ q\ (x) = deg\ r(x)$

Ans. (i) $deg\ p(x) = deg\ q(x)$

$p(x) = 2x^2 - 3$

$g(x) = 2$

$q(x) = x^2 - 1$

and $r(x) = -1$

Clearly $p(x) = q(x) - g(x) + r(x)$

(ii) $deg\ q(x) = 0$

$p(x) = 3x^3 - 5$

$g(x) = x^3$

$q(x) = 3$

and $r(x) = -5$

Clearly $p(x) = q(x) - g(x) + r(x)$

(iii) $deg\ q(x) = deg\ r(x)$

$p(x) = x - 5$

$g(x) = x - 2$

$q(x) = 1$

and $r(x) = -3$

Clearly $p(x) = q(x) - g(x) + r(x)$

EXERCISE 2.4

Q.1. Verify that the numbers given alongside of the cubic polynomials below are thier zeroes. Also, verify the relationship between the zeroes and coefficients in each case.

(*i*) $2x^3 + x^2 - 5x + 2;\ \frac{1}{2}, 1, -2$

(*ii*) $x^3 + 4x^2 + 5x - 2;\ 2, 1, 1$

Ans. (*i*) Let $p(x) = 2x^3 + x^2 - 5x + 2$

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

$a = 2, b = 1, c = -5, d = 2$

Now $p\left(\frac{1}{2}\right) = 2\times\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\times\left(\frac{1}{2}\right) + 2$

$$= 2\times\frac{1}{8} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= \frac{1 + 1 - 10 + 8}{4} = \frac{0}{4}$$

$p(1) = 2 \times (1)^3 + (1)^2 - 5 \times 1 + 2$
$= 2 + 1 - 5 + 2 = 0$

and $p(-2) = 2(-2)^3 + (-2)^2 - 5 \times (-2) + 2$
$= -16 + 4 + 10 + 2 = 0$

$\therefore$, 1 and (–2) are the zeroes of $2x^3 + x^2 - 5x + 2$.

Hence, verified

Here, we have

$$\alpha = \frac{1}{2}, \ \beta = 1 \text{ and } \Upsilon = -2$$

Now, $\alpha + \beta + \Upsilon = \frac{1}{2} + 1 + (-2) = \frac{1}{2} = -\frac{b}{a}$

$\alpha\beta + \beta\Upsilon + \Upsilon\alpha = \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2}$

$= \frac{1}{2} - 2 - 1 \frac{1-4-2}{2} = \frac{-5}{2} = \frac{c}{a}$

and $\alpha\beta\Upsilon = \frac{1}{2} \times 1 \times (-2)$

$= \frac{-2}{2} = \frac{-d}{a}$. Hence verified.

(ii) Let $p(x) = x^3 + 4x^2 + 5x - 2$

Comparing the given polynomial with $ax^3 + bx^2 +$
$-\frac{1}{1}$ $cx + d$, we get

$a = 1, b = -4, c = 5$ and $d = -2$

Now, $p(2) = (2)^3 - (2)^2 + 5 \times (2) - 2$
$= 8 - 16 + 10 - 2 = 18 - 18 = 0$

and $p(1) = (1)^3 - 4 \times (1)^2 + 5 \times 1 - 2$

$\therefore$ 2, 1, 1 are the zeroes of $x^3 - 4x^2 + 5x - 2$

Hence, Verified

Here, we have

$$\alpha = 2, \beta = 1 \text{ and } \gamma = 1$$

Now $\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2$

$= 2 + 1 + 2 = \frac{5}{1} = \frac{c}{a}$

and $\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, Verified

Q.2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Ans. Let the quadratic polynomial be $ax^3 + bx^2 + cx + d$ and its zeroes be α, β and γ.

We have

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{-7}{1} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \frac{c}{a}$$

and $$\alpha\beta\gamma = \frac{-14}{1} = \frac{\text{Constant t erm}}{\text{Coefficien t of x}^3} = \frac{-\text{d}}{\text{a}}$$

When we put $a = 1$, then $b = -2$, $c = -7$, $d = 14$.

Therefore, the required cubic polynomial be $x^3 - 2x^2 - 7x + 14$.

Q.3. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $(a - b)$, a, $(a + b)$, find 'a' and 'b'.

Ans. Since $a - b$, a and $a + b$ are the zeroes of $f(x)$.

We have

$$a - b + a + a + b = \frac{\text{Coeff. of } x^2}{Coeff\,.of\ x^3}$$

$$= \frac{-(-3)}{1}$$

and $3a = 3 \Rightarrow a = 1$

$\Rightarrow \quad (a - b)\, a\, (a + b) = -\frac{\text{Constant t erm}}{\text{Coeff. of } x^3}$

$\Rightarrow \qquad =$

$\Rightarrow \quad a(a^2 - b^2) = -1 \Rightarrow 1 - b^2 = -1$

$\Rightarrow \quad b^2 = 2 \Rightarrow b = \pm\sqrt{2}$

Hence, $a = 1$, $b = \pm\sqrt{2}$

Q.4. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other two zeores.

Ans. It is given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are two zeroes of $f(x)$.

$\therefore \ \{x - (2 + \sqrt{3})\}\ \{x - (2 - \sqrt{3})\}$

$= (x - 2 - \sqrt{3})\,(x - 2 + \sqrt{3})$

$= (x - 2)^2 - (\sqrt{3})^2$

$= x^2 - 4x + 1$ is a factor of $f(x)$

Let us now divide $f(x)$ by $x^2 - 4x + 1$

For $f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ to be exactly divisible by $x^2 - 2x + k$, we must have $(-10 + 2k)x + (10 - a - 8k + k^2) = 0$ for all x

$$\begin{aligned}
\Rightarrow \quad 10x + 2k &= 0,\ 10 - a - 8k + k^2 = 0\\
\Rightarrow \quad k &= 5,\ 10 - a - 40 + 25 = 0\\
\Rightarrow \quad k &= 5 \text{ and } a = -5
\end{aligned}$$

We have,

$$\begin{array}{r|l}
 & x^2 - 2x - 35 \\ \hline
x^2 + 4x + 1 & x^4 + 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 + 4x^3 + x^2 \\
 & - \quad + \quad - \\ \hline
 & -2x^3 + 27x^2 - 138x \\
 & -2x^3 - 8x^2 - 2x \\
 & + \quad - \quad + \\ \hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\ \hline
 & 0
\end{array}$$

$$\Rightarrow \quad f(x) = (x^2 - 4x + 1)\,(x^2 - 2x - 35)$$

Hence, other two zeroes of $f(x)$ are the zeroes of the polynomial $x^2 - 2x - 35$, we have

$$\begin{aligned}
x^2 - 2x - 35 &= x^2 - 7x + 5x - 35\\
&= (x - 7\,(x + 5)
\end{aligned}$$

Q.5. If the polynomial $x^4 - 6x^3 + 16x^2 - 26x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find 'k' and 'a'.

Ans. By division algorithm, we have

It is given that $f(x) = x^4 - 6x^3 + 16x^2 - 26x + 10$, when divided by $x^2 - 2x + k$ leaves $x + a$ as remainder.

$\therefore$ $f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ is exactly divisible by $x^2 - 2x + k$.

Let us now divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$.

For $f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ to be exactly divisible by $x^2 - 2x + k$, we must have $(-10 + 2k)\,(x + (10 - a - 8k + k^2) = 0$ for all x.

$\Rightarrow$ $-10 + 2k = 0,\ 10 - a - 8k + k^2 = 0.$

$\Rightarrow$ $k = 5,\ 10 - a - 40 + 25 = 0$

$\Rightarrow$ $k = 5$ and $a = -5$.

$$\begin{array}{r|l}
 & x^2 - 4x + (8 + k) \\ \hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\ \hline
 & -4x^3 + (16x - k)x^2 - 26x + 10 - a \\
 & -4x^3 + 8x^2 \qquad - 4kx \\
 & + \quad - \quad + \\ \hline
 & (8-k)x^2 - (26 - 4k)x + 10 - a \\
 & (8-k)x^2 - (16 - 2k)x + 8k + k \\ \hline
 & (-10 + 2k)\,x - (10 - a - 8k + k^2)
\end{array}$$

Additional Questions

Q.1. Answer the following and justify,

(i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5 ?

(ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s,\ p \neq 0$?

(iii) If on division of a polynomial *p(x)* by a polynomial *g(x)*,m the quotient is zero, what is the relation between the dgrees of *p(x)* and *g(x)*?

(iv) If on division of a non-zero polynomial *p(x)* by a polynomial *g(x)*, the remainder is zero, what is the relation between the degrees of *p(x)* and *g(x)?*

(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?

Ans. (i) Quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial of degree 5 cannot be $x^2 - 1$, as if we multiply any polynomial of dgree 5 with $x^2 - 1$, it comes a polynomial of degree 7.

(ii) We cannot divide any polynomial of lesser degree by a polynomial of greater degree.

(iii) If on division of a polynomial *p(x)* by a polynomial *g(x)*, the quotient is zero then the dgree of *p(x)* must be less than the degree of *g(x)*.

(iv) If on division of a non-zero polynomial *p(x)* by a polynomial *g(x)*, the remainder is zero then the degree of *g(x)* must be less than or equal to the degree of *p(x)* and *g(x)* must be a factor of *p(x)*.

(v) No, this is possible only when

$$\begin{aligned}
k^2 - 4k &= 0\\
\Rightarrow \quad k(k - 4) &= 0\\
\Rightarrow \quad k &= 0 \text{ or } 4
\end{aligned}$$

$\therefore$ $x^4 + kx + k$ can have equal zeroies for $k = 4$, which is greater than 1 but not an odd integer.

Q.2. The sum and the product of the zeroes of the polynomial $f(x) = 4x^2 - 27x + 3k^2$ are equal. Find the value (s) of m.

Ans. We have : $f(x) = 4x^2 - 27x + 3k^2$

Here, $a = 4, b = -27$ and $c = 3k^2$

$\therefore$ Sum of zeroes = Product of zeroes

i.e., $\frac{(-b)}{a} = \frac{c}{a}$

i.e., $\frac{-(-27)}{4} = \frac{3k^2}{4}$

i.e., $3k^2 = 27$

$\Rightarrow$ $k^2 = 9$

$\Rightarrow$ $k = +3$.

Q.3. Form a quadratic polynomial one of whose zeroes of $2 + \sqrt{5}$ and the sum of the zeroes is 4.

Ans. Sum of the zeroes = 4.

i.e. $\alpha + \beta = 4$

$(2 + \sqrt{5}) + b = 4$

$\Rightarrow$ $b = 4 - (2 + \sqrt{5})$

$= 2 - \sqrt{5}$

$\therefore$ Required quadratic polynomial

$= K[x^2 -$ sum of zeroes $x +$ Product of zeroes)

$= K[x^2 - (2 + \sqrt{5} + 2 - \sqrt{5})x + (2 + \sqrt{5})(2 - \sqrt{5})$

$= K[x^2 - 4x + (4 - 5]$

$= K(x^2 - 4x - 1)$; where. K is constant.

Q.4. Find the zeroes of the quadratic polynomial $f(x) = x^2 - 3x - 28$ and verify the relationship between the zeroes and the co-efficient of the polynomial.

Ans. We have :

$f(x) = x^2 - 3x - 28$

For zeroes $f(x) = 0$

i.e. $x^2 - 3x - 28 = 0$

$\Rightarrow$ $x^2 - 7x + 4x - 28 = 0$

$\Rightarrow$ $x(x - 7) + 4(x - 7) = 0$

$\Rightarrow$ $(x - 7)(x - 4) = 0$

$\Rightarrow$ $x = 7$

and $x = -4$

Also, $a = 1$, $b = -3$ and $c = -28$.

Now, Sum of zeroes $= 7 + (-4)$

$\frac{-(b)}{a} = \frac{-(-3)}{1} = 3$

Product of Zeroes $= 7 \times (-4) = \frac{c}{a}\ \frac{-28}{1} = -28$

Hence verified.

Q.5. If the zero of polynomials $3x^2 - px + 2$ and $4x^2 - qx - 10$ is 2, find the value of $2p - 3q$.

Ans. Let $f(x) = 3x^2 - px + 2$

$\therefore$ $f(2) = 3(2)^2 - p(2) + 2$

$= 12 - 2p + 2 = 14 - 2p$

$14 - 2p = 0$

$\Rightarrow$ $-2p = -14 \Rightarrow p = 7$

Let $g(x) = 4x^2 - qx - 10$

$g(2) = 4(2)^2 - q(2) - 10$

$= 16 - 2q - 10$

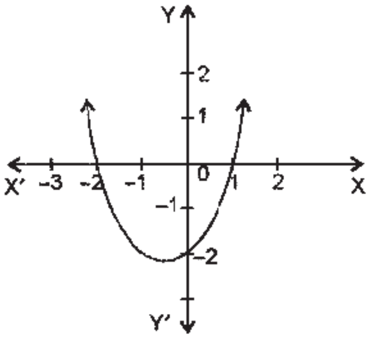
$= 6 - 2q$

$\Rightarrow$ $2q = 6 \Rightarrow q = 3$

Now $2p - 3q = 2(7) - 3(3)$

$= 14 - 9 = 5$.

Q.6. From the graph of a polynomial p(x) given below, find the zeroes and hence form the quadratic polynomial p(x)



Ans. Zeroes of p(x) are -2 and 1

Sum of zeroes, $S = -2 + 1 = -1$

Product of zeroes, $P = (-2)(1) = -2$

Quadratic polynomial is $k(x^2 - Sx + P)$

$\therefore$ $p(x) = k[x^2 - (-1)x + (-2)]$

$= k(x^2 + x - 2)$

Q.7. If a fifth degree polynomial is divided by a quadratic polynomial, write the possible degree of the (a) remainder and (b) quotient.

Ans. (a) Degree of remainder < Degree of divisor

So, Degree of remainder = 0 or 1

(b) Degree of quoient = Degree of dividend – Degree of divisor.

$= 5 - 2 = 3$.

Q.8. Find the value of k such that the quadratic polynomial $3x^2 + 2kx + x - k - 5$ has sum of zeroes as half of their product.

Ans. $3x^2 + 2kx + x - k - 5$

$3x^2 + (2k + 1)x - (k + 5)$

Here $a = 3$, $b = 2k + 1$, $c = -(k + 5)$

Sum of zeroes = $\frac{1}{2}$ (Product of zeroes) (Given)

$\therefore \quad \frac{-b}{a} = \frac{1}{2} \times \frac{c}{a}$

$\Rightarrow \quad \frac{-(2k+1}{3} = \frac{1}{2} \times \frac{-(k+5}{3}$

$\Rightarrow \quad 2k+2 = k+5$

$\Rightarrow \quad 4k - k = 5-2$

$\Rightarrow \quad 3k = 3 \qquad \therefore k = 1$

Q.9. If α and β are the zeroes of $x^2 - x - 2$, form a quadratic polynomial in x whose zeroes are $2\alpha + 1$ and $2\beta + 1$.

Ans. Part I. $x^2 - x - 2$

Here $a = 1, b = -1, c = -2$

Sum of zeroes, $\alpha + \beta = \frac{-b}{a} = \frac{-(-1)}{1} = 1$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{-2}{1} = (-2)$

Part II. For required polynomial

Sum of zeroes (S) $= (2\alpha+1) + (2\beta+1)$
$= 2(\alpha + b) + 2$
$= 2(1) + 2$
$= 4$

Product of zeroes (P) $= (2\alpha + 1)(2\beta + 1)$
$= 2\alpha\beta + 2\alpha + 2\beta + 1$
$= 2\alpha\beta + 2(\alpha + \beta) + 1$
$= 4(-2) + 2(1) + 1$
$= -8 + 2 + 1 = -5$

Required quadratic polynomial
$= K(x^2 - Sx + P)$, when K is a real number
$= K(x^2 - 4k - 5)$

Q.10. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other two zeores.

Ans. It is given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are two zeroes of $f(x)$.

$\therefore \; \{x - (2 + \sqrt{3})\} \{x - (2 - \sqrt{3})\}$
$= (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$
$= (x-2)^2 - (\sqrt{3})^2$
$= x^2 - 4x + 1$ is a factor of $f(x)$

Let us now divide f(x) by x2 – 4x + 1

For $f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ to be exactly divisible by $x^2 - 2x + k$, we must have $(-10 + 2k)x + (10 - a - 8k + k^2) = 0$ for all x

$\Rightarrow \quad 10x + 2k = 0,\; 10 - a - 8k + k^2 = 0$

$\Rightarrow \quad k = 5,\; 10 - a - 40 + 25 = 0$

$\Rightarrow \quad k = 5$ and $a = -5$

We have,

$$
\begin{array}{r|l}
 & 2x^2 - 2x + 35 \\
x^2 + 4x + 1 & x^4 + 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 + 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 + 27x^2 - 138x \\
 & -2x^3 - 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

$\Rightarrow \quad f(x) \; - (x^2 - 4x + 1)(x^2 - 2x - 35)$

Hence, other two zeroes of $f(x)$ are the zeroes of the polynomial $x^2 - 2x - 35$, we have

$x^2 - 2x - 35 = x^2 - 7x + 5x - 35$
$= (x - 7(x + 5)$

Multiple Choice Questions

Q.1. If one of the zeroes of the quadratic polynomials $(k-1)x^2 + kx + 1$ is -3, then the value of k is :

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$

(c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Ans. (a)

Q.2. The number of polynomials having zeroes as –2 and 5 is

(a) 1 (b) 2

(c) 3 (d) more than 3

Ans. (d)

Q.3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then :

(a) a = –7, b = –1 (b) a = 5, b = – 1

(c) a = 2, b = –6 (b) a = 9, b = –6

Ans. (d)

Q.4. Given the one of the zeroes of the cubic polynomial $ax^3 + bx^3 + cx + d$ is zero, the product of the other two zeroes is :

(a) $-\frac{c}{a}$ (b) $\frac{c}{a}$

(d) 0 (d) $-\frac{b}{a}$

Ans. (b)

Q.5. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is :

(a) $b - a + 1$ (b) $b - a - 1$

(c) $a - b + 1$ (d) $a - b - 1$

Ans. (a)

Q.6. If a, b are zeroes of p(x) = $x^2 - 5k - k$ and $a - b$ = 1, the value of 'k' is :

(a) 4 (b) –6

(c) 6 (d) 5

Ans.(c)

Q.7. The graph of the polynomial $f(x) = 2x - 5$ is a straight line which intersects the x = axis at exactly one point namely :

(a) $\left(\frac{-5}{2}, 0\right)$ (b) $\left(0, \frac{-5}{2}\right)$

(c) $\left(\frac{5}{2}, 0\right)$ (d) $\left(\frac{5}{2}, \frac{-5}{2}\right)$

Ans. (c)

Q.8. The product and sum of the zeroes of the quadratic polynomial $ax^2 + bx + c$ respectively a are :

(a) *–b/a' c/a* (b) *c/b'1*

(c) *c/a' b/a* (d) *c/a' – b/a*

Ans. (d)

Q.9. The number of zeroes that the polynomial *f(x)* = $(x - 2)^2 + 4$ can have is :

(a) 1 (b) 2

(c) 0 (d) 4

Ans. (b)

Q.10. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are :

(a) both positive

(b) both negative

(c) One positive and one negative

(d) Both equal

Ans. (b)