Chapter 1

Real Numbers

In the Chapter

In earlier classes you have studied about various collection of numbers and their properties. On account of various rules and numbers, we again give brief introduction of numbers.

NATURAL NUMBERS: The numbers that man first used were the numbers be needed for counting called Natural Numbers or Counting Numbers.

The natural numbers are denoted by the letters N={1,2,3,4...}. 1 is smallest natural number but there is no largest narural number.

REMARKS:

1. Natural numbers together with zero are called **whole numbers**.
2. Every natural number is a whole number.
3. The sum and multiplication of two natural numbers is always a natural number.
4. Subtraction of two natural numbers is not always a natural number. If a > b, then a - b is a natural number. But a > b, then a – b is a not a natural number.

INTEGERS: All natural number, 0 and negatives of numbers are called integers. These are denoted by letter Z.

REMARKS:

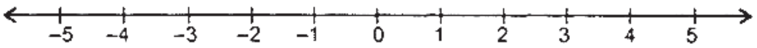
1. Every natural number is an integer. Natural numbers are also called positive integers.
2. Every whole number is an integer. Whole numbers are also called non- negative integers.
3. Integers are closed to operations of addition, subtraction and multiplication.

e.g., $a \in Z$ and $b \in Z$, then $a + b \in Z$ and $a - b \in Z$.

Also, $a \times b \in Z$.

But I is not closed to division.

Representation of integers on number line:



RATIONAL NUMBERS: The numbers which can be expressed in the form p/q, where p and q are both integers and $q \neq 0$ are called rational numbers. these are denoted by Q.

Note: The condition $q \neq 0$ is necessary, as division by zero is not defined.

REMARKS:

1. All the whole numbers are rational numbers since they can be represented as ratio. e.g.,

$$5=\frac{5}{1},\ 7\ \frac{7}{1} \text{ etc.}$$

2. Natural number and negative of natural numbers, zero and common fractions are rational numbers.

3. A rational number may be positive, negative or zero.

For any rational number $\frac{p}{q}$ with terminating decimal representation, the prime factorisation of q is of the form $2^n 5^m$ where n and m are non - negative integers.

For any rational number $\frac{p}{q}$ where prime factorisation of q is of the form $2^n 5^m$ where n and m are non - negative , the decimal representation is terminating.

For any rational number $\frac{p}{q}$ where prime factorisation of q is of the form $2^n 5^m$ where n and m are non - negative integers. the decimal representation is non– terminating and repeating.

IRRATIONAL NUMBERS: We have seen that every terminating decimal and non- terminating repeating decimals cannot be expressed in the form of p/q, where 'p' and 'q' are integers and $q \neq 0$. But there are some decimal numbers which are non- terminating and non- repeating. These decimal cannot be represented in the form of p/q

Exp. 0.51511511151111... is neither terminating nor repeating. So, it does not represent a rational number. thus, a number which can neither be expressed as a terminating decimal nor as a repeating decimal, is called irrational number.

REMARKS:

1. π is an irrational number.

[Note: $\pi = 3.1428571428...$]

2. $\sqrt{2}, \sqrt{2}, \sqrt{5}, \sqrt{7}, \sqrt{11}$, etc. all are irrational number.

REAL NUMBERS: The set of rational numbers and irrational number. together compose the set of real numbers.

Real Numbers (R)

Rational Number (Q) — Irrational Number. (Q)

EUCLID'S DIVISION LEMMA OR EUCLID'S DIVISION ALGORITHM

Note:

1. Algorithm: It is a series of well defined steps which gives a procedure for solving a type of problem.
2. Lemma: It is a Proven statement used for proving another statement.

Euclid's Division Lemma: For any positive integers a and b there exist unique whole numbers q and r such that $a = bq + r$, where $0 \leq r < b$

Note:

1. Euclid's division algorithm technique is used to compute the HCL of two given integers.
2. In the given condition, $a = bq+r$, we consider a as dividend, b as divisor, q as quotient and r as remainder.
3. Dividend= (Divisor × Quotient + Remainder.

FUNDAMENTAL THEOREM OF ARITHMETIC: Every composite number can be expressed (factorised) as a product of primes this factorisation is unique, apart from the order in which the prime factor occur.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 1.1

Q.1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and and 225 (ii) 196 and 38220

Ans. (i) Given integers are 135 and 225, clearly 225 > 135. Therefore, by applying Euclid's division lemma to 225 and 135, we get

I. $225 = 135 \times 1 + 90$

135) 225 (1
−135
90

II. Since, the remainder $90 \neq 0$, we apply division lemma to 135 and $90 \neq 0$, to get

$$\begin{array}{r|l} 90) \; 135 & (1 \\ -90 & \\ \hline 45 & \end{array}$$

III. We consider the new divisor 90 and new remainder 45 and apply division lemma to get

$$90 = 45 \times 2 + 0$$

$$\begin{array}{r|l} 45) \; 90 & (2 \\ -90 & \\ \hline 0 & \end{array}$$

The remainder of this step is zero. So, the divisor at this stage or the remainder at the previous stage i.e., 45 is the HCF of 135 and 225.

(ii) Given integers are 196 and 38220. Therefore by applying Euclid's division lemma to 196 and 38220, we get

I. $38220 = 196 \times 195 + 0$

$$\begin{array}{r|l} 196) \; 38220 & (195 \\ 38220 & \\ \hline 0 & \end{array}$$

The remainder at this step is zero. So, our procedure stops and divisor at this stage *i.e.* 195 is the HCF of 196 and 38220.

Q.2. Show that any positive integer is of the form 6*q* + 1 or 6*q* + 3 or 6*q* + 5, where *q* is some integer.

Ans. Let a be any odd positive integer and b = 6. Let q be a quotient and r be remainder.

Therefore, applying division lemma, we have

$a = 6q + r$, where $0 \le r < 6$

$\Rightarrow \quad a = 6q + 0$

or $\quad a = 6q + 1$

or $\quad a = 6q + 2$

or $\quad a = 6q + 3$

or $\quad a = 6q + 4 \qquad [0 \le r < 6]$

or $\quad a = 6q + 5$

$\Rightarrow \quad r = 0, 1, 2, 3, 4, 5$

$\Rightarrow$ But $\quad a = 6q, a = 6q + 2, a = 6q + 4$

given values of a.

Thus, when a is odd, it is at the form $(6q + 1)$ or $(6q + 3)$ or $(6q + 5)$ for some integer q.

Q.3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march ?

Ans. Given integers are 32 and 616.

Clearly 616 > 32. Therefore, applying Euclid's division lemma to 616 and 32, we get

$$\begin{array}{r|l} 32) \; 616 & (19 \\ 608 & \\ \hline 8 & \end{array}$$

Since, the remainder $8 \neq 0$, we apply the division lemma, to get

$$32 = 8 \times 4 + 0$$

$$\begin{array}{r|l} 8) \; 32 & (4 \\ 32 & \\ \hline 0 & \end{array}$$

The remainder has now become zero, so our procedure stops. Since the divisor at this stage is 8.

Therefore, the maximum number of columns in which both 616 members (army contingent) and 32 members (army band) can march is 8.

Q.4. Show that the square of any positive integer is of the form 3m or 3m + 1 for some integer.

[C.B.S.E. 2001]

Ans. Let a be any positive integer. Let q be the quotient and r be remainder. Then

$$a = bq + r$$

where q and r are also positive integers and

$$0 \le r < b$$

Taking $b = 3$, we get

$$a = 3q + r, \text{ where } 0 \le r < 3$$

When, $r = 0 \Rightarrow a = 3q$

When, $r = 1 \Rightarrow a = 3q + 1$

When, $r = 2 \Rightarrow a = 3q + 2$

Now, we have to show that the squares of positive integers $3q$, $3q + 1$ and $3q + 2$ can be expressed as $3m$ or $3m + 1$ for some integer m.

$\Rightarrow$ Squares of $3q = (3q)^2$

$= 9q^2 = 3(3q)^2 = 3m$

When m is some integer.

Square of $3q + 1 = (3q + 1)^2$

$= \quad 9q^2 + 6q + 1 = (3(3q^2 + 2q) + 1$

$= 3m + 1$, where m is some integer

Square of $3q + 2 = (3q + 1)^2$

$= (3q + 2)^2$

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1$ for some integer m.

$\therefore$ The square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

Q.5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9*m* + 1 or 9*m* + 8. [C.B.S.E. 2009 (C)]

Ans. Let a and b be two positive integers such that a is greater than b; then :

$$a = bq + r$$

Where q and r are positive integers $0 \leq r < b$

Taking $b = 3$, we get

$$a = 3q + r \text{ ; where } 0 \leq r < b.$$

$\Rightarrow$ Different values of integer a are $3q$, $3q + 1$ or $3q + 2$.

Cube of $3q = (3q)^3$

$= 27q^3 = 9(39^3) = 9m;$

where m is some integer.

Cube of $3q + 1 = (3q + 1)^3$

$= (3q)^3 + 3(3q)^2 \times 1 + 3(3q) \times 1^2 + 1^3$

$[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 1;$

where m is some integer.

Cube of $3q + 1 = (3q + 1)^3$

$= (3q)^3 + 3(3q)^2 \times 1 + 3(3q) \times 1^2 + 1^3$

$[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 1;$

where m is some integer.

Cube of $3q + 2 = (3q + 2)^3$

$= (3q)^3 + 3(3q)^2 \times 2 + 3 \times 3q \times 2^2 + 2^3$

$= 27q^3 + 54q^2 + 36q + 8$

$= 9(3q + 6q^2 + 4q) + 8 = 9m + 8$

where m is some integer.

$\therefore$ Cube of any positive integer is of the form 9m or $9m + 1$ or $9m + 8$.

EXERCISE 1.2

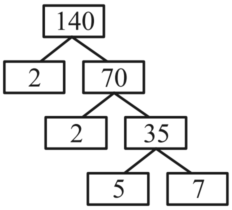
Q.1. Express each number as a product of its prime factors :

(*i*) 140 (*ii*) 156

(*iii*) 3825 (*iv*) 5005

(*v*) 7429

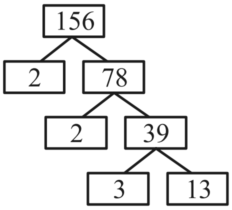
Ans. (*i*) 140



So, $140 = 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

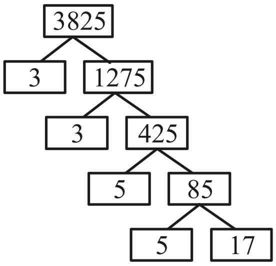
(*ii*) 156



So, $156 = 2 \times 2 \times 3 \times 13$

$= 2^2 \times 3 \times 13$

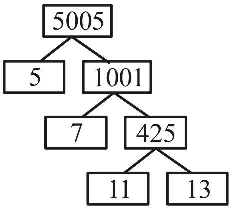
(*iii*) 3825



So, $3825 = 3 \times 3 \times 5 \times 17$

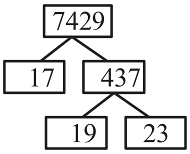
$3^2 \times 5^2 \times 17$

(*iv*) 5005



So, $5005 = 5 \times 7 \times 11 \times 13.$

(*v*) 7429



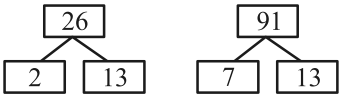
So, $7429 = 17 \times 19 \times 23$

Q.2. Find the L.C.M. and H.C.F. of the following pairs of integers and verify that L.C.M. × H.C.F.

(*i*) 26 and 91 (*ii*) 510 and 92

(*iii*) 336 and 54

Ans. (*i*)



So, $26 = 2 \times 13$

and $91 = 7 \times 13$

$\therefore$ H.C.F. (26, 91) $= 2 \times 7 \times 13$

$= 182$

Verification :

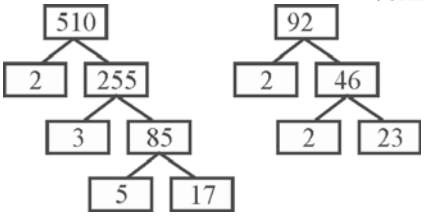
L.C.M. × H.C.F $= 182 \times 13 = 2366$

Procuct of the two nos. $= 26 \times 91 = 2366$

$\therefore$ L.C.M. × H.C.F. = Product of the two nos.

Verified.

(*ii*)



So, $510 = 2 \times 3 \times 5 \times 17$

and $92 = 2 \times 2 \times 23$

$\therefore$ H.C.F. (510, 92) $= 2$

and L.C.M. (510, 92) $= 2 \times 2 \times 5 \times 17 \times 23$

$= 23460$

Verification

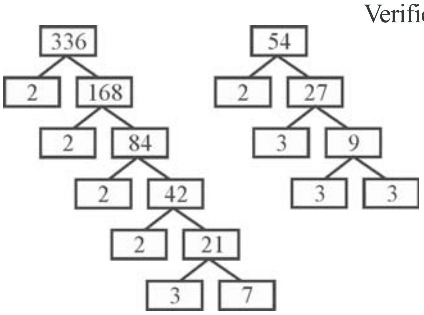
L.C.M. × H.C.F. $= 23460 \times 2 = 46920$

Product of the two nos. $= 510 \times 92 = 46920$

$\therefore$ L.C.M. × H.C.F. = Product of the nos.

Verified.

(iii)



So, $336 = 2^4 \times 3 \times 7$

and $54 = 2 \times 3^3$

$\therefore$ H.C.F. $= 2 \times 3 = 6$

and L.C.M. $= 2^4 \times 3^3 = 3024$

Verification

L.C.M. × H.C.F. $= 3024 \times 6 = 18144$

Product of the two nos. $= 336 \times 54 = 18144$

$\therefore$ L.C.M. × H.C.F. = Product of the nos.

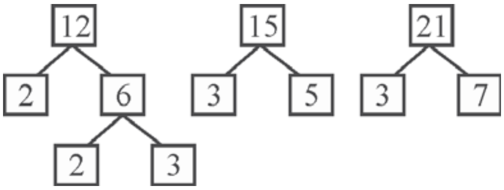
Verified.

Q.3. Find the L.C.M. and H.C.F. of the following integers by applying the prime factorisation method:

(*i*) 12, 15 and 21 (*ii*) 17, 23 and 29

(*iii*) 8, 9 and 25.

Ans. (*i*)



Now, $12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 2^2 \times 5,\ 21 = 3 \times 7$

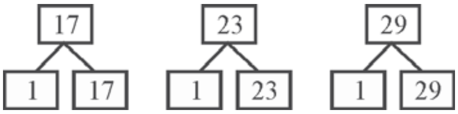
Here, 3 is the smallest common prime factor.

$\therefore$ H.C.F. (12, 15, 21) $= 3$

And 22, 31, 51, and 71 are the greatest powers of prime factors.

$\therefore$ L.C.M. (12, 15, 21) $= 2^2 \times 3^1 \times 5^1 \times 7^1 = 420$

(*ii*)



Now, $17 = 1 \times 17, 23 = 1 \times 23. 29 \times 1 \times 29$

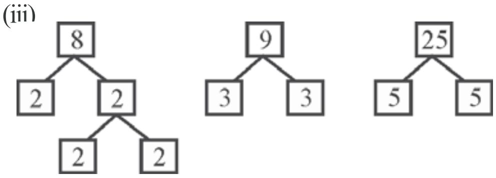
Here, 1 is the smallest common factor.

$\therefore$ H.C.F. (17, 23, 29) $= 1$

And 17^1, 23^1 and 29^1 are the greatest powers of prime factors.

$\therefore$ L.C.M. (17, 23, 29) $= 17 \times 23 \times 29 = 11339$

(iii)



Now, $8 = 2 \times 2 \times 2 = 2^3$,

$9 = 3 \times 3 = 3^2$

and $25 = 5 \times 5 = 52$

Since, there is no smallest common factor.

$\therefore$ H.C.F. (8, 9, 25) $= 1$

And 2^3, 3^2 and 5^s are greatest powers of prime factors.

$$\therefore \quad \text{L.C.M.}(8, 9, 25) = 2^3 \times 3^2 \times 5^2$$
$$= 1800$$

Q.4. Given that H.C.F. (306, 657) = 9, find L.C.M. (306, 657).

Ans. $\text{HCF}(306, 657) = 9$

$$\text{LCM}(306, 657) =$$

$$= \frac{201042}{9}$$
$$= 22338$$

Q.5. Check whether 6n can end with the digit '0' for any $n \in$N. [CBSE 2010]

Ans. If the number 6^n, for any natural number n, ends with digit 0, then it is divisible by 5. That is, the prime factorisation of 6^n contains the prime 5. This is not possible because the primes in the factorisation of 6^n are 2 and 3 the uniqueness of the Fundamental Theorem of Artihmetic guarantees that there are no other primes in the factorisation of 6^n.

So, there is no natural number n for which 6^n ends with digit zero.

Q.6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 + 5 are composite numbers.

Ans. Any positive number, which has more than two factors, is called a composite number.

Since,

$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1)$$
$$= 13 \times 78$$
$$= 13 \times 13 \times 3 \times 2$$

that is, the given number has more than two factors and it is a composite number.

Similarly

$$7 \times 6 \times 5 \times 4 \times 3 + 5$$
$$= 5 \times (7 \times 6 \times 4 \times 3 + 1)$$
$$= 5 \times 505 = 5 \times 5 \times 101$$

$\Rightarrow$ The given number is a composite number.

Q.7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?

Ans. Required number of minutes is the LCM of 18 and 12.

We have,

$$18 = 2 \times 3^2$$
and
$$12 = 2^2 \times 3$$

$\therefore$ LCM of 18 and 12 is $2^2 \times 3^2 = 36$

Hence, Ravi and Sonia will meet again at the starting point after 36 minutes.

EXERCISE 1.3

Q.1. Prove that $\sqrt{5}$ is irrational.

Ans. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find coprime integers a and b ($b \neq 0$) such that

$$\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{5}\,b = a$$

Squaring on both sides, we get

$$5b^2 = a^2$$

Therefore, 5 divides a^2

Therefore, 5 divides a

So, we can write

$$5b^2 = 25c^2$$
$$b^2 = 5b^2$$

This means that 5 divides b^2, and so 5 divides b.

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

Q.2. Prove that 3 + $2\sqrt{5}$ is irrational.

Ans. Let us assume, to the contrary that 3 + $2\sqrt{5}$ is rational.

That is, we can find coprime a and b ($b \neq 0$) such that $3 + 2\sqrt{5} = \frac{a}{b}$.

Therefore,
$$\frac{a}{b} = 3 + 2\sqrt{5}$$

$$\Rightarrow \quad \frac{a - 3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \quad \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that 3 + $2\sqrt{5}$ is rational.

So, we conclude that 3 + $2\sqrt{5}$ is irrational.

Q.3. Prove that the following are irrational :

(*i*) $\frac{1}{\sqrt{2}}$ (*ii*) $7\sqrt{5}$ (*iii*) $6+\sqrt{2}$

Ans. (*i*) $\mathbf{\frac{1}{\sqrt{2}}}$.

Let us assume, to the contrary, that $\frac{1}{\sqrt{2}}$ is rational.

So, we can find coprime integers *a* and *b* ($b \neq 0$) such that

64/455

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{a}{b} = \sqrt{2} \Rightarrow \frac{b}{a}$$

Since, *a* and *b* are integers, $\frac{a}{b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that $\frac{1}{\sqrt{2}}$ is irrational.

(*ii*) $\mathbf{7\sqrt{5}}$

Let us assume to the contrary, that $7\sqrt{5}$ is rational.

So, we can find coprime integers *a* and *b* ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{5} = \frac{a}{7b}$$

Since, *a* and *b* are integers, $\frac{a}{7b}$ is rational, and so, $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

So, we conclude that $7\sqrt{5}$ is irrational.

(*iii*) **6 +** $\mathbf{\sqrt{2}}$

Let us assume to the contrary, that $\sqrt{2}$ is rational.

Then, 6 + $\sqrt{2}$ is rational.

So, we can find coprime integers *a* and *b* ($b \neq 0$) such that

$$6+\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 6-\frac{a}{b} = \sqrt{2}$$

Since, a and b are integers, we get is rational and so, $6-\frac{a}{b}$ is rational and so, $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

So, we conclude that 6 + $\sqrt{2}$ is irrational.

EXERCISE 1.4

Q.1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non- terminating repeating decimal expansion :

(*i*) $\frac{13}{3125}$ (*ii*) $\frac{17}{8}$ (*iii*)

(*iv*) $\frac{15}{1600}$ (*v*) $\frac{29}{343}$ (*vi*) $\frac{23}{2^3 5^2}$

(*vii*) $\frac{129}{2^2 5^7 7^5}$ (*viii*) $\frac{6}{15}$ (*ix*) $\frac{35}{50}$

(*x*) $\frac{77}{210}$

Ans. (*i*) $\frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{2^5 \times 5^5}$

$\Rightarrow \frac{13}{3125}$ has a terminating decimal expansion.

[By NCERT Theorem 1.6]

(*ii*) $\frac{17}{8} = \frac{17}{2^3}$

$\Rightarrow \frac{17}{8}$ has a terminating decimal expansion.

[By NCERT Theorem 1.6]

(*iii*) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

$\Rightarrow \frac{64}{455}$ has a non-terminating repeating decimal expansion. [By NCERT Theorem 1.7]

(*iv*) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2}$

$\Rightarrow \frac{15}{1600}$ has a terminating decimal expansion. [By NCERT Theorem 1.6]

(*v*) $\frac{29}{343} = \frac{29}{7^3}$

$\Rightarrow \frac{29}{343}$ has a non-terminating decimal expansion. [By NCERT Theorem 1.7]

(*vi*) $\frac{23}{2^3 5^2} \Rightarrow$ has a terminating decimal expansion. [By NCERT Theorem 1.6]

(*vii*) $\frac{129}{2^2 5^7 7^5} \Rightarrow$ has a non-terminating repeating decimal expansion. [By NCERT Theorem 1.7]

(*viii*) $\frac{6}{15} = \frac{6}{3 \times 5} = \frac{2}{5} \Rightarrow$ has a terminating decimal expansion. [By NCERT Theorem 1.6]

(*ix*) $\frac{35}{50} = \frac{35}{2 \times 5^2} \Rightarrow$ has a terminating decimal expansion. [By NCERT Theorem 1.6]

(*x*) $\frac{77}{210} = \frac{77}{2 \times 3 \times 5 \times 7} \Rightarrow$ has a non-terminating decimal. [By NCERT Theorem 1.6]

Q.2. Write down the decimal expansions of those rational numbers in question 1 which have terminating decimal expansions.

Ans. (*i*) $\frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{2^5 \times 5^5}$

$= \frac{416}{(10)^5} = 0.00416$

(*ii*) $\frac{17}{8} = \frac{17}{2^3} = \frac{17 \times 5^3}{2^3 \times 5^3}$

$= \frac{2125}{(10)^6} = 2.125$

(*iv*) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^6}$

$= \frac{9375}{(10)^6} = 0.009375$

(*vi*) $\frac{23}{2^3.5^2} = \frac{23}{2^3 \times 5^3} = \frac{115}{(10)^3} = 0.115$

(*viii*) $\frac{6}{15} = \frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$

(*ix*) $\frac{35}{50} = \frac{35}{2 \times 5^2} = \frac{35 \times 2}{2^2 \times 5^2}$

$= \frac{70}{(10)^2} = 0.70$

Q.3. The following real numbers have decimal expansions as given below. In each case decide whether they are rational or not. If they are rational, and of the form *p/q*, what can you say about the prime factors of *q* ?

(*i*) $43.\overline{123456789}$

(*ii*) 0.120120012000120000 ...

(*iii*) $43.\overline{123456789}$

(*iv*) 123.096 (*v*) $21.23\overline{478}$

Ans. (*i*) $43.123456789 = \frac{43123456789}{1000000000}$

$= \frac{43123456789}{10^6}$

$= \frac{43123456789}{2^9 \times 5^9}$

43.123456789 is a rational number of the form *p/q*, where '*p*' and '*q*' are co-primes and the prime factors of '*q*' are of the form $2^n.5^n$, where *n* is non-negative integer.

(*ii*) Clearly, 0.120120012000120000 ... is a non-termating and non-repeating decimal and therefore, it is irrational.

(*iii*) $43.\overline{123456789} = 43.123456789123456789..$ which is non-termaing and repeating. Therefore, it is a rational number which can be expressed in the form *p/q*, where *p* and *q* are co-primes and the prime factors of '*q*' are of the form $2^n.5^n$, where '*n*' is non-negative integer.

(*iv*) $123.096 = \frac{123096}{1000}$

$= \frac{123096}{10^3} = \frac{123096}{2^3 5^3}$

$\therefore$ 123.096 is a rational number of the form *p/q* where the denominator *'q'* has the prime factors of the form $2^n.5^n$, where n is non-negative integer.

(v) $21.23\overline{478} = 21.234787878...$

which is non-terminating and repeating. Therefore, it is a rational number which can be expressed in the form p/q and the prime factors of *'q'* are of the form $2^n.5^n$, where n is a non-negative integer.

Additional Questions

Q.1. The number 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is the HCF (525, 3000)? Justify your answer.

Ans. Since, the HCF (525, 3000) = 75

Division

By Euclid's Lemma,

$$3000 = 525 \times 5 + 375$$
$$525 = 375 \times 1 + 150$$
$$375 = 150 \times 2 + 75$$
$$150 = 75 \times 2 + 0$$

And the numbers, 3, 5, 15, 25 and 75 divide the numbers 525 and 3000 that mean these terms are common in both 525 and 3000. So, the highest common factor among these is 75.

Q.2.Explain why 3 × 5 × 7 + 7 is composite number.

Ans. We have $3 \times 5 \times 7 + 7 = 105 + 7 = 112$

$$112 = 2 \times 2 \times 2 \times 2 \times 7 = 2^4 \times 7$$

So, it is the product of prime factors 2 and 7

Hence, it is a composite number.

Q.3. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of *q*, when this number is expressed in the form $\frac{p}{q}$? Give reasons.

Ans. 327.7081 is terminating. So, it represents a rational number.

Thus, $327.7081 = \dfrac{3277081}{10000} = \dfrac{p}{q}$

$\therefore \quad q = 10^4 \;= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
$= 2^4 \times 5^4 = (2 \times 5)^4$

So, the prime factors of q is 2 and 5.

Q.4. Prove that, if *x* and *y* are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Ans. Let $x = 2m + 1$ and $y = 2m + 3$ are odd positive integers, for every positive integer m.

Then, $x^2 + y^2 = (2m + 1)^2 + (2m + 3)^2$
$= 4m^2 + 1 + 4m + 4m^2 + 9 + 12m$
$= 8m^2 + 16m + 10 = \text{even}$
$= 2(4m^2 + 8m + 5)$ or
$4(2m^2 + 4m + 2) + 1$

$\therefore$ $x^2 + y^2$ is even for every positive integer m.

But not divisible by 4.

Q.5. Use Euclid's division algorithm to find the HCF of 441, 567, 693.

Ans. Let $a = 693$, $b = 567$, and $c = 441$

Now, by Euclid's algorithms

$$a = bq + r$$

First we take, $a = 693$ and $b = 567$, and find their HCF.

$$693 = 567 \times 1 + 126$$
$$567 = 126 \times 4 + 63$$
$$126 = 63 \times 2 + 0$$

$\therefore$ HCF (693, 567) = 63

Now, we take $c = 441$ and say $d = 63$, then find their HCF

$$c = dq + r$$

$\Rightarrow \quad 441 = 63 \times 7 + 0$

$\therefore$ HCF (693, 567, 441) = 63.

Q.6. Show that 12^n cannot end with the digit 0 or 5 for any natural number *n*.

Ans. If any number ends with the digit zero, it is always divisible by 5.

$\Rightarrow$ If 12^n ends with the digit zero it must be divisible by 5.

This is possible only if prime factorisation of 12^n contains the prime number 5.

Now, $\quad 12 = 2 \times 2 \times 3 = 2^2 \times 3$

$\Rightarrow \quad 12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

$\Rightarrow$ There is no value of $n \in N$ for which 12^n ends with digit zero or five.

Q.7. The product of three consecutive positive integers is divisible by 6. Is this statement true or false? Justify your answer.

Ans. Since three consecutive integers include an integer which is a multiple of 2 and an integer which is a multiple of 3.

$\therefore$ the product of three consecutive integers is divisible by 6.

$\therefore$ The statement is true.

Q.8. Write whether the square of any positive integer can be of the form 3*m* + 2, where *m* is a natural number. Justify your answer.

Ans. The square of any positive integer cannot be written in the form 3*m* + 2, where *m* is a natural number.

As for an example

Let the number is 3, its square is 9.

Now 9 cannot be writen as $3m+2$.

$3m+2=9$.

$\Rightarrow \quad m=\frac{7}{3}$, which is not a natural number.

Q.9. A positive integer is of the form 3*q* + 1, *q* being a natural number. Can you write its square in any form other than 3*m* + 1, *i.e.*, 3*m* + 2 for some integer *m* ? Justify your answer.

Ans. No. Since any positive integer is of the form $3q+1$, therefore, its square will be

$$(3q+1)^2 = 9q^2+6q+1$$
$$= 3(3q^2+2q)\ +1$$
$$= 3m+1$$

Q.10. Can two numbers have 18 as their HCF and 380 as their LCM ? Give reasons.

Ans. Since 18 does not divide 380.

$\therefore$ any two numbers cannot have 18 as their HCF and 380 as their LCM.

Multiple Choice Questions

Q.1. $\frac{3}{8}$ in decimal form is :

(a) 0.125 (b) 0.0125
(c) 0.0375 (d) 0.375

Ans. (d)

Q.2. For any positive integer a and 3, there exist unique integers *q* and *r* such that *a* = 3*q* + 4, where *r* mut satisfy.

(a) $0<r<3$ (b) $1<r<3$
(c) $0<r<3$ (d) $0<r<3$

Ans. (a)

Q.3. The product of two irrational numbers is :

(a) always a rational number
(b) always an irrational number
(c) sometimes a rational number, sometimes irrational
(d) not a real number.

Ans. (c)

Q.4. The prime factors of 98 are :

(a) $2^2 \times 7$ (b) $2^3 \times 7$
(c) 2×7^2 (d) $2^2 \times 7^2$

Ans. (c)

Q.5. The least positive integer divisible by 20 and 24 is :

(a) 240 (b) 480
(c) 120 (d) 960

Ans. (c)

Q.6. The HCF of the smallest composite number and the smallest prime number is :

(a) 1 (b) 3
(c) 2 (d) 4

Ans. (c)

Q.7. The product of two irrational numbers is :

(a) always rational
(b) always irrational
(c) one
(d) always a non-zero number

Ans. (d)

Q.8. The reciprocal of an irrational number is :

(a) an integer
(b) rational
(c) a natural number
(d) irrational

Ans. (d)

Q.9. A rational number can be expressed as a terminating decimal if the denominator has factors :

(a) 2, 3 (or) 5 only
(b) 2 (or) 3 only
(c) 3 (or) 5 only
(d) 2 (or) 5 only

Ans. (d)

Q.10. The decimal expansion of $\frac{21}{24}$ will terminate after how many places of decimal?

(a) 1 (b) 2
(c) 3 (d) 4

Ans. (6)