11

Chapter

Constructions

In the Chapter

In this chapter, you will learn the following constructions using a ruler and a compass:

- To bisect a given angle.
- To draw the perpendicular bisector of a given line segment.
- To construct an angle of 60° etc.
- To construct a triangle given its base, a base angle and the sum of the other two sides.
- To construct a triangle given its base, a base angle and the difference of the other two sides.
- To construct a triangle given its perimeter and its two base angles.
- **Need of Accurate Figures :** Sometimes one needs an accurate figure, for example to draw a map of a building to be constructed, to design tools and various parts of a machine, to draw road maps, etc. To draw such figures some basic geometrical instruments are needed.
- **Geometry Box :** A geometry box contains the following basic geometrical instruments.
  (i) A pair of dividers (or a divider) with adjustments.
  (ii) A pair of compasses (or a compass) with provision of fitting a pencil at one end.
  (iii) A protractor.
  (iv) A graduated scale, on one side of which centimetres and millimetres are marked off and on the other side inches and their parts are marked off.
  (v) A pair of set squares, one with angles 90°, 60° and 30° and other with angles 90°, 45° and 45°.
  Normally, all these instruments are needed in drawing a geometrical figure such as a triangle, a circle, a quadrilateral a polygon etc. with given measurements but a geometrical construction is the process of drawing a geometrical figure using two instruments – an ungraduated ruler, also called a straight edge and α-compass. In construction, where measurements are also required, we may use a graduated scale and protractor also.
- **Rules of Congruency of Two Triangles**
  (i) RHS : Two right triangles are congruent, if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.
  (ii) ASA : Two triangles are congruent, if any two angles and the included side of one triangle are equal to the two angles and the included side of the other triangle.
  (iii) SSS : Two triangles are congruent, if the three sides of one triangle are equal to the three sides of the other triangle.
  (iv) SAS : Two triangles are congruent, if any two sides and the included angle of one triangle are equal to any two sides and the included angle of the other triangle.
- **Uniqueness of triangle :** A triangle is unique, if
  (i) in a right triangle, hypotenuse and one side is given.
  (ii) three sides are given.
  (iii) two sides and the included angle is given.
  (iv) two angles and the included side is given.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 11.1

Q.1. Construct an angle of 90° at the initial point of a given ray and justify the construction.

Ans. Steps of construction :

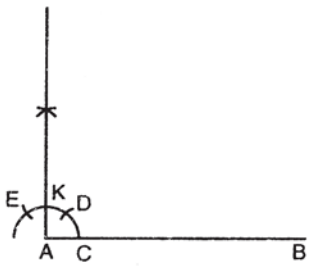
(i) Draw AB of any convenient length.

(ii) With A and centre and any convenient radius, draw an arc as shown intersecting AB and C.

(iii) With C as centre and radius = AC, draw an arc intersecting the previous arc at D.

(iv) Again with D as centre and radius = AC, draw an arc intersecting the first arc at E.

(v) Again with D and E as centres and radius > $\frac{1}{2}$ ED, draw two arc intersecting each other at K. Join AK.



(vi) < BAK = 90°.

Justification : Clearly CD makes an angle of 60° at A and DE also makes an angle of 60° at A. Since M is the mid point of ED.

∴ MD makes an angle of 30° at A.

Hence, ∠BAK = 90°.

Q.2. Construct an angle of 45° at the initial point of a given ray and justify the construction.

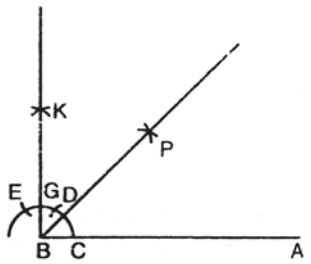
Ans. Steps of Construction :

(i) Draw a line BA.

(ii) With B as centre and any convenient radius, draw an arc intersecting AB at C.

(iii) With C as centre and radius = BC, draw an arc intersecting the previous arc at D.

(iv) With D as centre and radius = BC, draw an arc intersecting the first arc at E.



(v) With D and E as centres and radius > $\frac{1}{2}$ ED, draw two arcs intersecting each other at K and let BK intersect the arc ED at G.

Now, ∠ABK = 90°

(vi) With G and C as centres, and taking radius > $\frac{1}{2}$ CG, draw two arcs intersecting each other at P,

Now, ∠ABP = 45°.

Justification : We know that ∠ABK = 90° and BP is its bisector.

∴ ∠ABP = 45°

Q.3. Construct the angles of the following measurements:

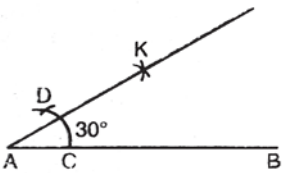
(i) 30° **(ii) $22\frac{1}{2}°$**

(iii) 15°

Ans. (a) Steps of construction :

(i) Draw a line segement AB of any Convenient length.

(ii) With A as centre and any convenient radius, draw an arc as shown in fig. which intersects AB at C.

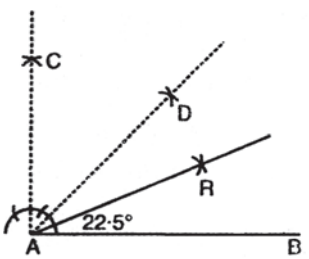


(iii) With C as centre and radius = AC, draw an arc which intersects the first arc at D.

(iv) With C and D as centre and radius > $\frac{1}{2}$ CD, draw two arcs intersecting each other at K.,

Now, ∠BAK = 30°.

(b) Steps of Construction :



(i) Draw a line segment AB of any convenient length.

(ii) Draw an angle BAC = 90°.

(iii) Draw the bisector AD of ∠BAC, so

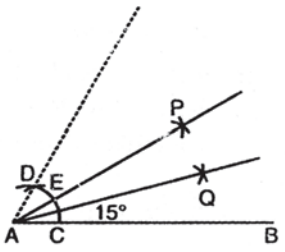
$$\angle BAR = \angle BAD = 22\frac{1}{2}°$$

Then, $\angle BAR = 22\frac{1}{2}°$

(c) Steps of Construction :

(i) Draw a line segment AB of any convenient length.

(ii) With A as centre and any convenient radius, draw an arc intersecting AB at C.



(iii) With C as centre and radius = AC, draw an arc such that it intersects the previous act at D. Now, ∠BAD = 60°.

(iv) With C and D as centres and radius > $\frac{1}{2}$ CD, draw two arcs intersecting each other at P and intersecting the previous arc at E.

Now, ∠BAP = 30°.

(v) With C and E as centres and radius > $\frac{1}{2}$ CE, draw two arcs intersecting each other at Q.

Now, ∠BAQ = 15°.

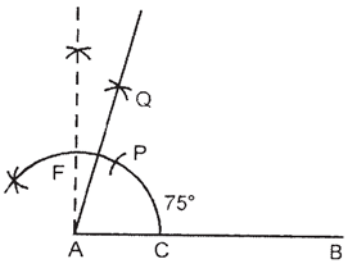
Q.4. Construct the following angles and verify by measuring them by a protractor:

(a) 75° (b) 105° (c) 135°

Ans. (a) Steps of Construction :

(i) Draw AB = 5 cm

(ii) With A as centre and any convenient radius, draw an arc which intersects AB at C.



(iii) With C as centre and radius = AC, draw an arc intersecting the previous arc at D.

(iv) With D as centre and radius = AC, draw an other arc intersecting the same arc at E.

(v) With D and E as centres, draw two arcs with radius > $\frac{1}{2}$ DE, which intersect at F.

Now ∠BAF = 90°.

(vi) Again with D and F as centres and radius > $\frac{1}{2}$ FD, draw two arcs intersecting each other at Q.

Join AQ.

Now ∠BAQ = 75°

(b) Steps of Construction :

(i) Draw AB = 5 cm.

(ii) At A, draw an ∠BAF = 90°, as discussed in Q.1.

(iii) With A centre and any convenient radius, draw an arc, intersecting AB at C.

(iv) With C as centre and the same radius, draw an arc, which cuts the previous arc at D.

(v) With D as centre and the same radius, draw an arc, which cuts the arc drawn in step (3) at E.

(vi) Draw rays AE.

(vii) With G and E as centres and radius more than $\frac{1}{2}$ GE, draw arcs to intersect each other at H.

(viii) Join AH. Then ∠BAH is the required angle of 105°.

On measuring by a protractor, we find that ∠BAH = 105°.

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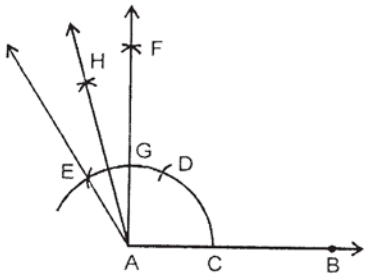
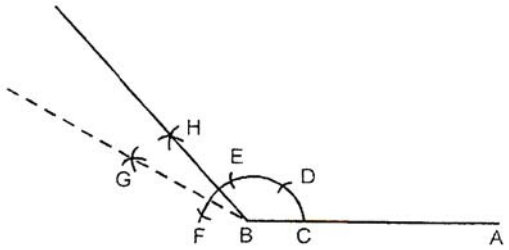


(c) Steps of Construction :

(i) Draw BA = 5 cm.

(ii) With B as centre and any convenient radius, draw an arc which intersects BA at C.



(iii) Now with C as centre and radius = BC, draw an arc which intersects the previous arc at D.

(iv) Again with D as centre and radius = BC, draw another arc which intersects the same arc at E.

Again with E as centre and same radius = BC draw another arc which cuts the same arc at F.

(v) With centres as E and F and radius $> \frac{1}{2}$ EF, draw two arcs intersecting each other at G.

(vi) With centre as K and E and radius $> \frac{1}{2}$ KE, draw two arcs intersecting each other at H.

Now $\angle ABH = 135°$.

Q.5. Construct an equilateral triangle, given its side and justify the construction.

Ans. Steps of Construction :

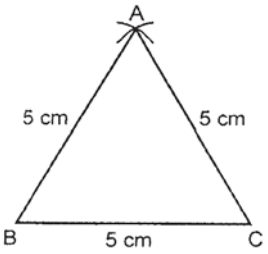
(i) Draw BC = 5 cm.

(ii) With B and C as centres and radius = 5 cm, draw two arcs intersecting each other at A ΔABC in the required equilateral triangle.

Justification : Since the three sides.

AB = BC = CA = 5 cm

Hence, ΔABC is an equilateral triangle.



EXERCISE 11.2

Q.1. Construct a triangle ABC in which BC = 7cm, $\angle B = 75°$ and AB + AC = 13 cm.

Ans. Given : In ΔABC,

BC = 7 cm, $\angle B = 75°$

and AB + AC = 13 cm

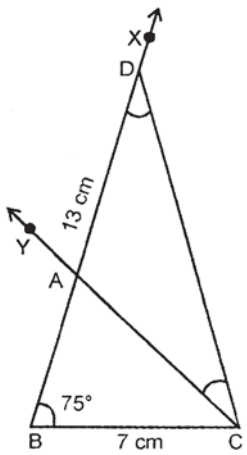
Required : To construct the triangle ABC.

Step of Construction :

(i) Draw the base BC = 7 cm

(ii) At the point B make an angle XBC = 75°.

(iii) Cut a line segment BD equal to AB + AC (= 13 cm) from the ray BX.



(iv) Join DC,

(v) Make an $\angle DCY = \angle BDC$.

(vi) Ley CY intersect BX at A.

Then, ABC is the required triangle.

Justification :

$\angle ACD = \angle ADC$ (By construction)

$\Rightarrow$ AC = AD

(Sides opposite to equal angles)

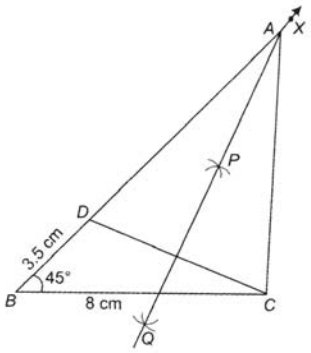
$\Rightarrow$ AB = BD – AD = BD – AC

$\Rightarrow$ AB + AC = BD.

Q.2. Construct a triangle ABC in which BC = 8cm, $\angle B = 45°$ and AB – AC = 3.5 cm.

Ans. Given that, ΔABC

BC = 8 cm, $\angle B = 45°$ and AB – AC = 3.5 cm



Steps of construction :

(i) Draw the base BC = 8 cm

(ii) At the point B make and ∠XBC = 45°.

(iii) Cut the line segment BD equal to AB – AC = 3.5 cm from the ray BX.

(iv) Join DC.

(v) Draw the perpendicular bisector, say PQ of C.

(vi) let it intersect BX at a point A.

(vii) Join AC.

Q.3. Construct a triangle PQR in which QR = 6cm, ∠Q = 60° and PR – PQ = 2cm.

Ans. Given that in ΔABC, QR = 6 cm, ∠Q = 60° and PR – PQ = 2 cm.

Steps of construction

(i) Draw the base QR = 6 cm

(ii) At the point Q make an ∠XQR = 60°.

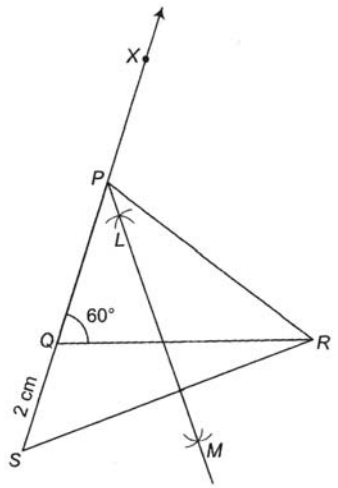
(iii) Cut line segment QS = PR – PQ (=2 cm) from the line QX extended on opposite side of line segment QR.

(vi) Join SR.

(v) Draw the perpendicular bisector LM of SR.

(vi) Let LM intersect QX at P.

(vii) Join PR.



Q.4. Construct a triangle XYZ in which ∠Y = 30°, ∠Z = 90° and XY + YZ + ZX = 11 cm.

Ans. Steps of Construction :

(i) Draw BC = 11 cm.

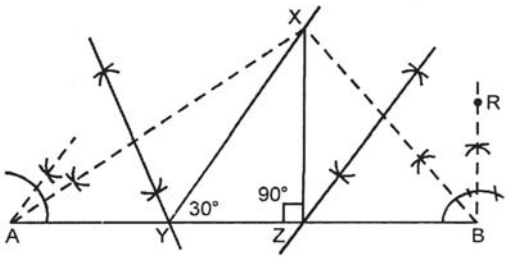
(ii) At B, draw an angle of $\frac{1}{2} \times 30° = 15°$. At C draw an angle of $\frac{1}{2} \times 90° = 45°$.

(iii) Let the arms meet at X.

(iv) Draw the right bisectors of BX and CX. Let the bisectors meet BC at Y and Z.

(v) Join X and Y and Z with X.

Thus we get the required ΔXYZ.



Justification : In ΔAXY,

we have AY = XY ...(i)

[Y lies on the ⊥ bisector of AY]

Similarly, ZB = XZ ...(ii)

∴ XY + YZ + ZX = AY + YZ + ZB = AB

[From (i) and (ii)]

From (i) we get

AY = XY

⇒ ∠YAX = ∠YXA ...(iii)

[Angles opposite to equal sides]

In ΔAXY, ∠XYZ = ∠YAX + ∠YXA

[By exterior angle Theorem]

⇒ ∠XYZ = 2∠YAX [From (iii)]

Q.5. Construct a right triangle whose base is 12cm and sum of its hypotenuse and other side is 18 cm.

Ans. Steps of Constructions :

(i) Draw BC = 12 cm.

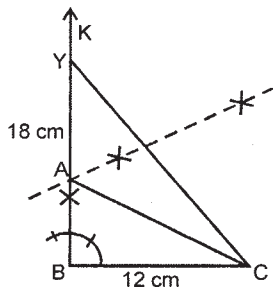
(ii) At B, draw ∠CBK = 90°

(iii) Along BK, cut off BY = 18 cm. Join C and Y.

(iv) Draw the right bisector of CY. Let the bisector meet BY at A.

Join A and C.

ΔABC is the required triangle.



Additional Questions

Q.1. Draw an angle of 80° with the help of a protractor. Then construct angles of (i) 40° (ii) 160° and (iii) 120°.

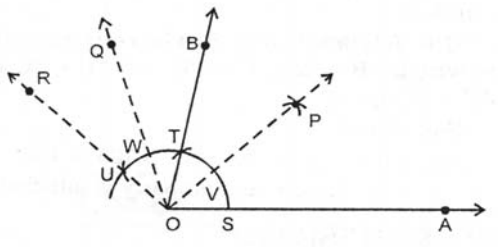
Ans. Make an ∠AOB = 80°.

(i) Bisect the ∠AOB (using compass)

∴ ∠AOP = ∠POB = 40°

(ii) Now measure the arc ST. and T as centre draw arc TS ≅ TU.

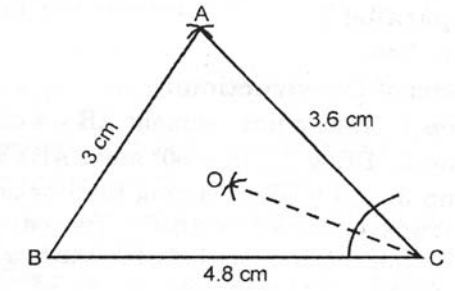
∴ ∠SOR = 80° + 80° = 160°.



(iii) With T as centre draw arc TV ≅ arc TW.

Hence, ∠AOQ = 80° + 40° = 120°.

Q.2. Construct a triangle whose sides are 3.6 cm, 3.0 cm and 4.8 cm. Bisect the smallest angle and measure each part.



Ans. Steps of Construction :

(i) Take a line segment BC = 4.8 cm.

(ii) With centre as B and C draw arcs of AB = 3 cm and AC = 3.6 cm.

(iii) Join AB and AC.

(iv) Now, bisect the ∠BCA

∠B = 55°, ∠C = 45° and ∠A = 85°

$$\therefore \quad \angle BCO = \angle ACO = \frac{1}{2}(45) = 22\frac{1}{2}°.$$

Q.3. A triangle ABC can be constructed in which ∠B = 60°, ∠C = 45° and AB + BC + AC= 12 cm. Is this statement true? Justify your answer.

Ans. Yes, Since, the base angles 60° and 45° can be constructed using ruler and compasses.

Q.4. Can we construct a triangle ABC in which ∠B = 105°, ∠C = 90° and AB + BC + AC = 10 cm.

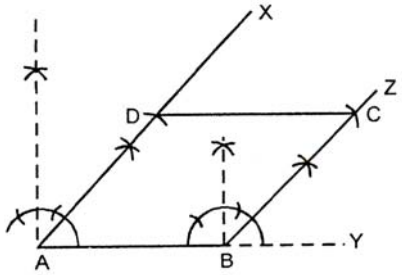
Ans. No, because ∠B + ∠C = 105° + 90° = 195°, which is not possible for a triangle.

Q.5. Construct a rhombus whose side is of length 3.4 cm and one of its angles is 45°.

Ans. Steps of construction :

(i) Draw AB = 3.4 cm.

(ii) At A, draw ∠BAX = 45°, From AX, cut off AD = 3.4 cm.



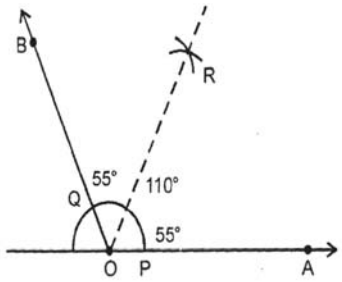
(iii) At B, draw ∠YBZ = 45°. From BZ cut off BC = 3.4 cm.

(iv) Join CD to get the required rhombus as ABCD.

Q.6. Draw an angle of 110° with the help of a protractor and bisect it. Measure each angle.

Ans. Steps of construction :

(i) Take a line segment OA.



(ii) Make an angle of AOB = 110°

(iii) With P and Q as centres and radius more than half of QP, draw two arcs.

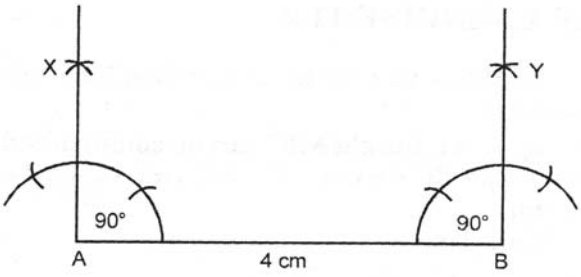
(iv) Join OR

(v) Hence ∠AOR = ∠BOR

$$= \frac{1}{2}(110°) = 55°.$$

Q.7. Draw a line segment AB of 4 cm in length. Draw a line perpendicular to AB through A and B, respectively. Are these lines parallel ?

Ans. Yes.



Steps of Construction :

(i) Take a line segment AB = 4 cm.

(ii) Draw $\angle BAX = 90°$ and $\angle ABT = 90°$.

(iii) AX || BY as sun of interior angles with transversal AB is 180°. You can also observe that the distance between the two lines AX and BY is always same.

Hence, AX || BY (True)

Q.8. A rhombus whose diagonals are 4 cm and 6 cm in lengths.

Ans. Steps of construction :

(i) Draw AC = 6 cm.

(ii) Draw the perpendicular bisector XY of AC i.e., AO = OC = 3 cm.

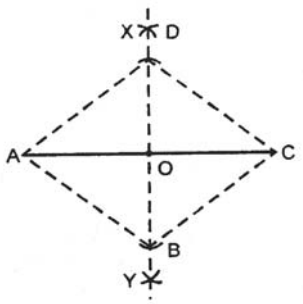
(iii) With O as centre cut two equal arcs of 2 cm on the opposite side of AC to intersect XY.

$$[OD = OB = \frac{1}{2}(BD) = \frac{1}{2} \times 4 = 2 \text{ cm.}]$$

(iv) Let these arc intersects XY at D and B.

(v) Join AD, DC, BC and AB.

Hence, ABCD is a required rhombus.



Justification : diagonals of rhombus bisect each other at 90°.

i.e., OA = OC = 3 cm

OB = OD = 2 cm

and $\angle DOC = \angle AOD$

$= \angle BOA = \angle BOC = 90°$

Q.9. Construct a triangle PQR in which QR = 6 cm, $\angle Q = 60°$ and PR – PQ = 2 cm.

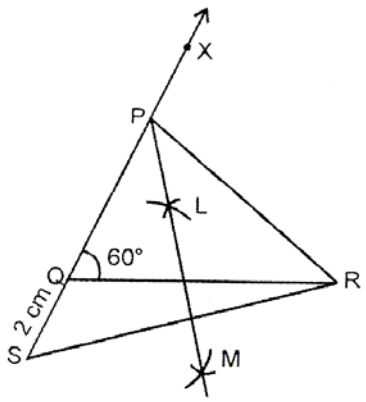
Ans. Given : In ΔPQR

QR = 6 cm

$\angle Q = 60°$

and PR – PQ = 2 cm.

Required : To construct the ΔPQR



Steps of construction :

(i) Draw the base QR = 6 cm.

(ii) At the point Q make an $\angle XQR = 60°$

(iii) Cut line segment QS = PR – PQ (= 2 cm) from the line QX extended on opposite side of line segment QR.

(iv) Join SR.

(v) Draw the perpendicular bisector LM or SR.

(vi) Let LM intersect QX at P.

(vii) Join PR.

Then, PQR is the required triangle.

Justification : In ΔPSR,

we have SP = PR

[P lies on the ⊥ bisector of SR]

$\therefore$ QS = PS – PQ

$\Rightarrow$ QS = PR – PQ.

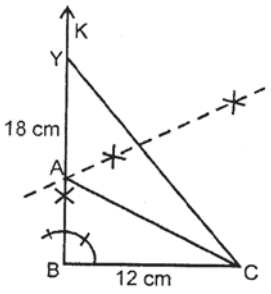
Q.10. Construct a right triangle whose base is 12 cm and sum of its hypotenuse and other side is 18 cm.

Ans. Steps of construction :

(i) Draw BC = 12 cm

(ii) At B, draw $\angle CBK = 90°$

(iii) Along BK, cut off BY = 18 cm. Join C and Y.



(iv) Draw the right bisector of CY. Let the bisector meet By at A.

Join A and C.

ΔABC is the required triangle.

Multiple Choice Questions

Q.1. With the help of a ruler and a compass, it is possible to construct an angle of:

(a) 35° (b) 37.5°

(c) 30° (d) 47.6°

Ans. (b) 37.5°

Q.2. The construction of a triangle ABC in which AB=4 cm, ∠A=60° is not possible when difference of BC and AC is equal to:

(a) 3.5cm (b) 4.5cm

(c) 3cm (d) 2.5cm

Ans. (b) 4.5cm

Q.3. Which of the following angles connot be contructed with help of a ruler and a compass?

(a) $7\frac{1}{2}°$ (b) $22\frac{1}{2}°$

(c) $30\frac{1}{2}°$ (d) $37\frac{1}{2}°$

Ans. (c) $30\frac{1}{2}°$

Q.4. If OA is the angle bisector of ∠ POQ then the value of ∠ AOP is:

(a) $33\frac{1}{2}°$ (b) $22\frac{1}{2}°$

(c) $30\frac{1}{2}°$ (d) $37\frac{1}{2}°$

Ans.(d) $37\frac{1}{2}°$

Q.5. Construction of a triangle is possible when:

(a) Sum of two sides is always greater than the third side

(b) angle sum property of a triangle is satisfied

(c) both (a) and (b) (d) None of these

Ans. (d) None of these

Q.6. In a triangle ABC; BC= 5.6 cm, AC-AB=1.6 cm and ∠ B=45° which would be the correct method:

(a) We have to produce AB, below BC

(b) We have to produce AB, above BC

(c) We have to bisect BC

(d) None of these

Ans. (a) We have to produce AB, below BC

Q.7. The construction of a triangle ABC, given that BC=4cm, ∠ C = 60° is possible when difference of AB and AC is equal to:

(a) 4.2 cm (b) 4.1 cm

(c) 4 cm (d) 3.8 cm

Ans. (d) 3.8 cm

Q.8. If AB+BC+AC=11.6 cm, ∠B=60° and ∠ C=45°, then to construct a ΔABC we will have to:

(a) bisect ∠B=60° and ∠ C=45°

(b) It is not necessary to bisect ∠B and ∠C

(c) first of all bisect XA and YA where X and Y lies on BC (produced) then join A to B and A to C

(d) Both (a) and (c) are the steps

Ans. (d) Both (a) and (c) are the steps

Q.9. Construction of a triangle is possible when:

(a) Sum of two side is always greater than the 3rd side

(b) angle sum property of a triangle satisfied

(c) both a and b

(d) none of these

Ans. (c) both a and b

Q.10. To construct a ΔABC in which BC= 6 cm AC + AB= 10 cm and ∠B= 90°, the wrong steps of construction may be:

(a) Draw ∠CBX=90°

(b) cut off BL =10cm

(c) Join CL

(d) It is not necessary to draw the perpendicular bisector of CL

Ans. (d) It is not necessary to draw the perpendicular bisector of CL