Chapter 10

Circles

In the Chapter

In this chapter, you will be studying the following points:

- A circle is the collection of all points in a plane, which are equidistant from a fixed point in the plane.
- Equal chords of a circle (or of congruent circles) subtend equal angles at the centre.
- If the angles subtended by two chords of a circle (or of congruent circles) at the centre (corresponding centres) are equal, the chords are equal.
- The perpendicular from the centre of a circle to a chord bisects the chord.
- The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
- There is one and only one circle passing through three non-collinear points.
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or corresponding centres).
- Chords equidistant from the centre (or corresponding centres) of a circle (or of congruent circles) are equal.
- If two arcs of a circle are congruent, then their corresponding chords are equal and conversely if two chords of a circle are equal, then their corresponding arcs (minor, major) are congruent.
- Congruent arcs of a circle subtend equal angles at the centre.
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- Angle in a semicircle is a right angle.
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
- If sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

Important Theorems

(i) The angle in a semi-circle is a right angle.

(ii) If two chords of a circle are equal, then their corresponding arcs (major, minor or semi-circle) are congruent and vice-versa.

(iii) The angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

(iv) Equal chords of a circle (or of congruent circles) are equidistant from the centre.

(v) If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

(vi) The perpendicular from the centre of a circle to a chord bisects the chord and it is vice-versa.

(vii) Congruent arcs of a circle subtend equal angles at the centre.

(viii) The sum of either pair of opposite angles of a cyclic quadrilatral is 180° and vice-versa.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 10.1

Q.1. Fill in the blanks:

(i) The centre of a circle lies in .... of the circle. (exterior/ interior)

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in .... of the circle. (exterior/ interior)

(iii) The longest chord of a circle is a .... of the circle.

(iv) An arc is a .... when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and .... of the circle.

(vi) A circle divides the plane, on which it lies, in .... parts.

Ans. (i) interior, (ii) exterior, (iii) diameter, (iv) semicircle, (v) the chord, (vi) three.

Q.2. Write True or False: Give reasons for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs, each is a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Ans. (i) True. Because all points are equidistant from the centre to the circle.

(ii) False. Because there are infinitely many points on the circle.

(iii) False. Because all three arcs are equal, so there is no difference between the major and minor arcs.

(iv) True. By the definition of diameter.

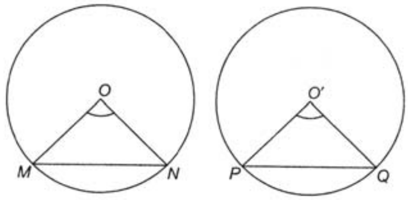
(v) False. Because the sector is the region between two radii and an arc.

(vi) True, as it is a part of a plane.

EXERCISE 10.2

Q.1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Ans. Given : MN and PQ are two equal chords of two congruent circles with centre at O and O'.



To prove : In ΔMON and DPO'Q, we have

MO = PO' (Radii of congruent circle)

NO = QO' (Radii of congruent circle)

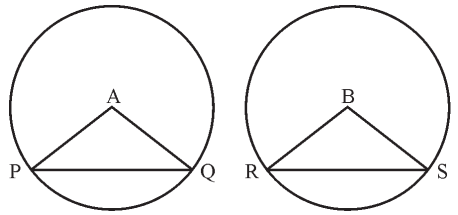
and MN = PQ (Given)

∴ By SSS criterion, we get

ΔMON = ΔPO'Q

Hence, ∠MON = ∠PO'O (By CPCT)

Q.2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.



Ans. Given : Two congruent circles with centre A and B. Chord PQ subtend angle ∠PAQ at the centre and also chord RS subtend ∠RBS at the centre. Also ∠PAQ = ∠RBS.

To prove : Chord PQ = Chord RS.

Proof : in ΔPAQ and ΔRBS

AP = BR (Radii of congruent circles)

AQ = BS (Raddi of congruent circles)

∠PAQ = ∠RBS (Given)

Thus Δ PAQ = ΔRBS

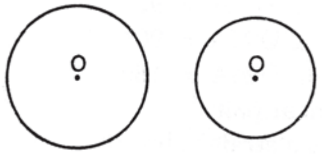
(SAS Congruence Rule)

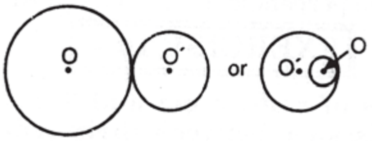
Hence PQ = RS(CPCT) Proved.

EXERCISE 10.3

Q.1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Ans. Fig. (i), The two circles have no common points.





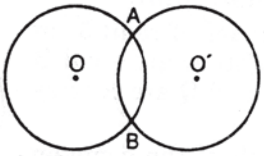


Fig. (ii), The two circles just touch each other. They have one common point.

Fig. (iii), The two circles intersect each other at two points.

The maximum number of common points between two circles is two.

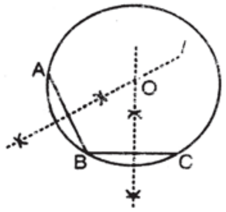
Q.2. Suppose you are given a circle. Give a construction to find its centre.

Ans. Steps of construction :

(i) Draw chords AB and BC.

(ii) Draw 'l', the right bisector of AB.

(iii) Draw 'm' the right bisector of BC.



(iv) Let l and m intersect each other at O.

Now, O is the required centre of the circle.

Q.3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Ans. Given : Two circles with centre O and O' intersect each other at A and B, thus, forming a common chord AB.

M is the mid-point of the common chord AB.

To prove : O O' is the perpendicular bisector of AB.

Proof : We know that the line joining of mid-point of a chord to the centre is perpendicular to the chord.

$\therefore$ $\angle OMA = 90°$

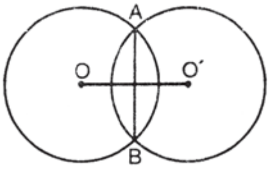
and $\angle O'MA = 90°$

Now, $\angle OMA + \angle O'MA = 180°$

But they form a linear pair.

Therefore, OMO' is a straight line.

Hence, OO' is the perpendicular bisector of AB.

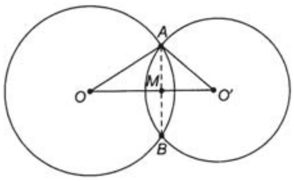


EXERCISE 10.4

Q.1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Ans. Let O and O' be the centres of the circles of radii 5 cm and 3 cm, respectively.

Let AB be their common chord.



Given, OA = 5 cm, O' A = 3 cm and OO' = 4 cm.

$\therefore \quad AO'^{2} + OO'^{2} = 3^2 + 4^2 = 9 + 16 = 25.$

$= OA^2$

$\therefore$ OO' A is a right angled triangle and right angles at O'

$$\text{Area of } \Delta OO'A = \frac{1}{2} \times O'A \times OO'$$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ sq units}$$

$$\text{Also, are of } \Delta OO'A = \frac{1}{2} OO' \times AM$$

$$= \frac{1}{2} \times 4 \times AM = 2AM \quad ...(ii)$$

From Eqs. (i) and (ii), we get

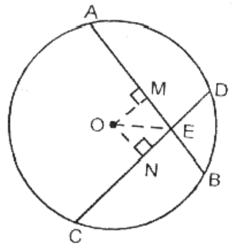
$$2\,AM = 6 \Rightarrow AM = 3$$

Since, when two cricles intersect at two points, then their centre lie on the perpendicular bisector of the common chord.

$\therefore \quad AM = 2 \times AM = 2 \times 3 = 6$ cm

Q.2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Ans. Given: AB and CD are two equal chords of a circle which meet at E.



To prove : AE = CE and BE = DE

Construction : Draw OM $\perp$ AB and ON $\perp$ CD and join OE.

Proof : In ΔOME and ΔONE

OM = ON

[Equal chords are equidistant]

OE = OE [Common]

$\angle$OME = $\angle$ONE [Each equal to 90°]

$\therefore$ ΔOME = ΔONE [RHS axiom]

$\Rightarrow$ EM = EN ...(i) (CPCT)

Now, AB = CD [Given]

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$$

$\Rightarrow$ AM = CN ...(ii)

[Perpendicular from centre bisects the chord]

Adding (i) and (ii), we get

EM + AM = EN + CN

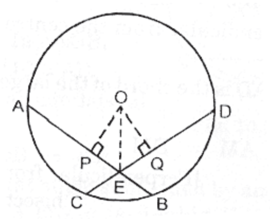
$\Rightarrow$ AE = CE ...(iii)

Now, AB = CD ...(iv)

$\Rightarrow$ AB – AE = CD – AE [From (iii)]

$\Rightarrow$ BE = CD – CE **Proved.**

Q.3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



Ans. A circle with centre O. Equal chords AB and CD intersect each other at E and AB = CD.

To prove : $\angle$OEA = $\angle$OEC

Construction : Draw OP $\perp$ AB and OQ $\perp$ CD.

Proof : in ΔOPE and ΔOQE,

OE = OE. (Common)

OP = OQ

(Equal chords are equidistant from the centre)

$\angle$OPE = $\angle$OQE

(By construction, each = 90°)

Thus, ΔOPE $\cong$ ΔOQE

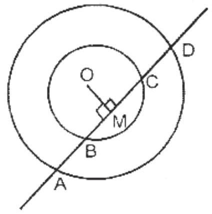
(RHS congruence rule)

Hence, $\angle$OEP = $\angle$OEQ

$\angle$OEA = $\angle$OEC. (CPCT)

Proved.

Q.4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig.).



Ans. Given : Two concentric circles with centre O. A line intersects the two circles at A, B, C and D.

To prove : AB = CD

Construction : Draw OM $\perp$ BC

Proof : Since, BC is the chord of the smaller circle and OM $\perp$ BC.

$\therefore$ BM = CM ...(i)

(Perpendicular from the centre bisects the chord)

Again AD is the chord of the larger circle and OM $\perp$ AD.

$\therefore$ AM = DM, ...(ii)

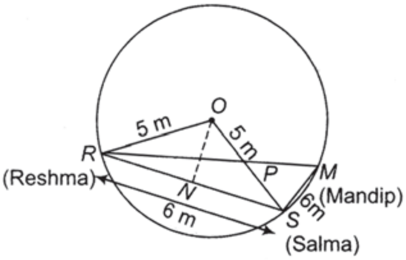
(Perpendicular from the centre bisects and chord)

Subtracting equaiton (i) from (ii), we get

AM – BM = DM – CM

Hence, AB = CD. Proved.

Q.5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?



Ans. Draw a figure as shown in the figure. Let the positions of Reshma, Salma and Mandip be marked as R, S and M. Draw ON ⊥ RS.

Now, join OS intersecting RM perpendicular at K. Let KR = x m.

$$\text{Area}(\Delta ORS) = \frac{1}{2}\,OS \times RK$$

$$= \frac{1}{2} \times 5 \times x$$

Also, $\quad \text{Area (ORS)} = \frac{1}{2}\,RS \times ON$

$$= \frac{1}{2} \times 6 \times 4 = 12$$

$$\left(\text{In rt. } \Delta ONR,\ ON = \sqrt{5^2 - 3^2} = 4\right)$$

$$\therefore \quad \frac{5}{2}x = 12$$

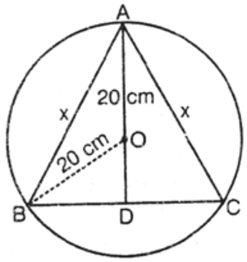
$$\Rightarrow \quad x = \frac{24}{5}$$

$$= 4.8$$

Hence, distance between Reshma and Mandip

$$= 2x = 2 \times 4.8 = 9.6 \text{ m.}$$

Q.6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.



Ans. Given : A circle park of radius = 20 m and centre O.

AB = BC = CA

To find : The distance between any two boys.

Solution : Let AB = BC = CA = x m.

Draw AOD ⊥ BC.

Now, $\quad BD = \frac{x}{2}$

Now using Pythagoras theorem,

$$OD^2 = OB^2 - BD^2$$

$$10^2 = (20)^2 - (x/2)^2$$

$$100 = 400 - \frac{x^2}{4}$$

$$\frac{x^2}{4} = 300$$

$$\Rightarrow \quad x^2 = 1200$$

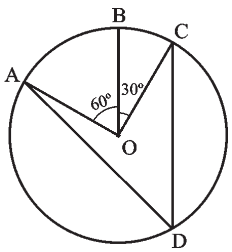
$$x = \sqrt{1200}$$

$$= 20\sqrt{3} \text{ m.}$$

Hence, length of the string of each phone = $20\sqrt{3}$ m.

EXERCISE 10.5

Q.1. In Fig., A,B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Ans. ∴ ∠AOC = ∠AOB + ∠BOC = 60° + 30° = 90°.

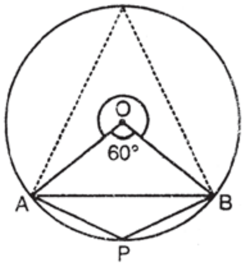
$\therefore$ Arc ABC makes 90° at the centre of the circle.

$$\therefore \quad \angle ADC = \frac{1}{2}\angle AOC$$

(The angles subtended by an arc at the centre is double the angle subtended by it any part of the circle.)

$$= \frac{1}{2} \times 90° = 45°$$

Q.2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



Ans. Given : A circle with centre O and in which chord AB = radius.

To find : $\angle APB$ and $\angle AQB$.

Solution : In ΔAOB, OA = OB = AB.

$\therefore$ ΔAOB is an equilatral triangle.

$\therefore$ $\angle AOB = 60°$

For major arc AQB, chord AB subtends $\angle AOB =$ 60° at the centre and $\angle AQB$ on the other part of the circumference.

$$\therefore \quad \angle AQB = \frac{1}{2}\angle AOB = \times 60° = 30°$$

Similarly for minor arc, chord AB subtends an $\angle APB$ on the minor arc and reflex angle $\angle AOB$ at the centre.

$$\therefore \quad \angle APB = \frac{1}{2} \times \text{reflex} \times \angle AOB$$

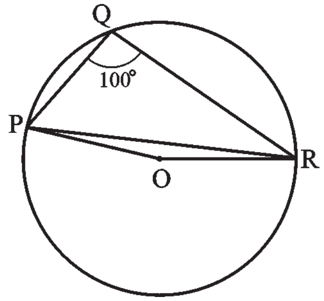
$$= \frac{1}{2} \times (360° - 60°)$$

$$= \frac{1}{2} \times 300° = 150°$$

Hence, $\angle APB = 150°$

and $\angle AQB = 30°$.

Q.3. In Fig., $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Ans. Given : A circle with centre O. P, Q and R are points on the circle as shown in the figure.

Arc PQR subtends reflex $\angle POR$ at the centre and $\angle PQR$ on the other part of the circumference.

$\therefore$ Reflex $\angle POR = 2\angle PQR$

$= 2 \times 100 = 200°$

Now $\angle POR = 360 - 200 = 160°$

In $\angle OPR$ OP = OR

[Radii of the same circle]

$\therefore$ $\angle OPR = \angle ORP$

[Angles opposite to equal sides]

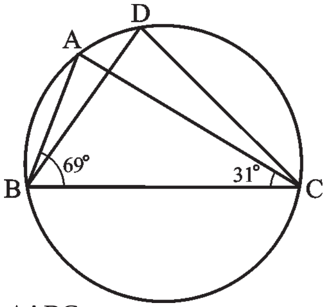
Now $\angle OPR + \angle ORP + \angle POR = 180°$

or $2\angle OPR + 160° = 180°$

or $2\angle OPR = 20°$

Hence $\angle OPR = 10°$

Q.4. In Fig. , $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$.



Ans. In ΔABC,

$\angle A + \angle ABC + \angle ACB = 180°$ [Angles of a Δ]

But $\angle ABC = 69°$

and $\angle ACB = 31°$ (Given)

$\therefore$ $\angle A + 69° + 31° = 180°$

or $\angle A + 100° = 180°$

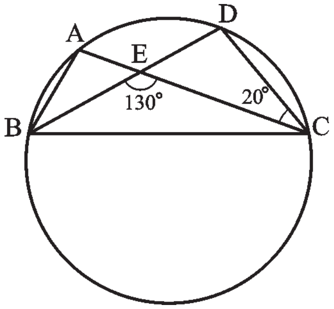
or $\angle A = 80°$

But $\angle A = \angle D$

[Angles in the same segement]

Hence $\angle D = 80°$

Q.5. In Fig., A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130°$ and $\angle ECD = 20°$. Find $\angle BAC$.



Ans. $\therefore$ $\angle AEB = 180° - 130° = 50°$

(Linear pair) ...(i)

$\Rightarrow$ $\angle CED = \angle AEB = 50°$

(Vertically opposite)

Again, $\angle ABD = \angle ACD$

[Since, the angles in the same segement are equal)

$\angle ABE = \angle ECD$

$\Rightarrow$ $\angle ABE = 180°$ ...(ii)

$\therefore$ In $\angle CDE$ $\angle A + 20° + 50° = 180°$

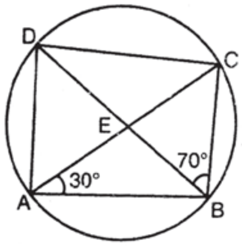
[From Eqs. (i) and (ii)]

$\angle A + 70° = 180°$

$\therefore$ $\angle A = 180° - 70° = 110°$

Hence $\angle BAC = 110°$

Q.6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is 30°, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.



Ans. Angles in the same segment are equal.

$\therefore$ $\angle BDC = \angle BAC$

$\therefore$ $\angle BDC = 30°$

In ΔBCD, we have

$\therefore$ $\angle BDC + \angle DBC + \angle BCD = 180°$

(Given, $\angle DBC = 70°$ and $\angle BDC = 30°$)

$\therefore$ $30° + 70° + \angle BCD = 180°$

$\therefore$ $\angle BCD = 180° - 30 - 70° = 80°$

If AB = BC, then $\angle BCA = \angle BAC = 30°$

(Angles opposite to equal sides in a triangle are equal)

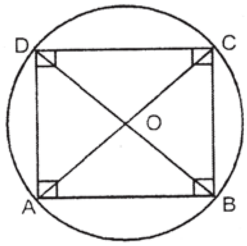
Now, $\angle ECD = \angle BCD - \angle BCA = 80° - 30° = 50°$

($\angle BCD = 80°$ and $\angle BCA = 30°$)

Hence, $\angle BCD = 80°$

and $\angle ECD = 50°$

Q.7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.



Ans. Given : A cyclic quadrilatral ABCD in which its diagonals AC and BD intersect at O, the centre of the circle.

To prove : ABCD is a rectangle.

Proof : $\angle DAB = 90°$

Similarly, $\angle ABC = 90°$

[Angles in the semi-circle]

$\angle BCD = 90°$, $\angle CDA = 90°$

[Angles in the semi-circle]

We know that if each angle of a ||gm is 90° then it is a rectangle.

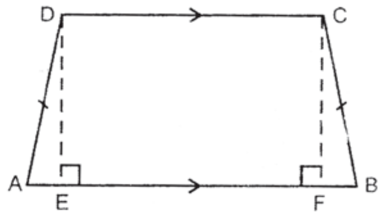
Hence ABCD is a rectangle.

Q.8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Ans. Given : a trapezium ABCD in which AB || CD and AD = BC.

To prove : ABCD is a cyclic trapezium.

Construction : Draw $DE \perp AB$ and $CF \perp AB$.



Proof : In ΔDEA and ΔCFB, we have

AD = BC [Given]

$\angle DEA = \angle CFB = 90°$

[$DE \perp AB$ and $CF \perp AB$]

DE = CF

[Distance between parallelel lines remains constant]

$\therefore$ $\Delta DEA \cong \Delta CFB$ [RHS axiom]

$\Rightarrow$ $\angle A = \angle B$ ...(i) (CPCT) and, $\angle ADE = \angle BCF$ ...(ii) (CPCT)

Since, $\angle ADE = \angle BCF$ [From (ii)]

$\Rightarrow \angle ADE + 90° = \angle BCF + 90°$

$\Rightarrow \angle ADE + \angle CDE = \angle BCF + \angle DCF$

[$\angle CDE = \angle DCR = 90°$]

$\Rightarrow \angle D = \angle C$ ...(iii)

[$\angle ADE + \angle CDE = \angle D, \angle BCF + \angle DCF = \angle C$]

$\therefore \angle A = \angle B$ and $\angle C = \angle D$

[From (i) and (iii)]

$\angle A + \angle B + \angle C + \angle D = 360°$ ...(iv)

[Sum of the angles of a quadrilateral is 360°]

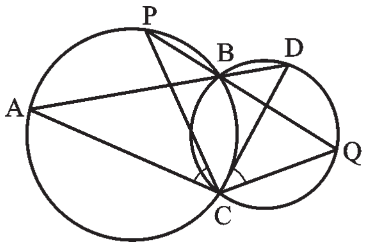
$\Rightarrow 2(\angle B + \angle D) = 360°$ [Using (iv)]

$\Rightarrow \angle B + \angle D = 180°$

$\Rightarrow$ Sum of a pair of opposite angles of quadrilateral ABCD is 180°.

$\Rightarrow$ ABCD is a cyclic trapezium. **Proved.**

Q.9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig.). Prove that $\angle ACP = \angle QCD$.



Ans. Given : Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively

To prove : $\angle ACP = \angle QCD$

Proof : $\angle ACP = \angle ABP$ ...(i)

[Angles in the same segment]

$\angle QCD = \angle QBD$ ...(ii)

[Angles in the same segment]

But, $\angle ABP = \angle QBD$ ...(iii)

[Vertically opposite angles]

By (i), (ii), and (iii), we get

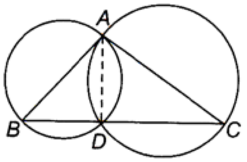
$\angle ACP = \angle QCD$ **Proved.**

Q.10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans. Given : Two circles are drawn with sides AC and AB of ΔABC as diameters. Both circles intersect each other at D.

To prove : D lies on BC.

Construction : Join AD.



Proof : Since, AC and AB are the diameters of the two circles.

$\angle ADB = 90°$ ..(i)

(Angles in a semi-circle)

and $\angle ADC = 90°$ ...(ii)

(Angles in a semi-circle)

On adding Eqs. (i) and (ii), we get

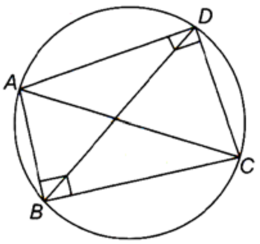
$\angle ADB + \angle ADC = 90° + 90° = 180°$

Hence, BCD is a straight line.

So, D lies on BC.

Q.11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Ans. Since ΔADC and ΔABC are right angled triangles with common hypotenuse.



Draw a circle with AC as diameter passing through B and D. Join BD.

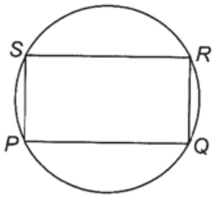
Angles in the same segment are equal.

$\therefore \angle CBD = \angle CAD$

Q.12. Prove that a cyclic parallelogram is a rectangle.

Ans. Given : PQRS is a parallelogram inscribed in a circle.

To prove : PQRS is a rectangle.



Proof : Since, PQRS is a cyclic quadrilatral.

$\therefore \angle P + \angle R = 180°$

[Sum of opposite angles in a cyclic quadrilateral is 180°) ...(i)

But $\angle P = \angle R$

(In a || gm opposite angles are equal) ...(ii)

From Eqs. (i) and (ii), we get

$\angle P = \angle R = 90°$

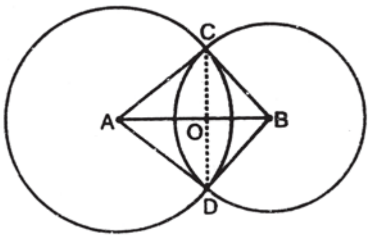
Similarly, $\angle Q = \angle S = 90°$

$\therefore$ Each angle of PQRS is 90°

Hence, PQRS is a rectangle.

EXERCISE 10.6 (Optional)

Q.1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.



Ans. Given : Two circles with centres A and B intersect each other at C and D.

To prove : $\angle ACB = \angle ADB$

Construction : Join C and D intersecting AB at O.

Proof : In the circle with centre A.

AC = AD, (Radii of the same circle)

$\therefore$ $\angle ACO = \angle ADO$ ...(i)

(Angles opposite to equal sides)

Adding equations (i) and (ii), we get

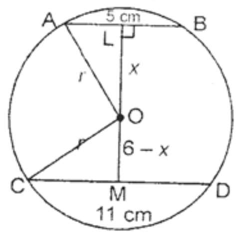
$\therefore$ $\angle ACO + \angle BCO = \angle ADO + \angle BDO$

Hence, $\angle ACB = \angle ADB$. **Proved.**

Q.2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Ans. Given : In a circle with centre O. Chord AB = 5 cm. Chord CD = 11 cm.

Draw $OL \perp AB$ and $OM \perp CD$.



As perpendicular from the centre bisects a chord, therefore

AL = 2.5 cm

and CM = 5.5 cm

Let the radius of the circle = r

In ΔALO, $r^2 = x^2 + (2.5)^2$ ...(i)

In ΔCMO, $r^2 = (6 - x)^2 + (5.5)^2$ ...(ii)

From (i) and (ii), we get

$x^2 + (2.5)^2 = (6 - x)^2 + (5.5)^2$

or $x^2 + 6.25 = 36 + x^2 - 12x + 30.25$

or $6.25 = 36 - 12x + 30.25$

or $12x = 66.25 - 6.25$

or $12x = 60$

or $x = 5$

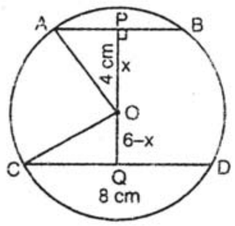
$\therefore$ $r^2 = (5)^2 + (2.5)^2$

or $r^2 = 25 + \dfrac{25}{4}$

$r^2 = \dfrac{125}{4}$

$r = \dfrac{5\sqrt{5}}{2}$ cm

Q.3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?



Ans. Given : A circle with centre O. Chord AB = 6 cm. Chord CD = 8 cm.

$OP \perp AB$ and $OQ \perp CD$

AP = 3 cm, OP = 4 cm

Let OQ = x

Let radius = r cm

In ΔAOP, $r^2 = 4^2 + (3)^2$

$\Rightarrow$ $r^2 = 25 \Rightarrow r = 5.$

Now in ΔOCQ,

$r^2 = x^2 + 4^2$

or $5^2 = x^2 + 4^2$

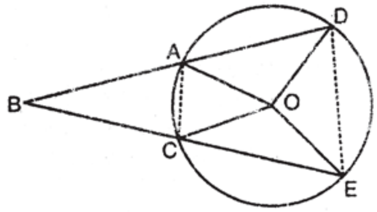
or $25 = x^2 + 16$

or $x^2 = 9$

or $x = \sqrt{9} = 3.$

Hence distance of the other chord from the centre = 3 cm.

Q.4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.



Ans. Given : The vertex B of an ÐABC lies outside the circle. The sides BA and BC of the angle intersect chords AD and CE with the circle and AD = CE.

To prove :

$\angle ABC = \frac{1}{2}$ [ÐDOC – ∠AOC]

Proof : Let ∠AOC = x and ∠DOE = y

Let ∠AOD = z

Now, ∠EOC = z and $x + y + 2z = 360°$

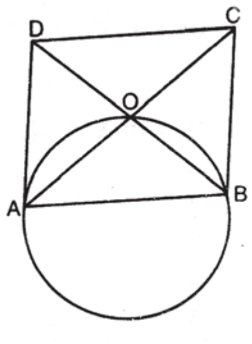
∠ODB = ∠OAD + ∠DOA

$= 90° - \frac{1}{2}z + z$

$= 90° + \frac{1}{2}z$

Also, ∠OEB = $90° + \frac{1}{2}z$ **Proved.**

Q.5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.



Ans. Given : A rhombus ABCD in which a circle is drawn with AB as diameter. Diagonals AC and BD intersect at O.

To prove : The circle passess through O.

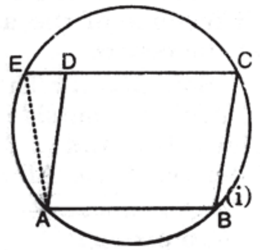
Proof : We know that angles in a semicircle is a right angle.

Now ∠AOB = right angle

[Diagonals of a rhombus bisect at 90°]

Therefore the circle passes through the point of intersection of its diagonals *i.e.*, the point O.

Q.6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.



Ans. Given : ABCD is a parallelogram. A circle passes through A, B and C and intersects CD (or CD produced) at E.

To prove : AE = AD

Construction : Join A and E.

Proof : ABCE is a cyclic quadrilateral.

∴ ∠AED + ∠ABC = 180° ...(i)

Now EDC is a straight line.

∴ ∠ADE + ∠ADC = 180°

or ∠ADE + ∠ABC = 180° ...(ii)

[∠ADC = ∠ABC opposite angles of a || gm]

Comparing (i) and (ii), we get

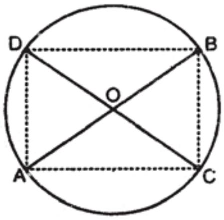
∠AED + ∠ABC = ∠ADE + ∠ABC

or ∠AED = ∠ADE

As sides opposite to equal angles of ΔAED are equal.

Therefore AE = AD.

Q.7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters, (ii) ABCD is a rectangle.



Ans. Given: A circle whose chords AC and BD bisect each other at O.

To prove : (i) AC and BD are diameters.

(ii) ABCD is a rectangle.

Proof : (i) In ΔOAB and ΔOCD.

OA = OC, (Given)

OB = OD (Given)

∠AOB = ∠COD,

(Vertically opp. angles)

ΔOAB = ΔOCD

(SAS congruence rule)

AB = CD

arc AB = arc CD,

(Arcs opp. to equal chords)

arc AB + arc BC = arc CD + arc BC

arc ABC = arc BCD

AC = BD

(Chords opp. to equal arcs)

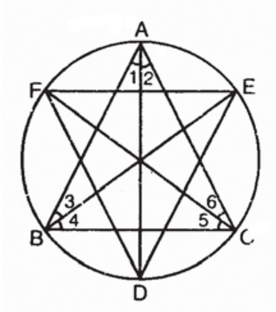
∴ AC and BD are diameters as only diameters can bisect each other as chords of a circle.

(ii) AB = CD (Proved above)

and diagonals AC and BD are equal and bisect each other.

Hence, ABCD is a rectangle. **Proved.**

Q.8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}$ A, $90° - \frac{1}{2}$ B and $90° - \frac{1}{2}$ C.



Ans. ∠EDF = ∠EDA + ∠ADF

∠EDA and ∠EBA are the angles in the same segment of the circle.

∴ ∠EDA = ∠EBA

and similarly ∠ADF and ∠FCA are the angles in the same segment and hence.

∠ADF = ∠FCA

∴ From Eq. (i) $\angle EDF = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

$$\Rightarrow \quad \angle D = \frac{\angle B + \angle C}{2}$$

Similarly, $$\angle F = \frac{\angle A + \angle B}{2}$$

$$\angle E = \frac{\angle C + \angle A}{2}$$

So, $$\angle D = \frac{\angle B + \angle C}{2}$$

$$= \frac{180° - \angle A}{2}$$

($\angle A + \angle B + \angle C = 180°$)

$$\angle E = \frac{180° - \angle B}{2}$$

and $$\angle F = \frac{180° - \angle C}{2}$$

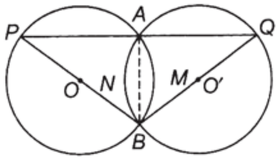
$$\Rightarrow \quad \angle D = 90° - \frac{\angle C}{2}$$

$$\Rightarrow \quad \angle E = 90° - \frac{\angle B}{2}$$

and $$\angle F = 90° - \frac{\angle C}{2}$$

Q.9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Ans. Let O' and O be the centres of two congruent circles.



Since, AB is a common chord of these circles.

∴ ∠BPA = ∠BQA

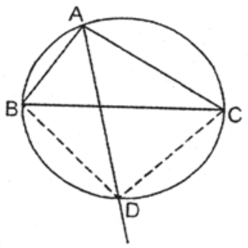
[Angle subtended by equal chords are equal]

⇒ BP = BQ

Q.10. In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Ans. Given : A ΔABC in which bisector of ∠A and perpendicular bisector of BC intersect each other at OD.

To prove : D lies on the circumcircle of ΔABC.



Construction : Join BD and CD.

Proof : $\angle BCD = \angle BAD = \frac{1}{2}\angle A$

(Angles in the same segment)

And ∠DBC = ∠DAC

$= \frac{1}{2}\angle A$

(Angles in the same segment)

Therefore, $\angle BCD = \angle DBC$

$\therefore$ $DB = DC$

[Sides opposite to equal angles]

Thus D lies on the perpendicular of BC.

As D is equidistant from B and C.

Hence, angle bisector of ÐA and perpendicular bisector of BC intersect on the circumcircle of a ΔABC.

Additional Questions

Q.1. Two congruent circles with centre O and O' intersect at two points A and B. Then $\angle AOB = \angle AO'B$.

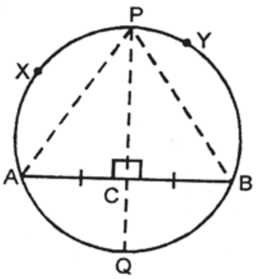
Ans. True, because equal chords of congruent circles subtend equal angles at the respective centres.

Q.2. Through three collinear points a circle can be drawn.

Ans. False, because a circle through two points cannot pass through a point which is collinear to these two points.

So, circle can pass through only three non-collinear points.

Q.3. If the perpendicular bisector of a chord AB of a circle. PXAQBY intersects the circle at P and Q, prove that arc PXA ≅ arc PYB.



Ans. Since PQ is the perpendicular bisector of AB.

$\therefore$ $AC = CB$

Now in ΔACP and ΔBCP, we have

	AC	$= CB$	(Given)
	CP	$= PC$	(Common)
	$\angle PCA$	$= \angle PCB$	(Each 90°)
Hence,	ΔACP	$= \Delta BCP$	[By SAS]
$\therefore$	AP	$= PB$	

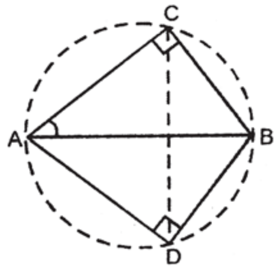
$\Rightarrow$ arc PXA $\cong$ arc PYB

[If chords are equal then their corresponding arcs are also equal] Proved.

Q.4. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that

$\angle BAC = \angle BDC$.

Ans. Given : ACB and ADB are two right triangles and AB is common hypotenuse.



To prove : $\angle BAC = \angle BDC$

Proof : $\angle ACB = 90°$ [Given]

$\angle ADB = 90°$ [Given]

$\therefore$ $\angle C + \angle D = 90° + 90° = 180°$

If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

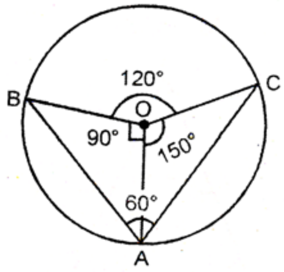
i.e., A, C, B, D are lie on the circle.

Hence $\angle BAC = \angle BDC$.

[Angle in the same segment) **Proved.**

Q.5. Two chords AB and AC of a circle subtend angles equal to 90° and 150°, respectively at the centre. Find $\angle BAC$, if AB and AC lie on the opposite sides of the centre.

Ans. Reflex $\angle BOC = 90° + 150° = 240°$



$\angle BOC = 360° - 240°$

$\angle BOC = 120°$

$\Rightarrow \quad \angle BOC = 2\angle BAC$

or $\quad 2\angle BAC = 120°$

$$\angle BAC = \frac{120}{2}$$

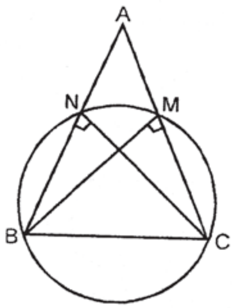
[By angle subtended theorem]

$\Rightarrow \quad \angle BAC = 60°.$

Q.6. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.

Ans. Given : $BM \perp AC$

and $\quad CN \perp AB$



To prove : B, C, M and N are concyclic.

Proof : $\angle BMC = 90°$ [Given]

and $\quad \angle BNC = 90°$ [Given]

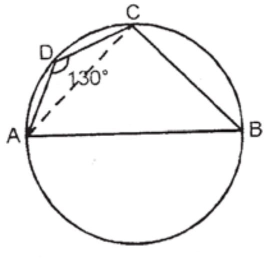
Since BC is a line segment, which subtends equal angle at two points M and N i.e.,

$$\angle BMC = \angle BNC = 90°$$

Hence, the points B, C, M and N are concyclic.

(By theorem 10.10)

Q.7. A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and ∠ADC=130°. Find ∠BAC.



Ans. Since ABCD is a cyclic quadrilateral.

$\therefore \quad \angle ADC + \angle ABC = 180°$

$\Rightarrow \quad 130° + \angle ABC = 180°$

$\Rightarrow \quad \angle ABC = 50°$

In DACB, we have

$\angle BAC + \angle ACB + \angle ABC = 180°$

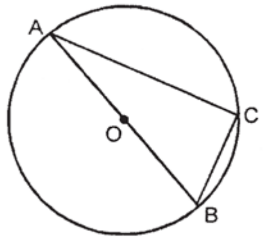
$\Rightarrow \quad x° + 90° + 50° = 180°$

[$\angle ACB = 90°$ (Angle in semicircle)]

$\Rightarrow \quad x = 40$

$\Rightarrow \quad \angle BAC = 40°$. Ans.

Q.8. In fig. AB is the diameter of a circle with radius 6.5 cm. If AC= 12 cm, then find the length of the chord BC.



Ans. In DABC, $\angle C = 90°$

[Angle in a semi-circle is a right angle]

We have, $\quad AC = 12$ cm

and $\quad AB = 2 \times 6.5$ cm $= 13$ cm

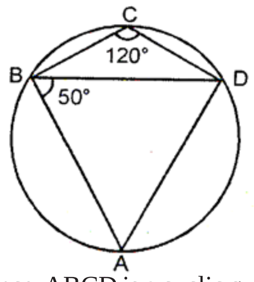
$\therefore$ In DABC $\quad BC^2 = AB^2 - AC^2$

$\Rightarrow \quad BC^2 = 169 - 144$

$\Rightarrow \quad BC^2 = 25$

$\Rightarrow \quad BC = 5$ cm. Ans.

Q.9. In fig. ABCD is a cyclic quadrilateral. If ∠BCD = 120° and ∠ABD = 50°, then find ∠ADB.



Ans. Since, ABCD is a cyclic quadrilateral.

$\therefore \quad \angle BCD + \angle BAD = 180°$

$\Rightarrow \quad 120° + \angle BAD = 180°$

$\Rightarrow \quad \angle BAD = 60°$

Now, in DABD, we have

$\angle ABD + \angle BAD + \angle ADB = 180°$

$\Rightarrow \quad 50° + 60° + \angle ADB = 180°$

$\Rightarrow \quad \angle ADB = 70°$

Multiple Choice Questions

Q.1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB 30 cm, the distance of AB from the centre of the circle is :

(a) 17 cm (b) 15 cm
(c) 4 cm (d) 8 cm

Ans. (d) 8 cm

Q.2. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribign it and $\angle ADC = 140°$, then $\angle BAC$ is equal to :

(a) 80° (b) 50°
(c) 40° (d) 30°

Ans. (b) 50°

Q.3. There are three non-collinear points. The number of circles passing through them are :

(a) 2 (b) 1
(c) 3 (d) 4

Ans. (b) There is one and only one cirlce passing through three given non-collinear points.

Q.4. Three chords AB, CD and EF of a circle are respectively 3 cm, 3.5 cm and 3.8 cm away from the centre. Then which of the following relatiosn is correct ?

(a) $AB > CD > EF$ (b) $AB < CD < EF$
(c) $AB = AD = EF$ (d) None of these

Ans. (c) We kow that longer the chord, shorter is its distance from the centre.

Q.5. Given three collinear points, then the number of circles which canbe drawn through three points is :

(a) Zero (b) One
(c) Two (d) Infinite

Ans. (a) Zero

Q.6. The distance of a chord 8 cm long from the centreof a circle of radius 5 cm is :

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 9 cm

Ans. (b) 3 cm

Q.7. The distance of chord length 16 cm from the centre of the circle of radius 10 cm is :

(a) 6 cm (b) 8 cm
(c) 10 cm (d) 12 cm

Ans. (a) 6 cm

Q.8. The distance of a chord 8 cm long from the centre of a cirlc eof radius 5 cm is :

(a) 4 cm (b) 3 cm
(c) 2 cm (d) 9 cm

Ans. (b) 3 cm

Q.9. Chord AB subtends $\angle AOB = 60°$ at centre. If OA = 5 cm then length of AB (in cm) is :

(a) $\frac{5}{2}$ cm (b) $\frac{5\sqrt{3}}{2}$ cm

(c) 5 cm (d) $\frac{25\sqrt{3}}{4}$

Ans. (c) 5 cm

Q.10. The region between an arc and two radii, joining the centre to the end points of the arc is called :

(a) Sector (b) Segment
(c) Semicircle (d) None of the above

Ans. (a) Sector