Chapter 6

Lines and Angles

In the Chapter

- In this chapter, you have studied the following points:
  1. If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and vice-versa. This property is called as the Linear pair axiom.
  2. If two lines intersect each other, then the vertically opposite angles are equal.
  3. If a transversal intersects two parallel lines, then
     (i) each pair of corresponding angles is equal,
     (ii) each pair of alternate interior angles is equal,
     (iii) each pair of interior angles on the same side of the transversal is supplementary.
  4. If a transversal intersects two lines such that, either
     (i) any one pair of corresponding angles is equal, or
     (ii) any one pair of alternate interior angles is equal, or
     (iii) any one pair of interior angles on the same side of the transversal is supplementary, then the lines are parallel.
  5. Lines which are parallel to a given line are parallel to each other.
  6. The sum of the three angles of a triangle is 180°.
  7. If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
- **Segment :** A part of a line with two end points is called a *line segment.*
  A line-segment is denoted by $\overline{AB}$ and its length is denoted by AB.
- **Ray :** A part of a line with one end-point, is called a ray.
  A ray AB is denoted by $\overrightarrow{AB}$.
  We can denote a line segement AB, ray AB and length AB and line AB by the same symbol AB. Sometimes small letters *i, m, n* etc. will be used to denote lines.
- **Angle :** An angle is formed by two rays originating from the same end point.
  The rays making an angle are called the arms of the angle and the end-point is called the vertex of the angle.
- **Collinear Points :** If three or more points lie on the same line, then they are called *collinear points,* otherwise they are called *non-collinear points.*
- **Types of Angles :**
  (i) Acute angles : An angle whose measure lies between 0° and 90°, is called an acute angle.
  (ii) Right angle : An angle whose measure is equal to 90°, is called a *right angle*.
  (iii) Obtuse angle : An angle whose measure lies between 90° and 180°, is called an *obtuse angle.*

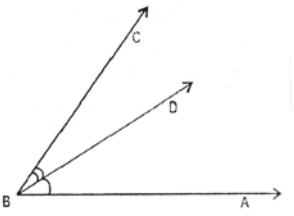
(iv) Straight angle : The measure of a straight angle is 180°.

(v) Reflex angle : An angle which is greater than 180° and less than 360°, is called a reflex angle.

(vi) Complimentary angles : Two angles whose sum is 90°, are called *complimentary angles*.

(vii) Supplementary angles : Two angles who sum is 180°, are called *supplementary angles*.

(viii) Adjacent angles : Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm.

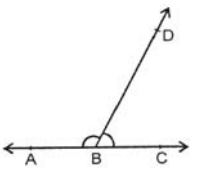


In the fig. ∠ABD and ∠DBC are adjacent angles. Ray BD is their common arm and point B is their common vertex. Ray BA and ray BC are non-common arms.

When the two angles are adjacent, then their sum is always equal to the angle formed by the two non-common arms.

Thus $\angle ABC = \angle ABD + \angle DBC$

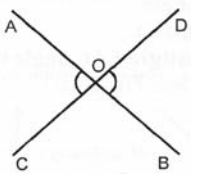
We see that ∠ABC and ∠DBC are not adjacent angles, because their non-common arms BD and AB lie on the same side of the common arm BC.



(ix) Linear pair of angles : If the sum of two adjacent angles is 180°, then their non-common arms are in the same straight lines and the two adjacent angles form a linear pair of angles.

In the fig. ∠ABD and ∠CBD form a linear pair of angles because

$\angle ABD + \angle CBD = 180°$

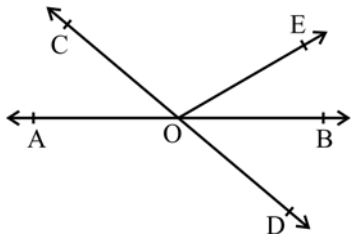


(x) Vertically opposite angles : When two lines AB and CD intersect at a point O, then vertically opposite angles are formed.

Here are two pairs of vertically opposite angles. One pair is ∠AOC and ∠BOD and the second pair is ∠AOD and ∠BOC.

EXERCISE 6.1

Q.1. In Fig. lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Ans. Given :

$\angle AOC + \angle BOE = 70°$

and $\angle BOD = 40°$

$\angle AOC = \angle BOD$ (Vertically opp. side)

Now,

$\angle AOC + \angle BOE + \angle COE = 180°$ (Angles on a line)

$\therefore \quad 70° + \angle COE = 180°$

or $\angle COE = 180° - 70°$

$\angle COE = 110°$ ...(i)

Reflex $\angle COE = 360° - 110° = 250°$.

Again $\angle AOC + \angle BOC = 70°$

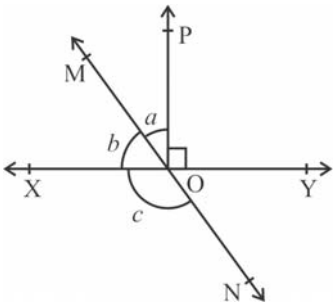
$\therefore \quad 40° + \angle BOE = 70°$

or $\angle BOE = 70° - 40° = 30°$

Hence $\angle BOE = 30°$

and reflex $\angle COE = 250°$.

Q.2. In the given fig., line XY and MN intersect at O. If ∠POY = 90° and *a* : *b* = 2 : 3, find c.



Ans. We have given that lines XY and MN intersect at O and ∠POY = 90°.

Also, given $a : b = 2 : 3$

Let $a = 2x$ and $b = 3x$

Since, ∠XOM + ∠POM + ∠POY = 180°

[Linear pair axiom]

$\Rightarrow 3x + 2x \times 90° = 180°$

$\Rightarrow 5x = 180° - 90°$

$\Rightarrow x = \frac{90°}{5} = 18°$

$\therefore$ ∠XOM = $b = 3x$

$= 3 \times 18° = 54°$

and ∠POM = $a = 2x$

$= 2 \times 18° = 36°$

Now, ∠XON = c = ∠MOY

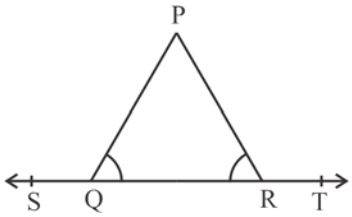
= ∠POM + ∠POY

[Vertically opposite angles]

= 36° + 90° = 126°

Hence, $c = 126°$

Q.3. In Fig., ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



Sol. Given : As in the figure,

∠PQR = ∠PRQ

To prove : ∠PQS = ∠PRT

Proof : ∠PQR + ∠PQS = 180° ...(i)

(Linear Pair)

and ∠PRA + ∠PRT = 180° ... (ii)

(Linear Pair)

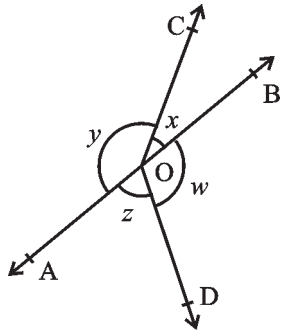
From (i) and (ii), we get

∠PQR + ∠PQS = ∠PRQ + ∠PRT

But ∠PQR = ∠PRQ (Given)

Hence ∠PQS = ∠PRT. Proved.

Q.4. In fig., if x + y = w + z, then prove that AOB is a line.



Ans. Given : AOB in the figure,

$x + y = w + z$

To prove. AOB is a straight line.

Proof. $x + y = w + z$

But $x + y + w + z = 360°$

[Angles around a point]

$\therefore (x + y) + (x + y) = 360°$

$2x + 2y = 360°$

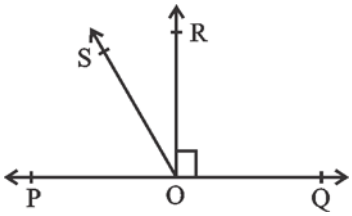
or $x + y = 180°$

But x and y form a linear pair.

Hence AOB is a straight line. **Proved.**

Q.5. In Fig , POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR.

Prove that ∠ROS = $\frac{1}{2}$ (∠ QOS – ∠ POS).



Ans. Given : In the adjacent figure POQ is a straight line. Another ray meets line PQ at O.

To prove. $\angle RPS = \frac{1}{2}(\angle QOS - \angle POS)$

Proof. ∠QOS = ∠SOR + ∠ROQ ... (i)

and ∠POS = ∠POR – ∠SOR ... (ii)

From (i) and (ii) on subtraction.

∠QOS – ∠POS = ∠SOR + ∠ROQ – ∠POR + ∠SOR

or ∠QOS – ∠POS = ∠ROQ – ∠POR + 2∠SOR

But ∠ROQ = ∠POR = 90° (Given)

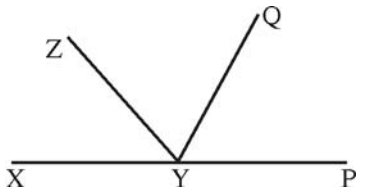
$\therefore$ ∠QOS – ∠POS = 2∠SOR

or $2\angle ROS = \angle QOS - \angle POS$

Hence $\angle SOR = \frac{1}{2}[\angle QOS - \angle POS]$.

Q.6. It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Ans.



Given : $\angle XYZ = 64°$

Now $\angle XYZ + \angle ZYP = 180°$ (Linear pair)

$64° + \angle ZYP = 180°$

$\angle ZYP = 180° - 64°$

$\angle ZYP = 116°$

But YQ is the bisector of $\angle QYP$

$\therefore \angle ZYQ = \angle PYQ = \frac{116°}{2} = 58°$

$\therefore \angle XYQ = 64° + 58°$

$= 122°$

Now reflex $\angle QYP = 360° - 58°$

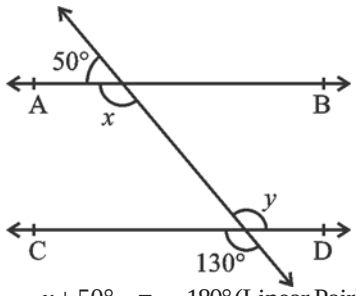
$= 302°$

Hence $\angle XYQ = 122°$

and reflex $\angle QYP = 302°$

EXERCISE 6.2

Q.1. In Fig., find the values of *x* and *y* and then show that AB || CD.



Ans. $x + 50° = 180°$ (Linear Pair)

$\therefore x = 180° - 50°$

$x = 130°$

$\therefore y = 130°$

(Vertically opp. Angles)

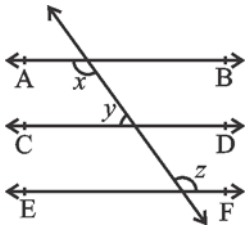
Hence $x = 130°$ and $y = 130°$

Now $x = y$

But these are alternate interior angles,

Threfore, AB || CD.

Q.2. In Fig., if AB || CD, CD || EF and $y : z = 3 : 7$, find *x*.



Ans. In the given Fig., AB || CD, CD || EF and $y : z = 3 : 7$.

Let $y = 3a$ and $z = 7a$

$\angle DHI = y$

[Vertically opp. angles]

$\angle DHI + \angle FIH = 180°$

[Interior angles on the same side of the transversal]

$\Rightarrow y + z = 180°$

$\Rightarrow 3a + 7a = 180°$

$\Rightarrow 10a = 180°$

$\Rightarrow a = 180°$

$\therefore y = 3 \times 18° = 54°$

and $z = 18° \times 7 = 126°$

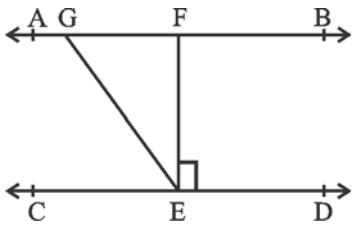
Also, $x + y = 180°$

$\Rightarrow x + 54° = 180°$

$\therefore x = 180° - 54° = 126°$

Hence, $x = 126°$

Q.3. In Fig. 6.30, if AB || CD, EF ⊥ CD and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$ and $\angle FGE$.



Ans. $\angle DEF = 90°$

and $\angle GED = 126°$

$\therefore \angle DEF + \angle GEF = 126°$

$90° + \angle GEF = 126°$

$\angle GEF = 126° - 90° = 36°$

$\angle AGE = \angle GEF + \angle EFG$ (Exterior Angle Theorem)

But $\angle EFG = \angle FED$ (Alternate Angles)

But $\angle FED = 90°$

$\therefore \angle EFG = 90°$

$\therefore \angle AGE = 36° + 90° = 126°$

Also, $\angle AGE + \angle EGF = 180°$ (Linear Pair)

$126° + \angle EGF = 180°$

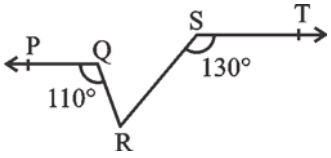
$\angle EGF = \angle 180° - 126° = 54°$

Hence, $\angle AGE = 126°$

$\angle GEF = 36°$ and $\angle FGE = 54°$

Q.4. In Fig., if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

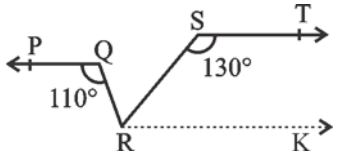
[**Hint :** Draw a line parallel to ST through point R.]



Ans. Given. PQ || ST,

$\angle POR = 110°$

$\angle RST = 130°$



Required. To Find ∠QRS.

Construction. Draw RK || RK

Steps of Solution.

ST ‖ RK

$\therefore \angle TSR + \angle KRS = 180°$ (Interior angles on the same side of transversal)

But $\angle TSR = 130°$

$\therefore 130° + \angle KRS = 180°$

$\angle KRS = 180° - 130° = 50°$

Again PQ || RK as PQ || ST.

$\therefore \angle PQR = \angle QRK$

$110° = \angle QRS + \angle SRK$

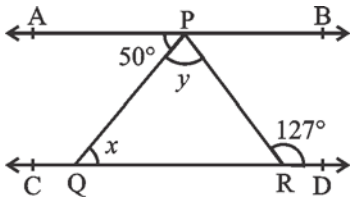
$110° = \angle QRS + 50°$

$\therefore 110° - 50° = \angle QRS$

or $\angle QRS = 60°$

Hence $\angle QRS = 60°$

Q.5. In Fig., if AB || CD, ∠ APQ = 50° and ∠PRD = 127°, find *x* and *y*.



Ans. AB ‖ CD

$\therefore x = 50°$ (Alternate Angles)

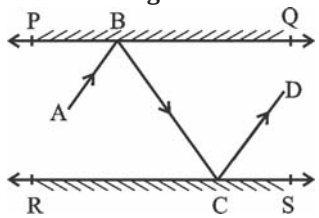
Also $x + y = 127°$ (Exterior angle theorem)

or $50° + y = 127°$

or $y = 127° - 50° = 77°$

Hence $x = 50°, y = 77°$

Q.6. In Fig., PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



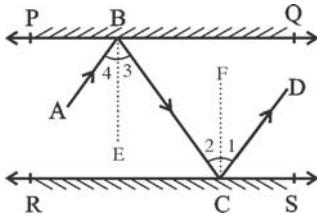
Ans. Const. At point B, draw BE || PQ and at point C, draw CF || RS.

$\angle 1 = \angle 2$ ...(i)

[Angle of incidence is equal to angle of reflection]

$\angle 3 = \angle 4$ ...(ii)

[Same reasons]



Also $\angle 2 = \angle 3$ ...(iii)

[Alternate angles]

$\Rightarrow \angle 1 = \angle 4$

[From (i), (ii) and (iii)]

$\Rightarrow 2\angle 1 = 2\angle 4$

$\Rightarrow \angle 1 + \angle 1 = \angle 4 + \angle 4$

$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$

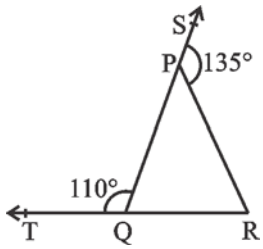
[From (i) and (ii)]

$\Rightarrow \angle BCD = ABC$

Hence, AB || CD [Alternate angles are equal.]

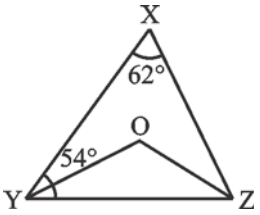
EXERCISE 6.3

Q.1. In Fig., sides QP and RQ of ∠PQR are produced to points S and T respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.



Ans. ∠TPQ + ∠PQR = 180° (Linear pair)
But ∠TQP = 110°
∴ 110° + ∠PQR = 180°
or ∠PQR = 180° – 110°
or ∠PQR = 70°
Now in ΔPQR, QP is produced to S forming ∠SPR, an exterior angle.
∴ ∠PQR + ∠PRQ = ∠SPR
70° + ∠PRQ = 135°
or ∠PRQ = 135° – 70° = 65°
Hence ∠PRQ = 65°.

Q.2. In Fig., ∠ X = 62°, ∠ XYZ = 54°. If YO and ZO are the bisectors of ∠ XYZ and ∠ XZY respectively of ∠ XYZ, find ∠OZY and ∠ YOZ.



Ans. In ΔXYZ,
∠X + ∠XYZ + ∠XYZ = 180°
But ∠X = 62°
and ∠XYZ = 54°
62° + 54° + ∠XYZ = 180°
or ∠XYZ = 180° – 62° – 54
or ∠XYZ = 180° – 116° = 64°
Now ZP is the bisector of ∠XZY

$$\angle OZY = \frac{1}{2}\angle XZY \quad \frac{1}{2} \times 64° = 32°$$

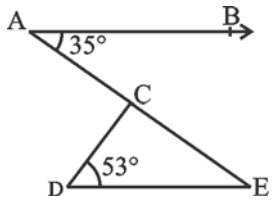
Also YO is the bisector of ∠XYZ

$$\angle OYZ = \frac{1}{2}\angle XYZ$$

$$= \frac{1}{2} \times 54° = 27°$$

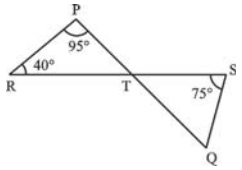
Now in ΔOYZ,
∠OYZ + ∠YOZ + ∠OZY = 180°
or 27° + ∠YOZ + 32° = 180°
or ∠YOZ + 59° = 180°
or ∠YOZ = 180° – 59°
or ∠YOZ = 121°
Hence ∠OZY = 32°
and ∠YOZ = 121°

Q.3. In Fig., if AB || DE, ∠ BAC = 35° and ∠CDE = 53°, find ∠ DCE.



Ans. Construction : Through C, draw CE || CD
As AB ‖ DE (Given)
∴ CD ‖ DE (By construction)
Now, AB || EC and AC intersects them,
∠1 = 35° ...(i)
(Alternate angles)
Also CE || DE and CD intersects them,
∠2 = 53° ...(ii)
(Alternate angles)
Now ACE is a straight line,
∠1 + ∠2 + ∠DCE = 180°
or 35° + 53° + ∠DCE = 180°
or 88° + ∠DCE = 180°
or ∠DCE = 180° – 88° = 92°
Hence ∠DCE = 92°

Q.4. In Fig., if lines PQ and RS intersect at point T, such that ∠PRT = 40°, ∠ RPT = 95° and ∠TSQ = 75°, find ∠SQT.



Ans. In ΔPRT

$\angle P + \angle R + \angle PTR = 180°$

But $\angle P = 95°$

$\angle R = 40°$

$95° + 40° = 180°$

or $135° + \angle PTR = 180°$

or $\angle PTR = 180° - 135°$

or $\angle PTR = 45°$

Also $\angle PTR = \angle STQ$ (Vertically opposite angles)

$\therefore \angle STQ = 45°$

Again in ΔTSQ,

$\angle STQ + \angle SQT + \angle S = 180°$

But $\angle S = 75°$

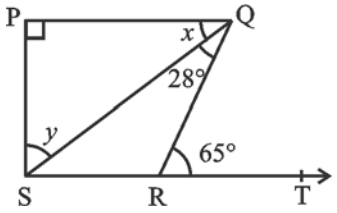
or $\angle 45° + \angle SQT + 75° = 180°$

or $\angle SQT + 120° = 180°$

or $\angle SQT = 180° - 120° = 60°$

Hence $\angle SQT = 60°$

Q.5. In Fig. 6.43, if PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°, then find the values of *x* and *y*.



Ans. In the given Fig., lines PQ ⊥ PS, PQ || SR, ∠SQR = 28° and ∠QRT = 65°

$\angle PQR = \angle QRT$ [Alternate angles]

$\Rightarrow x + 28° = 65°$

$\Rightarrow x = 65° - 28° = 37°$

$\angle SQR + \angle PQS + \angle QSP = 180°$ [Angle sum property of a triangle]

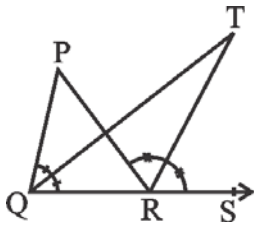
$\Rightarrow 90° + 37° + y = 180°$ [PQ ⊥ PS, $\angle PQS = x = 37°$ and $\angle QSP = y$]

$\Rightarrow 127° + y = 180°$

$\Rightarrow y = 180° - 127° = 53°$

Hence, $x = 36°$ and $y = 53°$

Q.6. In Fig., the side QR of ΔPQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠ QTR = $\frac{1}{2}$ ∠QPR.



Ans. Given : The side QR of ΔPQR is produced to a point S. The bisectors of ∠PQR and ∠PRS meet at point T.

To prove : $\angle QTR = \frac{1}{2}\angle QPR$

Proof : In ΔPQR, its side QR is produced to S.

$\therefore \angle PRS = \angle P + \angle PQR$ [Exterior Angle Theorem]

or $\angle 3 + \angle 4 = \angle 1 + \angle 2 + \angle P$

or $2\angle 3 = 2\angle 2 + \angle P$ [$\angle 3 = \angle 4$ and $\angle 1 = \angle 2$]

or $\angle P = 2(\angle 3 - \angle 2)$ ...(i)

Now in ΔQRT, its side QR is produced to S.

$\therefore \angle 4 = \angle 2 + \angle QTR$ (Exterior Angle Theorem)

or $\angle 3 = \angle 2 + \angle QTR$ [$\angle 4 = \angle 3$]

or $\angle T = \angle 3 - \angle 2$ ... (ii)

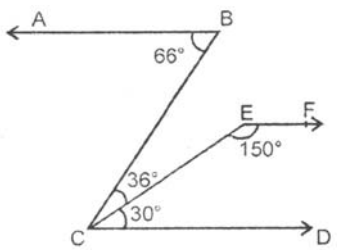
From (i) and (ii), we get

$\angle P = 2\angle T$

Hence $\angle T = \frac{1}{2}\angle P$ Proved.

Additional Questions

Q.1. In the given fig., prove that : AB || EF.



Ans. AB ∥ CD

[66° = 36° + 30° (Alternate angles)]

CD ∥ EF [30° + 150° = 180°]

[Sum of interior angles in 180°]

∴ AB ∥ EF

[If two lines are parallel to the same line then they are parallel to each other]

Q.2. Let OA, OB, OC and OD are rays in the anticlockwise direction such that ∠AOB = ∠COD

= 100°, ∠BOC = 80° and ∠AOD = 78°. Is it true to say that AOC and BOD are lines ?

Ans. AOC is not a line, because ∠AOB + ∠COB = 100° + 82° = 182°, which is not equal to 180°. Similarly, BOD is also not a line.

Q.3. A transversal intersects two lines in such a way that the two interior angles on the same side of the transversal are equal. Will the two lines always be parallel?

Ans. In general, the two lines will not be parallel, because the sum of the two equal angles will not always be 180°. Lines will be parallel when each equal angle is equal to 90°.

Q.4. If the angles of a triangle are in the ratio 2 : 3 : 4, find the angles of the triangle.

Ans. Let the angles of a triangle are $2x$, $3x$ and $4x$ respectively. By angle sum property of a triangle.

$\therefore \quad 2x + 3x + 4x = 180°$

$\Rightarrow \quad 9x = 180° \Rightarrow x = 20°$

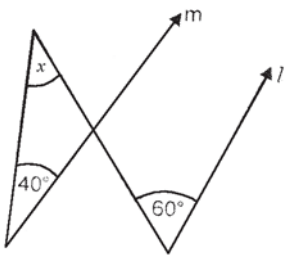
Thus, angles of triangle are :

1^{st} angle $= 2x = 2 \times 20 = 40°$

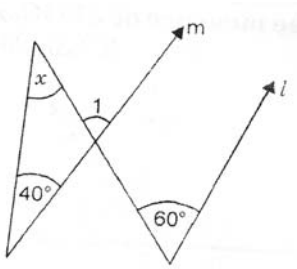
2^{nd} angle $= 3x = 3 \times 20 = 60°$

3^{rd} angle $= 4x = 4 \times 20 = 80°$

Q.5. In figure, if *l* || *m*, then find the value of *x*.



Ans. $l \parallel m$



$\therefore \quad \angle 1 = 60°$ (Corresponding angle)

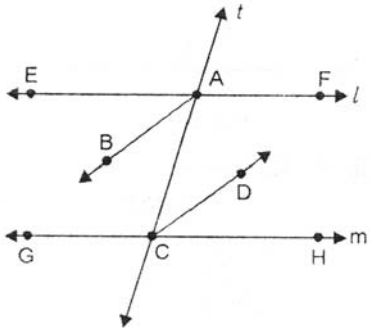
Now $\quad \angle 1 = x + 40°$

[Exterior angle property]

$\Rightarrow \quad 60° = x + 40°$

$\Rightarrow \quad x = 20°$

Q.6. AB and CD are the bisectors of the two alternate interior angles formed by the intersection of a transversal 't' with parallel lines *l* and *m* (Shown in figure). Show that AB || CD.



Ans. Given : $l \parallel m$, AB and CD are bisectors of alternate angles.

To Prove : AB || CD.

Proof : $\angle EAC = \angle HCA$

[Alternate interior angles]

$\Rightarrow \quad \frac{1}{2}\angle EAC = \frac{1}{2}\angle HCA$

$\Rightarrow \quad \angle BAC = \angle ACD$

[AB and CD are bisectors]

But this is a pair of alternate angles with transversal '*t*'.

Hence, AB || CD. Proved.

Q.7. How many triangles can be drawn having its angles as 53°, 64° and 63°? Given reason for your answer.

Ans. Infinitely many triangles, because sum of all interior angles of every triangle is always 180°.

Q.8. Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer.

Ans. No, because each of these will be a right angle only when they form a linear pair.

Q.9. If one of the angles formed by two interesecting lines is a right angle, what can you say about the other three angles? Give reason for your answer.

Ans. Each will be 90° because of linear pair axiom.

Q.10. Two lines *l* and *m* are perpendicular at the same line *n*. Are *l* and *m* perpendicular to each other? Give reason for your answer.

Ans. No, they are parallel.

Multiple Choice Questions

Q.1. An exterior angle of a triangle is 110° and the two interior opposite angles are equal. Each of these angles is :

(a) 70° (b) 55°
(c) 35° (d) 110°

Ans. (b)

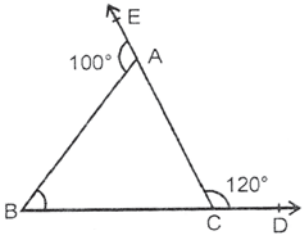
Q.2. If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3, then the greater of the two angles

(a) 54° (b) 108°
(c) 120° (d) 136°

Ans. (b)

Q.3. In the given Fig., if ∠BAE = 110°, ∠DCA = 120°, then measure of ∠ABC is :

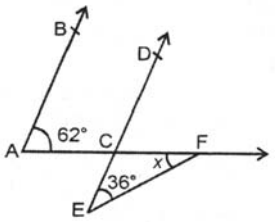
(a) 45° (b) 50°
(c) 120° (d) 30°



Ans. (b)

Q.4. In the given figure, AB || ED, the value of *x* is :

(a) 36° (b) 26°
(c) 62° (d) 54°



Ans. (b)

Q.5. The complement of an angle *m* is :

(a) m (b) $90° + m$
(c) $90° - m$ (d) $m \times 90°$

Ans. (c)

Q.6. The complementary angles are in the ratio 1 : 5. Find the measures of the angles:

(a) 15,75 (b) 75,15
(c) 12,60 (d) 60, 12

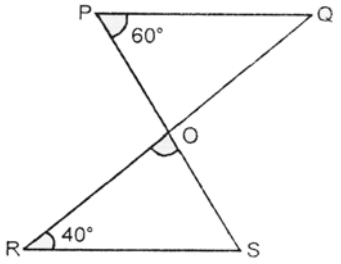
Ans. (a)

Q.7. If the measure of an angle is twice the measure of its supplementary angle then the measure of the angle is :

(a) 60° (b) 90°
(c) 120° (d) 130°

Ans. (a)

Q.8. In the given figure, OP = OQ, ∠P = 60° and ∠R = 40°, then measure of ∠SQR is :

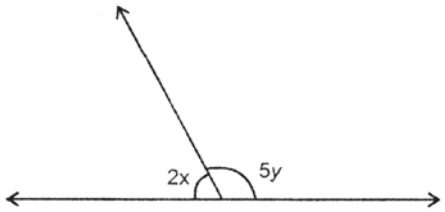


(a) 100° (b) 80°
(c) 40° (d) 50°

Ans. (b)

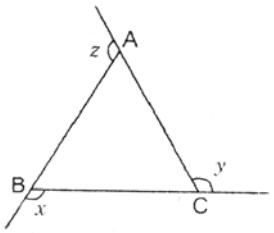
Q.9. In the fig, a ray stands on a line. If *x* = 30°, then *y* is equal to :

(a) 20° (b) 24°
(c) 25° (d) 36°



Ans. (b)

Q.10. In the figure below, if *x*, *y* and *z* are exterior angles of ΔABC, then $x + y + z$ is :



(a) 180° (b) 360°
(c) 270° (d) 90°

Ans. (b)