Chapter 4

Linear Equations in Two Variables

In the Chapter

- An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in two variables.
- A linear equation in two variables has infinitely many solutions.
- The graph of every linear equation in two variables is a straight line.
- $x = 0$ is the equation of the y-axis and $y = 0$ is the equation of the x-axis.
- The graph of $x = a$ is a straight line parallel to the y-axis.
- The graph of $y = a$ is a straight line parallel to the x-axis.
- An equation of the type $y = mx$ represents a line passing through the origin.
- Every point on the graph of a linear equation in two variables is a solution of the linear equation. Moreover, every solution of the linear equation is a point on the graph of the linear equation.
- The pair of values of x and y which satisfies the given equation is called solution of the equation in two variables.

  Example : Consider the equation

  $$2x + 3y = 5$$

  We see that $x = 1$, $y = 1$ satisfies the equation

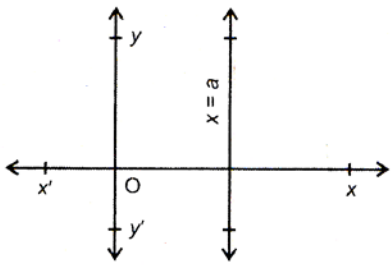
  i.e. $\text{L.H.S.} = 2 \times 1 + 3 \times 1 = 5 = \text{R.H.S.}$

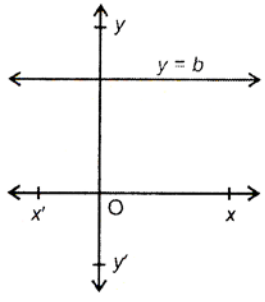
  $\therefore$ The solution of the above equation is the ordered pair (1, 1)

  Also putting $x = 5y = -\frac{5}{3}$ in $2x + 3y = 5$

  We have $\text{L.H.S.} = 2 \times 5 + 3\left(\frac{-5}{3}\right) = 10 - 5 = 5 = \text{R.H.S.}$

- Equation of a line parallel to the y-axis at a distance a from it is $x = a$.





- Equation of a line parallel to the x-axis at a distance b from it is $y = b$.

NCERT TEXT BOOK QUESTION (SOLVED)

EXERCISE 4.1

Q.1.The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Hint. Take the cost of a notebook to be Rs. *x* and that of a pen to be Rs. *y*).

Ans. According to the question, the cost of a notebook is twice the cost of a pen.

$\therefore \quad x = 2y$

$\Rightarrow \quad x - 2y = 0$

Q.2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of *a, b* and *c* in each case:

(i) $2x + 3y = 9.\overline{35}$ (ii) $x - \frac{y}{5} - 10 = 0$

(iii) $-2x + 3y = 6$ (iv) $x = 3y$

(v) $2x = -5y$ (vi) $3x + 2 = 0$

(vii) $y - 2 = 0$ (viii) $5 = 2x$

Ans. (i) $2x + 3y = 9.35$

$2x + 3y - 9.35 = 0$

Comparing with $ax + by + c = 0$, we have

$\boldsymbol{a = 2, b = 3, c = 9.\overline{35}}$

(ii) $x - \frac{y}{5} - 10 = 0$

Comparing with

$ax + by + c = 0$

$a = 1, b = -\frac{1}{5}, c = 10.$

(iii) $-2x + 3y = 6$

$-2x + 3y - 6 = 0$

Comparing with $ax + by + c = 0$, we have

$\boldsymbol{a = 2, b = 3, c = -6}$

(iv) $x = 3y$

$x - 3y = 0$

Comparing with ax + by + c = 0. we have

$\mathbf{a = 1, b = -3, c = 0}$

(v) $2x = -5y$

$2x = -5$

$\Rightarrow \quad 2x + 5y + 0 = 0$

Comparing with $ax + by + c = 0$, we have

$\boldsymbol{a = 2, b = 5, c = 0}$

(vi) $3x + 2 = 0$

$3x + 0y + 2 = 0$

Comparing with $ax + by + c = 0$, we have

$\boldsymbol{a = 3, b = 0, c = 2}$

(vii) $y - 2 = 0$

$0x + 1y - 2 = 0$

Comparing with $ax + by + c = 0$, we have

$\boldsymbol{a = 0, b = 1, c = -2}$

(viii) $5 = 2x$

or $-2x - 0y + 5 = 0$

Comparing with $ax + by + c = 0$, we have

$\boldsymbol{a = -2, b = 0, c = 5}$

EXERCISE 4.2

Q.1. Which one of the following options is true, and why?

$y = 3x + 5$ has

(i) a unique solution,

(ii) only two solutions,

(iii) infinitely many solutions

Ans. $y = 3x + 5$

It is a linear equation in two variables. We know that a linear equation in two variables has infinite solutions.

Hence option (iii) is correct.

Q.2. Write four solutions for each of the following equations:

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$

Ans. (i) $2x + y = 7$

or $y = 7 - 2x$

When $x = 0, y = 7$

When $x = 1, y = 7 - 2 \times 1 = 5$

When $x = 2, y = 7 - 2 \times 2 = 7 - 4 = 7$

When $x = 3, y = 7 - 2 \times 3 = 1$

Hence four solutions are (0, 7), (1, 5), (2, 3) and (3, 1)

(ii) $\pi x + y = 9$

or $y = 9 - \pi x$

When $x = 0, y = 9$

When $x = -1, y = 9 - 17 \times 1 = 9 - \pi$

When $x = -1, y = 9 - \pi(-1) = 9 + \pi$

When $x = 2, y = 9 - \pi(2) = 9 - 2\pi$

Hence four solutions are $(0, 9), (0, 9 - \pi), (-1, 9 + \pi)$ and $(2, 9 - 2\pi)$

(iii) $x = 4y$

When $y = 0, x = 0$

When $y = 1, x = 4$

When $y = -1, x = -4$

When $y = 2, x = 8$

Hence four solutions of the given equations are (0, 0), (4, 1), (–4, –1) and (8, 2)

Q.3. Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:

(i) (0, 2) (ii) (2, 0) (iii) (4, 0)

(iv) $(\sqrt{2}, 4\sqrt{2})$ (v) (1, 1)

Ans. (i) $x - 2y = 4, (0, 2)$

L.H.S. $= (x - 2y)$

$= 0 - 2 \times 2$

$= -4 \neq$ R.H.S.

$\therefore$ (0, 2) is not a solution of $x - 2y = 4$.

(ii) $x - y = 4, (2, 0)$

L.H.S. $= x - 2y$

$= 2 - 2 \times 0 = 2$

$=$ R.H.S.

$\therefore$ (2, 0) is a solution of $x - 2y = 4$.

(iii) $x - 2y = 4, (4, 0)$

L.H.S. $= x - 2y$

$= 4 - 2 \times 0 = 4$

$=$ R.H.S.

$\therefore$ (4, 0) is a solution of $x - 2y = 4$.

(iv) $x - 2y = 4, (\sqrt{2}, 4\sqrt{2})$

L.H.S. $= x - 2y$

$= \sqrt{2} - 2 \times 4\sqrt{2}$

$= 2 - 8\sqrt{2} = -7\sqrt{2}$

$=$ R.H.S.

$\therefore$ $(2, 4\sqrt{2})$ is not a solution of $x - 2y = 4$.

(v) $x - 2y = 4, (1, 1)$

L.H.S. $= x - 2y$

$= 1 - 2 \times 1$

$= 1 - 2 = -1$

$=$ R.H.S.

Hence (1, 1) is not a solution of $x - 2y = 4$.

Q.4. Find the value of k, if $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$.

Ans. $x = 2, y = 1$ is a solution of the equation

$2x + 3y = k$

$\therefore \quad 2 \times 2 + 3 \times 1 = k$

$\Rightarrow \quad 4 + 3 = k$

$\Rightarrow \quad 7 = k$

Hence $k = 7$.

EXERCISE 4.3

Q.1. Draw the graph of each of the following linear equations in two variables:

(i) $x + y = 4$ (ii) $x - y = 2$

(iii) $y = 3x$ (iv) $3 = 2x + y$

Ans. (i) $x + y = 4$

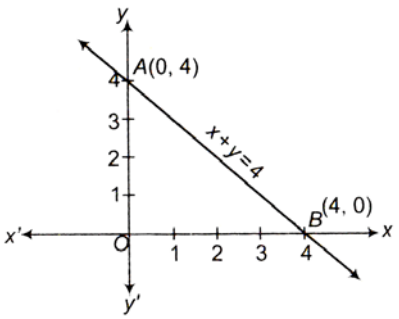
or $y = 4 - x$

Now we take certain values of x and find their corresponding values of y

For $x = 0, y = 4 - 0 = 4$

$x = 1, y = 4 - 1 = 3$

$x = 2, y = 4 - 2 = 2$



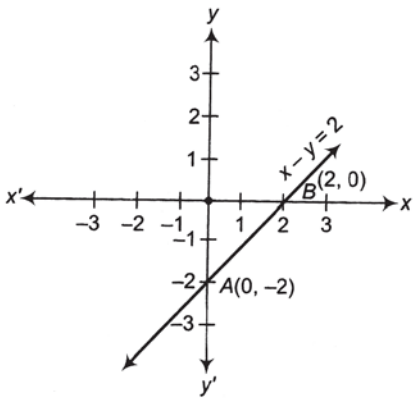
$\therefore$ Table is given below :

	A	B	C
x	0	1	2
y	4	3	2

Now we plot the points A(0,4), B(1, 3) and C(2, 2) on the graph and join them to get the straight line which represents the linear equation $x + y = 4$.

(ii) $x - y = 2$

$x = 2 + y$



Now consider certain values of y and find their corresponding values of x.

When $y = 0, x + 2 + 0 = 2$

$y = 1, x = 2 + 1 = 3$

$y = 2, x = 2 + 2 = 4.$

The corresponding table is

	P	Q	R
x	2	3	1
y	0	1	2

Now we plot the points P(2, 0), Q(3, 1) and R(4, 2) on the graph paper and thus we get the corresponding line of the linear equation $x - y = 2$.

(iii) $y = 3x$

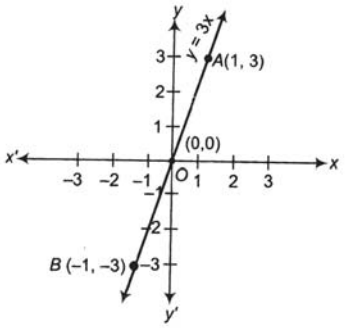
It is a line passing through the origin O(0, 0). Now we consider certain values of x and find the corresponding values of y

When $x = 0, y = 3 \times 0 = 0$

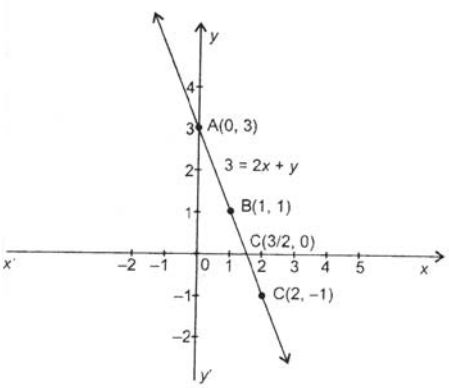
$x = 1, y = 3 \times 1 = 3$

$x = -13, y = 3 \times -1 = -1$

	O	A	B
x	0	1	–1
y	0	3	–3



(iv) We have: $3 = 2x + y$



Put $x = 0, 3 = 0 + y$

$\Rightarrow y = 3$

Put $x = 1, 3 = 2 \times 1 + y$

$\Rightarrow y = 1$

Put $x = \frac{3}{2}, 3 = 2 \times \frac{3}{2} + y$

$\Rightarrow y = 0$

	A	B	C
x	0	1	3/2
y	3	1	0

Q.2. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

Ans. Clearly $x = 2, y = 14$

$\therefore$ One equation is $y = 7x$

Second equation is $x + y = 16$

In fact we can find infinite equations because through one point infinite lines pass.

Q.3. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.

Ans. Hence, $3(4) - 1(3) - 7 = 0$

$\Rightarrow 12 - 3a - 7 = 0$

$\Rightarrow 5 - 3a = 0$

$\Rightarrow 3a = 5$

$\Rightarrow a = \frac{5}{3}$

Q.4. The taxi fare in a city is as follows: For the first kilometre, the fare is Rs. 8 and for the subsequent distance it is Rs. 5 per km. Taking the distance covered as x km and total fare as Rs y, write a linear equation for this information, and draw its graph.

Ans. Taxi fare for first kilmoter = Rs. 8

Taxi fare for subsequent distance = Rs. 5

Total distance covered = x

Total fare = y

Fare for first kilometer = Rs. 8

Fare for (x –1) kilometer = $5(x–1)$

Total fare = $5(x–1) + 8$

$\therefore y = 5(x–1) + 8$

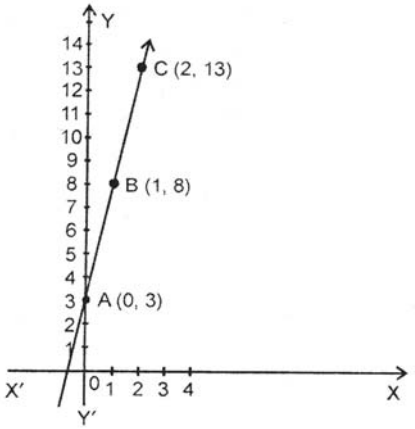
or $y = 5x – 5 + 8$

Hence $y = 5x + 3$ is the required linear equation.

Now we draw the graph of

$y = 5x + 3$

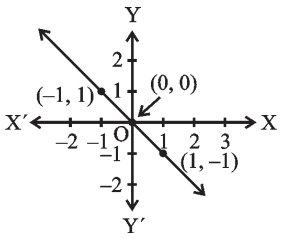
	A	B	C
x	0	1	2
y	3	8	13



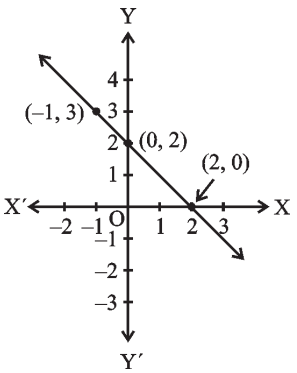
Q.5. From the choices given below, choose the equation whose graphs are given in Fig. (a) and Fig. (b).

For Fig. (a)
(i) $y = x$
(ii) $x + y = 0$
(iii) $y = 2x$
(iv) $2 + 3y = 7x$

For Fig. (b)
(i) $y = x + 2$
(ii) $y = x - 2$
(iii) $y = -x + 2$
(iv) $x + 2y = 6$



(a)



(b)

Ans. Fig. (a), $x + y = 0$ and for Fig. (b), $y = -x + 2$.

Q.6. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is

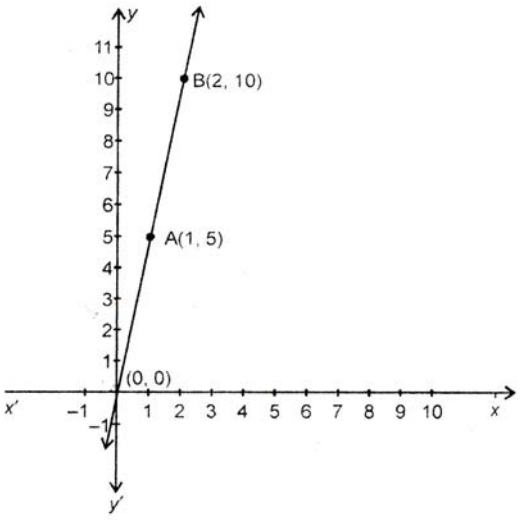
(i) 2 units (ii) 0 unit

Ans. [Hint : Assume the work done by constant force as y units and distance travelled by the body as x units to get $y = 5x$]

It is a straight line passing through the origin.

Now to plot the graph, we take some values of x and find the corresponding values of y.

	O	A	B	C
x	0	1	2	3
y	0	5	10	15



(i) When x = 2 units (distance)
$\therefore$ $y = 5x$
$\Rightarrow$ $y = 5 \times 2 = 10$
$\therefore$ Work done = 10.
(ii) When x = 0 units
$\therefore$ $y = 5x$
$\Rightarrow$ $y = 5 \times 0 = 0$
Hence work done = =

Q.7. Yamini and Fatima, two students of Class IX of a school, together contributed Rs. 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions

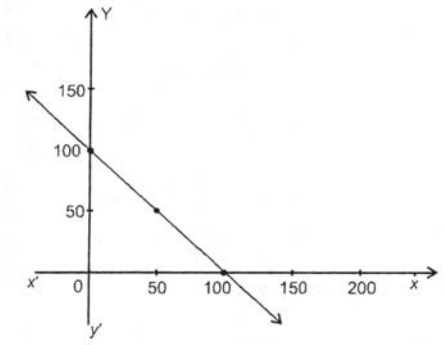
as Rs. *x* and Rs. *y*.) Draw the graph of the same.

Ans. Let the contribution of Yamini = Rs. *x*
and contribution of Fatima = Rs. *y*
Given both contribution = Rs. 100
According to question

$$x + y = 100 \quad \text{(Linear equation)}$$
$$y = 100 - x$$

We suppose certain values of *x* and find the corresponding values of *y*.

	A	B	C
x	0	50	100
y	100	50	0



We plot the point A(0, 100), B (50, 50) and C(100, 0) on the graph and get the corresponding graph of the above obtained linear equation $x + y = 100$.

Q.8. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$\mathbf{F = \left(\frac{9}{5}\right)C + 32}$$

(i) Draw the graph of the linear equation above using Celsius for *x*-axis and Fahrenheit for *y*-axis.

(ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

(iii) If the temperature is 95°F, what is the temperature in Celsius?

(iv) If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

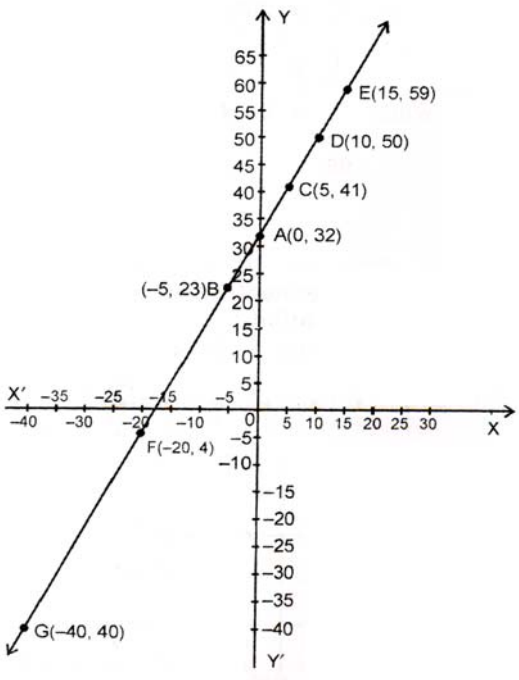
Ans. Given linear equation :

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) We have to take Celsius along *x*-axis a Fahrenheit along *y*-axis.

Let $C = x$ and $F = y$

$$\therefore \quad y = \frac{9}{5}x + 32$$



Now we consider certain values of *x* and find the corresponding values of *y*.

When $x = 0, y = \frac{9}{5} \times 0 + 32 = 32$

When $x = -5, y = \frac{9}{5}(-5) + 32 = 23$

When $x = 5, y = \frac{9}{5}(5) + 32 = 41$

When $x = 10, y = \frac{9}{5}(10) + 32 = 50$

When $x = 15, y = \frac{9}{5}(15) + 32 = 59$

When $x = -20, y = \frac{9}{5}(-20) + 32 = -4$

When $x = -40, y = \left(\frac{9}{5}\right)(-40) + 32 = -40$

Now we plot the points A(0, 32), B(–5, 23), C(5, 41), D(10, 50), E (!5, 59), F(–20, –4) and G(–40, –40) on the graph paper.

(ii) When $C = 30°$

$$F = \frac{9}{5} \times 30 + 32 = 86°$$

(iii) When $F = 95°$ F,

$$95 = \frac{9}{5} \times C + 32$$

$$\Rightarrow \quad 95 - 32 = \frac{9}{5} C$$

$$\Rightarrow \quad \frac{9}{5} C = 63$$

$$\Rightarrow \quad C = \frac{63 \times 5}{9} = 35°$$

(iv) When $C = 0$,

$$F = \frac{9}{5} \times 0 + 32 = 32°$$

When $F = 0°$

$$0 = \frac{9}{5} C + 32$$

$$\Rightarrow \quad C = -32 \times \frac{5}{9}$$

$$= \frac{-160}{9} = -17.8°C$$

(v) When $x°$ F $= x°$F

$$\therefore \quad F = \left(\frac{9}{5}\right) C + 32$$

$$x = \frac{9}{5} x + 32$$

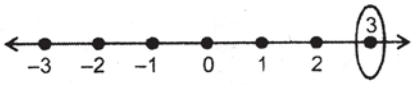
$$-32 = \frac{9}{5} x - x = \frac{9x - 5x}{5} = \frac{4x}{5}$$

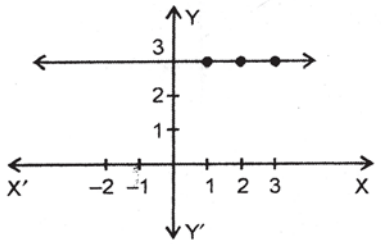
$$\Rightarrow \quad x = \frac{-32 \times 5}{4} = -40°$$

Hence $-40°F = -40°C$

EXERCISE 4.4

Q.1. Give the geometric representations of $y = 3$ as an equation
(i) in one variable
(ii) in two variables





(Fig. (b)

Ans. $y = 3$

(i) As an equation in one variable, it is the number 3 on the number line.

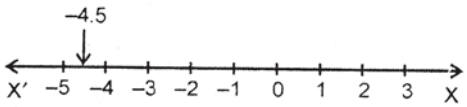
(ii) As an equation in two variables, we can write it as $0x + y = 3$. Value of x can be any number but of y will be 3 only. It is a line parallel to x-axis and 3 units above x-axis.

Q.2. Give the geometric representations of $2x + 9 = 0$ as an equation
(i) in one variable
(ii) in two variables

Ans. $2x + 9 = 0$

(i) As an equation in one variable $2x = -9$

$$\Rightarrow \quad x = \frac{-9}{2} \text{ or } -4.5$$



(ii) We know that

$$2x + 9 = 0$$

$$2x + 0y + 9 = 0$$

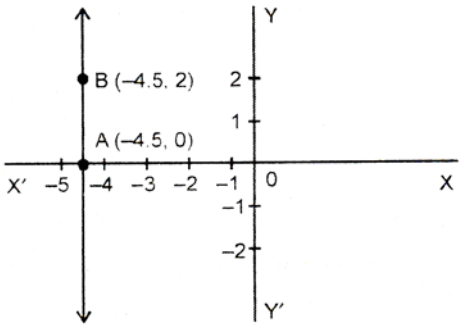
We know that it is actually

$$2x + 9 = 0$$

$$\Rightarrow \quad x = -\frac{9}{5}$$

$$\Rightarrow \quad x = -4.5$$

It is line parallel to *y*-axis at a negative distance we have the two points lying it, the points as A(–4.5, 0), B(–4.5, 2).



Additional Questions

Q.1. Determine the point on the graph of the linear equation $2x + 5y = 19$, whose ordinate is $1\frac{1}{2}$ times its abscissa.

Ans. Given equation is :

$$2x + 5y = 19 \qquad \text{.... (i)}$$

Ordinate (*y*-co-ordinate) = $1\frac{1}{2}$ times it abscissa

i.e. $y = \frac{3}{2}x$

Put the value of *'y'* in equation (i)

$$2x + 5 \times \frac{3}{2}x = 19$$

$$2x + \frac{15x}{2} = 19$$

$$\Rightarrow \quad \frac{19x}{2} = 19$$

$$\Rightarrow \quad x = \frac{19 \times 2}{19} = 2$$

From equation (ii)

$$y = \frac{3}{2} \times 2 = 3$$

$\therefore$ Required point is (2, 3).

Q.2. If the point (3, 4) lies on the graph of $3y = ax + 7$, then find the value of *a*.

Ans. We have :

$$3y = ax + 7 \qquad \text{...(i)}$$

Since the point (3, 4) lies on the graph of equation (i) (given)

$$\therefore \quad 3 \times 4 = a \times 3 + 7 = 0$$

[Point 3, 4) must satisfy (i)]

$$\Rightarrow \quad 12 = 3a + 7$$

$$\Rightarrow \quad 3a = 5$$

$$\Rightarrow \quad a = \frac{5}{3}$$

Q.3. How many solution(s) of the equation $2x + 1 = x - 3$ are there on the :

(i) Number line

(ii) Cartesian plane ?

Ans. (i) We have :

$$2x + 1 = x - 3$$

$$\Rightarrow \quad 2x - x = -3 - 1$$

$$\Rightarrow \quad x = -4$$

This is in the form of linear equation in one variable.

So only one solution is possible on the number line.

(ii) We can write

$$x = -4$$

as a linear equation in two variable

i.e. $1.x + 0.y = -4$

Since any value of 'y' is permissible.

So, Infinitely many solutions are possible.

Q.5. Find the solution of the linear equation $x + 2y = 8$ which represents a point on :

(i) *x*-axis (ii) *y*-axis

Ans. (i) on *x*-axis

$$y = 0$$

$$\Rightarrow \quad x + 2 \times 0 = 8$$

$$\Rightarrow \quad x = 8$$

Therefore, the required point is (8, 0)

(ii) On *y*-axis

$$x = 0$$